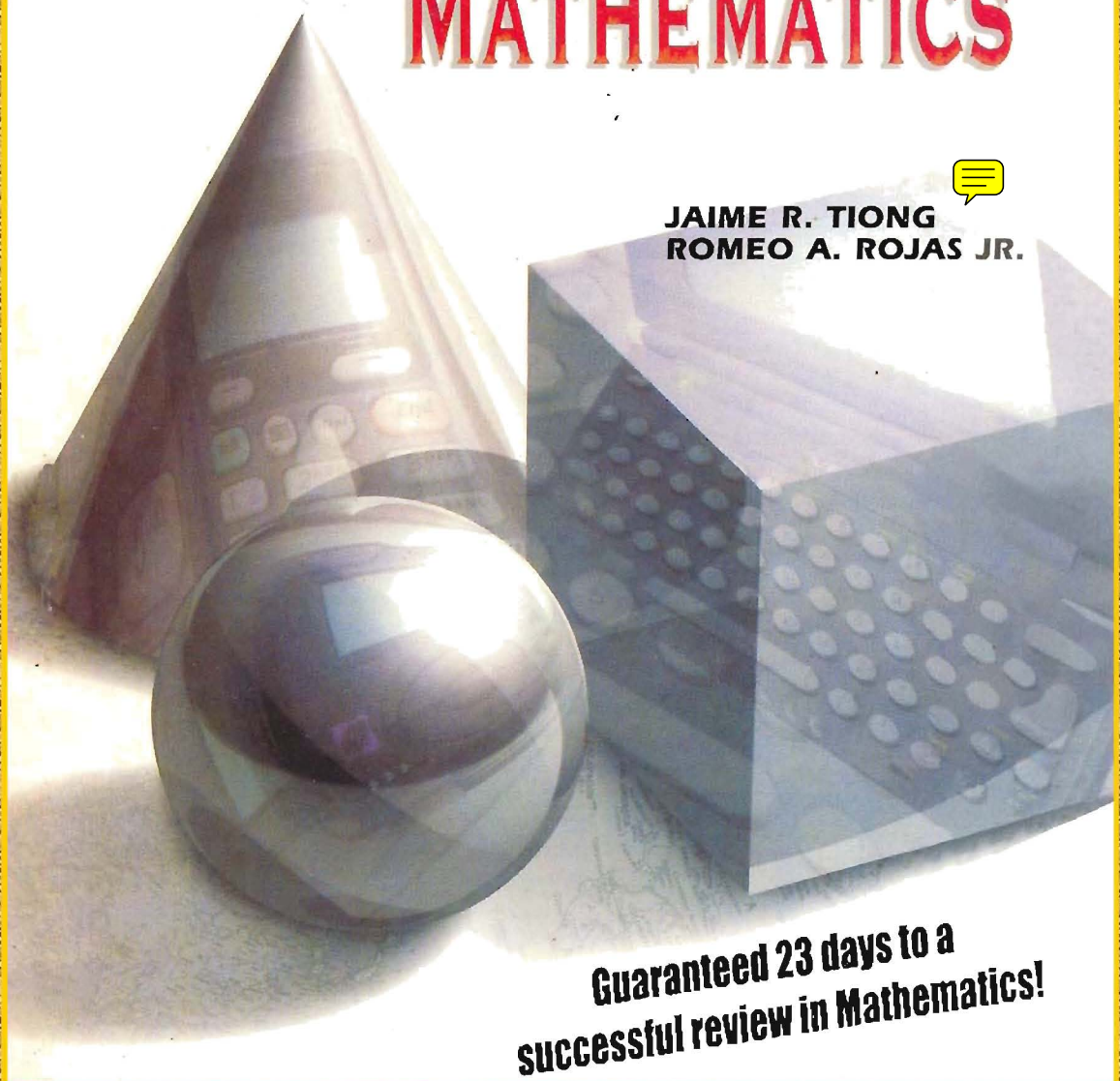
1001 Solved Problems in Engineering Mathematics

JAIME R. TIONG
ROMEO A. ROJAS JR.



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DAY 1

SYSTEMS OF NUMBERS & CONVERSION

SYSTEMS OF NUMBERS

Number is an item that describes a magnitude or a position. Numbers are classified into two types, namely cardinal and ordinal numbers. **Cardinal numbers** are numbers which allow us to count the objects or ideas in a given collection. Example, 1,2,3... while **ordinal numbers** states the position of individual objects in a sequence. Example, First, second, third...

Numerals are symbols, or combination of symbols which describe a number. The most widely used numerals are the **Arabic numerals** and the **Roman numerals.** Arabic numerals were simply a modification of the Hindu-Arabic number signs and are written in Arabic digits. Taken singly, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and in combination 20, 21, 22, ... 1999, ... The Roman numerals are numbers which are written in Latin alphabet. Example MCMXCIV.

The following are Roman numerals and their equivalent Arabic numbers:

I	= 1	C	= 100
V	= 5	D	= 500
X	= 10	M	= 1000
L	= 50		

To increase the number, the following are used:

1. Bracket – to increase by 100 times.
   $|X| = 1000$
2. Bar above the number – to increase by 1000 times.
   $\overline{X} = 10000$
3. A "doorframe" above the number – to increase by 100000 times.
   $\boxed{X} = 1\,000\,000$

Digit is a specific symbol or symbols used alone or in combination to denote a number. For example, the number 21 has two digits, namely 2 and 1. In Roman numerals, the number 9 is denoted as IX. So the digits I and X were used together to denote one number and that is the number 9.

In mathematical computations or engineering applications, a system of numbers using cardinal numbers was established and widely used.

The number system is divided into two categories: **Real numbers and Imaginary number.**

❶ **REAL NUMBERS:**

A. **Natural numbers** - are numbers considered as "counting numbers".
Examples: 1, 2, 3, ...

B. **Integers** – are all the natural numbers, the negative of the natural numbers and the number zero.

C. **Rational numbers** – are numbers which can be expressed as a quotient (ratio) of two integers. The term "rational" comes from the word "ratio".

Example: 0.5, $\frac{2}{3}$, -3, 0.333...

In the example, 0.5 can be expressed as $\frac{1}{2}$ and –3 can be expressed as $\frac{-6}{2}$, hence rational numbers. The number 0.333... is a repeating and non-terminating decimal. As a rule, a non-terminating but repeating (or periodic) decimals is always a rational number. Also, all integers are rational numbers.

D. **Irrational numbers** – are numbers which cannot be expressed as a quotient of two integers.

Example: $\sqrt{2}, \pi, e, ...$

The numbers in the examples above can never be expressed exactly as a quotient of two integers. They are in fact, a non-terminating number with non-terminating decimal.

❷ **IMAGINARY NUMBER:**

An **imaginary number** is denoted as i. In some other areas in mathematical computation, especially in electronics and electrical engineering it is denoted as j.

Imaginary number and its equivalent:

$i = \sqrt{-1}$
$i^2 = -1$
$i^3 = -i = -\sqrt{-1}$
$i^4 = 1$

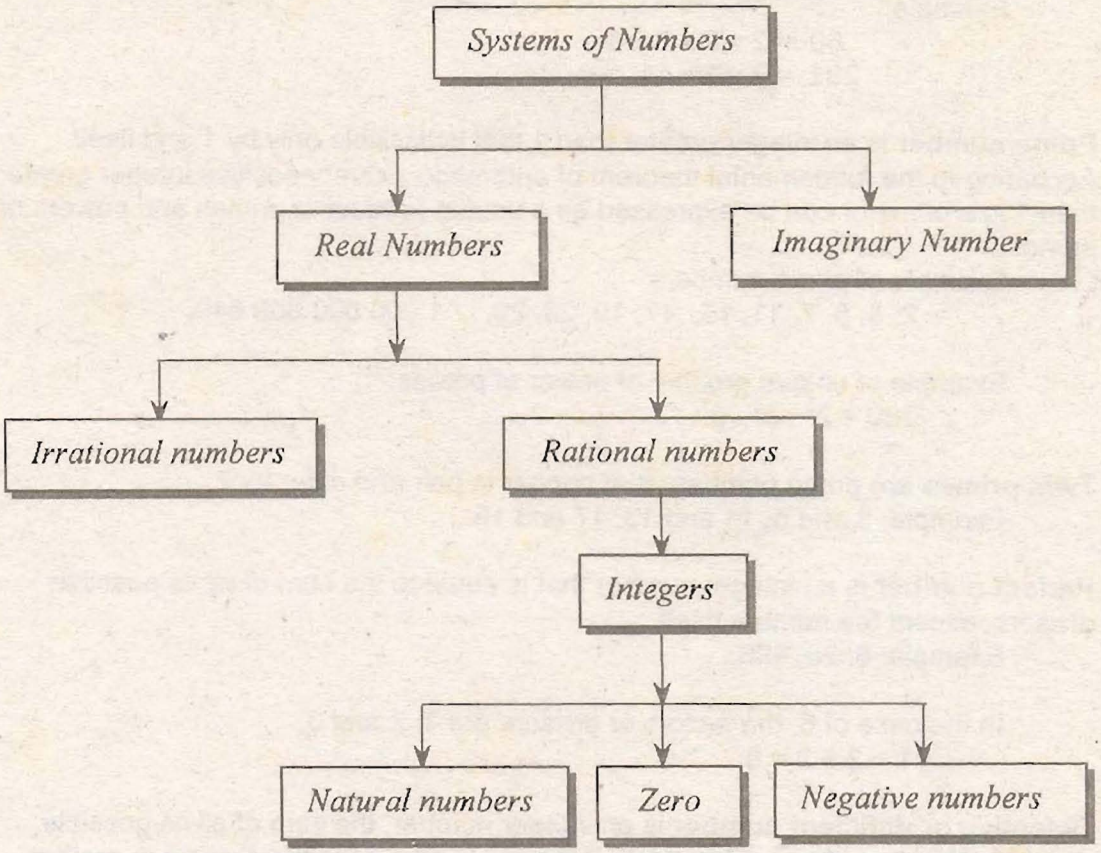


Diagram showing the Systems of Numbers

Complex number is an expression of both real and imaginary number combined. It takes the form of a + bi, where a and b are real numbers. If a = 0, then **pure imaginary number** is produced while **real number** is obtained when b = 0.

Absolute value of a real number is the numerical value of the number neglecting the sign. For example, the absolute value of –5 is 5 while of –x is x. The absolute value $|a|$ is either positive or zero but can never be negative.

Common fractions are numbers which are in the form of $\frac{a}{b}$ or a/b, where a is the numerator which may be any integer while b is the denominator which may be any integer greater than zero. If the numerator is smaller than the denominator, it is called as **proper fraction** while **improper fraction** is when the numerator is greater than the denominator.

Unit fractions are common fractions with unity for numerator and positive integer for the denominator.

Example: $\frac{1}{5}, \frac{1}{25}$

Composite number is a number that can be written as product of two or more integers, each greater than 1. It is observed that most integers are composite numbers.

Example:

$$60 = 2 \times 2 \times 3 \times 5$$
$$231 = 3 \times 7 \times 11$$

Prime number is an integer greater than 1 that is divisible only by 1 and itself. According to the fundamental theorem of arithmetic, " Every positive integer greater than 1 is a prime or can be expressed as a unique product of primes and powers of primes".

Example of prime numbers:

2, 3, 5, 7, 11, 13, ,17, 19, 23, 29, ... 1 000 000 009 649, ...

Example of unique product of power of primes:

$360 = 2^3 . 3^2 . 5^1$

Twin primes are prime numbers that appear in pair and differ by 2.

Example: 3 and 5, 11 and 13, 17 and 19...

Perfect number is an integer number that is equal to the sum of all its possible divisors, except the number itself.

Example: 6, 28, 496...

In the case of 6, the factors or divisors are 1, 2 and 3.

$1 + 2 + 3 = 6$

Defective or deficient number is an integer number, the sum of all its possible divisor is less than the number itself. If the sum of the possible divisors is greater than the number, it is referred to as **abundant number.** There are around 30 numbers known today as perfect number and all of which are even numbers.

Amicable numbers or **friendly numbers** refers to two integer numbers where each is the sum of all the possible divisors of the other. The smallest known amicable numbers are 220 and 284.

The number 220 has the following factors/divisors: 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, & 110 which when added sums up to 284, while the number 284 has the following divisors 1, 2, 4, 71, and 142 which adds up to 220.

Factorial denoted as n!, represents the product of all positive integers from 1 to n, inclusive.

Example: $n! = n(n-1)...3, 2, 1$

If n = 0, by definition:

$$(n!)(n + 1) = (n + 1)!$$ ← *This is known as recursion formula*

$$(0!)(0 + 1) = (0 + 1)!$$
$$0!\,(1) = 1!$$
$$0! = 1$$

The factorial symbol (!) was introduced by **Christian Kramp** in 1808.

Significant figures or digits are digits that define the numerical value of a number. A digit is considered significant unless it is used to place a decimal point.

The significant digit of a number begins with the first non-zero digit and ends with the final digit, whether zero or non-zero.

Examples:

❶ 16.72	4 significant figures
❷ 1.672×10^3	4 significant figures
❸ 0.0016	2 significant figures

Example 2 is expressed in scientific notation and figures considered significant are 1, 6, 7 and 2 excluding 10^3. Example 3 has 2 significant figures only because the 3 zeros are used only to place a decimal.

The number of significant digits is considered the place of accuracy. Hence, a number with 3 significant digits is said to have a **three place accuracy** and a number with 4 significant figures is said to have a **four place accuracy**.

Rounding and Truncating:

The two forms of approximations are known as **rounding** and **truncation.**

Rounding of a number means replacing the number with another number having fewer significant decimal digits, or for integer number, fewer value-carrying (non-zero) digits.

Example: ❶ 3.14159 shall be rounded up to 3.1416
❷ 3.12354 shall be rounded down to 3.1235

Truncation refers to the dropping of the next digits in order to obtain the degree of accuracy beyond the need of practical calculations. This is just the same as **rounding down** and truncated values will always have values lower than the exact values.

Example: 3.14159 is truncated to 4 decimal as 3.1415



Tips:

1. **Revolution and its equivalent in units of angle.**
   1 revolution = 360 degrees
   = 2π radians
   = 400 grads
   = 6400 mils = 6400 centissimal degree
   = 6400 gons

2. **Temperature.**
   Relation between °Celsius and °Fahrenheit

$$°C = \frac{5}{9}(°F - 32)$$

$$°F = \frac{9}{5}°C + 32$$

   Absolute temperature:
   °K = °C + 273 °R = °F + 460

   Kelvin was named after British physicist, **William Thompson** (1824-1902) the First Baron, Kelvin.

   Rankine was named after Scottish engineer and physicist, **William John Macquom Rankine** (1820-1872).

   Fahrenheit was named after German physicist, **Gabriel Daniel Fahrenheit** (1686 – 1736).

   Celsius (or Centigrade) was named after Swedish astronomer, **Anders Celsius** (1701 – 1744).

3. Density of water = 1000 kg / m^3
   = 62.4 lb/cu. ft.
   = 9810 N/m^3
   = 1 gram / cc

4. Read **"Appendix B – Conversion"** and **Appendix D – "Prefixes"** at the last part of this book and be familiar with the values.

Did you know that... the symbol π (pi), which is the ratio of the circumference of a circle to its diameter was introduced by William Jones in 1706 after the initial letter of the Greek word meaning "periphery".

Proceed to the next page for your first test. GOODLUCK! ☞

Test 1

Time element: 3.0 hours

Problem 1: ME Board October 1996
How many significant digits do 10.097 have?

A. 2
B. 3
C. 4
D. 5

Problem 2: ECE Board April 1991
Round off 0.003086 to three significant figures.

A. 0.003
B. 0.00309
C. 0.0031
D. 0.00308

Problem 3: ECE Board April 1991
Round off 34.2814 to four significant figures.

A. 34.2814
B. 34.281
C. 34.28
D. 34.0

Problem 4: ME Board April 1996
Which number has three significant figures?

A. 0.0014
B. 1.4141
C. 0.01414
D. 0.0141

Problem 5: ECE Board April 1991
Round off 149.691 to the nearest integer

A. 149.69
B. 149.7
C. 150
D. 149

Problem 6: ECE Board April 1991

Round off 2.371×10^{-8} to two significant figures.

A. 2.4×10^{-8}
B. 2.37×10^{-8}
C. 0.2371×10^{-9}
D. 0.002371×10^{-11}

Problem 7: EE Board October 1994

$7 + 0i$ is

A. irrational number
B. real number
C. imaginary number
D. a variable

Problem 8: ECE Board March 1996

The number 0.123123123123..... is

A. irrational
B. surd
C. rational
D. transcendental

Problem 9: ECE Board April 1991

Round off 6785768.342 to the nearest one-tenth.

A. 6785768
B. 6785768.4
C. 6785768.3
D. None of these

Problem 10: EE Board April 1993

Express decimally: Fourteen Ten thousandths

A. 0.0014
B. 0.00014
C. 0.014
D. 0.14

Problem 11: ECE Board March 1996

MCMXCIV is equivalent to what number?

A. 1964
B. 1994
C. 1984
D. 1974

Problem 12: EE Board April 1993
Express decimally: Fourty-Sevenmillionth

A. 0.00000047
B. 0.0000047
C. 0.000047
D. 0.000000047

Problem 13: EE Board April 1993
Express decimally: Seven hundred twenty-five hundred thousandths

A. 0.000725
B. 0.00725
C. 0.0725
D. 0.725

Problem 14: EE Board April 1993
Express decimally: Four and two tenth

A. 0.042
B. 4.02
C. 4.2
D. 0.42

Problem 15: ECE Board November 1995
Express 45° in mils

A. 80 mils
B. 800 mils
C. 8000 mils
D. 80000 mils

Problem 16: ME Board April 1997
What is the value in degrees of 1 radian?

A. 90
B. 57.3
C. 100
D. 45

Problem 17: CE Board May 1993
3200 mils is equal to how many degrees?

A. 45°
B. 90°
C. 180°
D. 270°

Problem 18: ECE Board November 1995

An angular unit equivalent to 1/400 of the circumference of a circle is called

A. mil
B. degree
C. radian
D. grad

Problem 19: ECE Board April 1999

4800 mils is equivalent to ____ degrees.

A. 135
B. 270
C. 235
D. 142

Problem 20: ME Board April 1996

How many degrees Celsius is 100 degrees Fahrenheit?

A. 2.667° C
B. 1.334° C
C. 13.34° C
D. 37.8° C

Problem 21: EE Board October 1990

What is the absolute temperature of the freezing point of water in degree Rankine?

A. 492
B. 0
C. 460
D. 273

Problem 22: ME Board October 1994

What is the Fahrenheit equivalent of 100 degrees Celsius?

A. 200
B. 180
C. 212
D. 100

Problem 23: EE Board April 1993

The temperature 45° C is equal to

A. 45° F
B. 113° F
C. 57° F
D. 81° F

Problem 24: ME Board October 1994

How many degrees Celsius is 80 degrees Fahrenheit?

A. 13.34
B. 1.334
C. 26.67
D. 2.667

Problem 25: ME Board October 1996

10 to the 12th power is the value of the prefix

A. micro
B. femto
C. teta
D. atto

Problem 26: EE Board October 1994

The micro or μ means

A. 10^{-2}
B. 10^{-6}
C. 10^{-3}
D. 10^{-12}

Problem 27: RME Board October 1994

The prefix pico means

A. 10^{-12} of a unit
B. 10^{-6} of a unit
C. 10^{-15} of a unit
D. 10^{-9} of a unit

Problem 28: ME Board April 1999

The prefix nano is opposite to

A. mega
B. giga
C. tera
D. hexa

Problem 29:

1 foot is to 12 inches as 1 yard is to ____ spans.

A. 4
B. 6
C. 9
D. 24

Problem 30: EE Board June 1990

A one-inch diameter conduit is equivalent to

A. 254 mm
B. 25.4 mm
C. 100 mm
D. 2.54 mm

Problem 31:

If a foot has 12 inches, then how many hands are there in one foot?

A. 3
B. 4
C. 6
D. 8

Problem 32:

How many feet difference is 1 nautical mile and 1 statute mile?

A. 100 feet
B. 200 feet
C. 400 feet
D. 800 feet

Problem 33:

In a hydrographic survey, a certain point below the surface of the water measures 12 fathoms. It is equivalent to a deep of how many feet?

A. 72
B. 60
C. 48
D. 36

Problem 34:

The legendary ship, Titanic that sunk in 1912 was estimated to be at the sea bottom at a deep of 18 cables. How deep it is in feet?

A. 12,000
B. 12,343
C. 12,633
D. 12,960

Problem 35: ME Board October 1994

How many square feet is 100 square meters?

A. 328.1
B. 929
C. 32.81
D. 1076

Problem 36:

A certain luxury ship cruises Cebu to Manila at 21 knots. If it will take 21 hours to reach Manila from Cebu, the distance traveled by the ship is nearly

A. 847.5 km
B. 507.15 statute mile
C. 441 statute mile
D. 414 nautical mile

Problem 37: EE Board October 1994

Carry out the following multiplication and express your answer in cubic meter: 8 cm x 5 mm x 2m.

A. 8×10^{-2}
B. 8×10^{2}
C. 8×10^{-3}
D. 8×10^{-4}

Problem 38:

Which of the following is equivalent to 1 hectare?

A. 100 ares
B. 2 acres
C. 1000 square meters
D. 50000 square feet

Problem 39:

Ten square statute miles is equivalent to ____ sections.

A. 100
B. 5
C. 10
D. 20

Problem 40:

The land area of the province of Cebu is 5088.39 sq. km. This is equivalent to

A. 5088.39 hectares
B. 1964.64 sq. miles
C. 2257907.2 acres
D. 5.08839 acres

Problem 41: ME Board October 1994

How many cubic feet is 100 gallons of liquid?

A. 74.80
B. 1.337
C. 13.37
D. 133.7

Problem 42: ME Board October 1994, ME Board April 1998
How many cubic meters is 100 gallons of liquid?

A. 1.638
B. 3.785
C. 0.164
D. 0.378

Problem 43: ME Board October 1994
How many cubic meters is 100 cubic feet of liquid?

A. 3.785
B. 28.31
C. 37.85
D. 2.831

Problem 44:
Ten (10) cubic meter is equivalent to how many stere?

A. 5
B. 10
C. 20
D. 100

Problem 45: ME Board April 1995
The standard acceleration due to gravity is

A. 32.2 ft/s^2
B. 980 ft/s^2
C. 58.3 ft/s^2
D. 35.3 ft/s^2

Problem 46: ME Board October 1996
A 7kg mass is suspended in a rope. What is the tension in the rope in SI?

A. 68.67 N
B. 70 N
C. 71 N
D. 72 N

Problem 47:
A 10-liter pail is full of water. Neglecting the weight of the pail, how heavy is its water content?

A. 5 kg
B. 6.67 kg
C. 10 kg
D. 12.5 kg

Problem 48:

The unit of work in the mks system is known as joule (J) and the unit of work in the cgs system is erg. How many ergs are there in one joule?

A. 10^6
B. 10^7
C. 10^5
D. 10^4

Problem 49: ME Board April 1998

One horsepower is equivalent to

A. 746 watts
B. 7460 watts
C. 74.6 watts
D. 7.46 watts

Problem 50: ME Board October 1994

How many horsepower is 746 kilowatts?

A. 500
B. 74.6
C. 100
D. 1000

ANSWER KEY

1. D	14. C	27. A	40. B
2. B	15. B	28. B	41. C
3. C	16. B	29. A	42. D
4. D	17. C	30. B	43. D
5. C	18. D	31. A	44. B
6. A	19. B	32. D	45. A
7. B	20. D	33. A	46. A
8. C	21. A	34. D	47. C
9. C	22. C	35. D	48. B
10. A	23. B	36. B	49. A
11. B	24. C	37. D	50. D
12. C	25. C	38. A	
13. B	26. B	39. C	

RATING

- ☐ 43-50 Topnotcher
- ☐ 30-42 Passer
- ☐ 25-29 Conditional
- ☐ 0-24 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 1

1. The number 10.097 has 5 significant figures.

2. The number 0.003086 when rounded off to three significant digit becomes 0.00309.

3. The number 34.2814 when rounded off to four significant digit becomes 34.28.

4. 0.0014 has two significant figures
1.4141 has five significant figures
0.01414 has four significant figures
0.0141 has three significant figures ☞ *Answer*

5. The number 149.691 when rounded off to the nearest integer becomes 150.

6. The number 2.371×10^{-8} when rounded off to two significant digit becomes 2.4×10^{-8}.

7. $7 + 0i = 7$ thus, the answer is, " real number".

8. Repeating decimal number is a "rational number".

9. The number 6785768.342 when rounded off to the nearest one-tenth becomes 6785768.3.

10. Fourteen Ten thousandths $= \dfrac{14}{10000} = 0.0014$

11. MCMXCIV = M CM XC IV
$1000 + 900 + 90 + 4$
$= 1994$

12. Fourty-seven millionth $= \dfrac{47}{1000000} = 0.000047$

13. Seven hundred twenty-five hundred thousandths $= \dfrac{725}{100000} = 0.00725$

14. Four and two tenth = 4.2

15. By ratio and proportion:

$$\frac{x}{6400 \text{ mils}} = \frac{45°}{360°}$$

$$x = 800 \text{ mils}$$

16. By ratio and proportion:

$$\frac{x}{360°} = \frac{1\text{rad}}{2\pi\,\text{rad}}$$

$$x = 57.3°$$

17. By ratio and proportion:

$$\frac{x}{360°} = \frac{3200\,\text{mils}}{6400\,\text{mils}}$$

$$x = 180°$$

18. Grad

19. By ratio and proportion:

$$\frac{x}{360°} = \frac{4800\,\text{mils}}{6400\,\text{mils}}$$

$$x = 270°$$

20. Using the formula, $°C = \frac{5}{9}(F - 32)$

$$°C = \frac{5}{9}(100 - 32) = 37.8°C$$

21. The freezing point of water is equal to 32°F or 0°C.

$°R = °F + 460$

$°R = 32 + 460 = 492°R$

22. Using the formula,

$$°F = \frac{9}{5}°C + 32$$

$$°F = \frac{9}{5}(100) + 32 = 212°F$$

23. Using the formula,

$$°F = \frac{9}{5}°C + 32$$

$$°F = \frac{9}{5}(45) + 32 = 113°F$$

24. Using the formula,

$$°C = \frac{5}{9}(F - 32)$$

$$°C = \frac{5}{9}(80 - 32) = 26.67°C$$

25. The prefix tera is equivalent to 10^{12} of a unit.

26. 10^{-6} means micro

27. The prefix pico is equivalent to 10^{-12} of a unit

28. The prefix nano is equivalent to 10^{-9} of a unit
while the prefix giga is equivalent to 10^{9} of a unit

29. 1 span is equivalent to 9 inches
1 yard = 3 ft = 36 inches, thus

$$36 \text{ inches} \times \frac{1 \text{ span}}{9 \text{ inches}} = 4 \text{ spans}$$

30.
$$1 \text{ inch} = 2.54 \text{ centimeters} \times \frac{10 \text{ millimeters}}{1 \text{ centimeter}}$$
$$= 25.4 \text{ mm}$$

31. 1 hand is equivalent to 4 inches, thus

$$1 \text{ foot} = 12 \text{ inches} \times \frac{\text{hand}}{4 \text{ inches}}$$
$$1 \text{ foot} = 3 \text{ hands}$$

32. 1 statute mile = 5280 ft. and 1 nautical mile = 6080 ft
Let: x = the difference between a nautical mile and a statute mile
$x = 6080 - 5280 = 800$ feet

33. 1 fathom is equivalent to 6 feet, thus
12 fathoms = 6(12) = 72 feet

34. 1 cable is equivalent to 120 fathoms, thus
18 cables = 120(18) = 2160 fathoms

$$18 \text{ cables} = 2160 \text{ fathoms} \times \frac{6 \text{ feet}}{\text{fathom}} = 12{,}960 \text{ feet}$$

35. 1 meter is equivalent to 3.281 ft, thus
$1 \text{ m}^2 = (3.281)^2 \text{ ft}^2$
$1 \text{ m}^2 = 10.76 \text{ ft}^2$
$100 \text{ m}^2 = 100(10.76) \text{ ft}^2$
$100 \text{ m}^2 = 1076 \text{ ft}^2$

36. Solving for distance, D = Vt

$$V = 21 \text{ knots} = 21 \frac{\text{nautical miles}}{\text{hour}}$$

$$D = 21(21) = 441 \text{ nautical miles} \times \frac{1.15 \text{ statute mile}}{\text{nautical mile}} = 507.15 \text{ statute mile}$$

37. $8 \text{ cm} \times \frac{1 \text{m}}{100 \text{ cm}} = 0.8 \text{ m}$

$5 \text{ mm} \times \frac{1 \text{m}}{1000 \text{ mm}} = 0.005 \text{ m}$

$0.08(0.005)(2) = 8 \times 10^{-4} \text{ m}^3$

38. 1 hectare = 100 ares = 10,000 sq. meters

39. 1 square statute mile is equivalent to 1 section, thus
10 square statute mile = 10(1) = 10 sections

40. 1 square km is equivalent to 0.386102 sq. miles

$$A = 5088.39 \text{ km}^2 \times \frac{0.386102 \text{ mile}^2}{\text{km}^2} = 1964.64 \text{ sq. miles}$$

41. 1 cubic ft. = 7.48 gallons

$$100 \text{ gallons} \times \frac{\text{ft}^3}{7.48 \text{ gallons}} = 13.37 \text{ ft}^3$$

42. 1 gallon = 3.79 liters
1000 liters = 1 cubic meters

$$100 \text{ gallons} \times \frac{3.79 \text{ liters}}{\text{gallon}} \times \frac{\text{m}^3}{1000 \text{ liters}} = 0.379 \text{ m}^3$$

43. Given volume is 100 cu. ft.

$$V = 100 \text{ ft}^3 \times \left(\frac{\text{m}}{3.281 \text{ ft}}\right)^3 = 2.831 \text{ m}^3$$

44. 1 cubic meter = 1 stere, thus,
$10 \text{ m}^3 = 10$ steres

45. The following are the standard gravitational acceleration:
32.2 ft/s^2; 981 cm/s^2; 9.81 m/s^2

46. The unit of force (tension) in the SI system is newtons (N).

$$\text{Tension} = 7 \text{ kg} \left(\frac{9.81 \text{m}}{\text{s}^2}\right) = 68.67 \text{ N}$$

47. Density of water (ρ) is 1000 $\frac{kg}{m^3}$ or 1 $\frac{kg}{liter}$

$W = \rho \cdot V$

$W = 1 \frac{kg}{liter} \times 10 \text{ liters} = 10 \text{ kg}$

48. 1 joule = 10^7 ergs

49. 1 horsepower = 746 watts

50. 1 hp = 746 watts = 0.746 kilowatts

$746 \text{ kW} \times \frac{\text{hp}}{0.746 \text{ kW}} = 1000 \text{ hp}$

DAY 2

FUNDAMENTALS IN ALGEBRA

BASIC RULES OF ALGEBRA

Let a, b, and c be real numbers, variables or algebraic expressions.

❶	Closure property	: Addition	$a + b$
		: Multiplication	$a \cdot b$
❷	Commutative property	: Addition	$a + b = b + a$
		: Multiplication	$a \cdot b = b \cdot a$
❸	Associative property	: Addition	$(a + b) + c = a + (b + c)$
		: Multiplication	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
❹	Identity property	: Addition	$a + 0 = 0 + a = a$
		: Multiplication	$a \cdot 1 = 1 \cdot a = a$
❺	Inverse property	: Addition	$a + (-a) = 0 = (-a) + a$
		: Multiplication	$a\left(\frac{1}{a}\right) = 1 = \left(\frac{1}{a}\right)a$ with $a \neq 0$
❻	Distributive property	: Left	$a(b + c) = ab + ac$
		: Right	$(a + b)c = ac + bc$

In item 4, 0 is called an **additive identity** while 1 is called a **multiplicative identity**. In item 5, - a is known as **additive inverse** of a while 1/a is the **multiplicative inverse** of a.

PROPERTIES OF EQUALITY

Let a, b and c be real numbers, variables of algebraic expression

❶	Reflexive property	: $a = a$
❷	Symmetric property	: If $a = b$, then $b = a$
❸	Transitive property	: If $a = b$ and $b = c$, then $a = c$
❹	Substitution property	: If $a = b$, then a can be replaced by b in any expression involving a
❺	Addition/Subtraction	: If $a = b$, then $a + c = b + c$
		: If $a = b$, then $a - c = b - c$
❻	Multiplication/Division	: If $a = b$, then $ac = bc$
		: If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, with $c \neq 0$

❼ Cancellation property : If $a + c = b + c$, then $a = b$
: If $ac = bc$ and $c \neq 0$, then $a = b$

PROPERTIES OF ZERO

Let a and b real numbers, variables or algebraic expression.

❶ $a + 0 = a$ and $a - 0 = a$

❷ $a \quad 0 = 0$

❸ $\frac{0}{a} = 0, \quad a \neq 0$

❹ $\frac{a}{0}$ is undefined

❺ If $ab = 0$, then $a = 0$ or $b = 0$. This is known as **Zero-Factor Property**

PROPERTIES OF EXPONENTS

The exponential notation states that if a is a real number, variable, or algebraic expression and n is a positive number, then

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{\text{n factors}}$$

	Properties:	**Examples:**
❶	$a^m \cdot a^n = a^{m+n}$	$4^2 + 4^3 = 4^{2+3} = 4^5$
❷	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^8}{x^3} = x^{8-3} = x^5$
❸	$(a^m)^n = a^{mn}$	$\left(y^6\right)^2 = y^{6(2)} = y^{12}$
❹	$(ab)^m = a^m b^m$	$(2x)^4 = 2^4 x^4 = 16x^4$
❺	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{2}{x}\right)^4 = \frac{2^4}{x^4} = \frac{16}{x^4}$
❻	$a^{m/n} = \sqrt[n]{a^m}$	$4x^{5/3} = \sqrt[3]{(4x)^5}$
❼	$a^{-m} = \frac{1}{a^m}$	$x^{-5} = \frac{1}{y^5}$
❽	$a^0 = 1, \quad a \neq 0$	$(x^5+2)^0 = 1$

PROPERTIES OF RADICALS

In the expression, $\sqrt[n]{a}$, n is called the **index**, a the **radicand** while the symbol $\sqrt{\ }$ is called the **radical** or **radical symbol.**

	Properties:	**Examples:**	
❶	$\sqrt[n]{a^m} = (\sqrt[n]{a})^m$	$\sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$	
❷	$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$	$\sqrt[3]{5} \cdot \sqrt[3]{675} = \sqrt[3]{5(675)} = \sqrt[3]{3375} = 15$	
❸	$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}, \quad b \neq 0$	$\frac{\sqrt[3]{50}}{\sqrt[3]{10}} = \sqrt[3]{\frac{50}{10}} = \sqrt[3]{5}$	
❹	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$	$\sqrt{\sqrt[4]{15}} = \sqrt[8]{15}$	
❺	$\left(\sqrt[n]{a}\right)^n = a$	$\left(\sqrt[5]{2x}\right)^5 = 2x$	
❻	$\sqrt[n]{a^n} = \lvert a \rvert$	$\sqrt[4]{(-12)^4} = \lvert -12 \rvert = 12,$	For n = even number
	$\sqrt[n]{a^n} = a$	$\sqrt[3]{(-15)^3} = -15,$	For n = odd number

Surd is a radical expressing an irrational number. The surd is described after the index of the radical. For example, $\sqrt{3}$ is a **quadratic surd**, $\sqrt[3]{3}$ is a **cubic surd**, $\sqrt[4]{3}$ is a **quartic surd** and so on.

Pure surd, sometimes called as **entire surd** contains no rational number and all its terms are surds. For example, $\sqrt{3} + \sqrt{2}$. A **mixed surd** contains at least one rational number. $5\sqrt{3}$ is a mixed surd because 5 is rational number while $\sqrt{3}$ is a surd. A **binomial surd** is an expression of two terms with at least one term is a surd. For example, $5 + \sqrt{2}$. A **trinomial surd** is an expression of three terms with at least two of them are surds and cannot be expressed as a single surd, otherwise it will become a binomial surd. Example, $5 + \sqrt{2} + \sqrt{3}$.

SPECIAL PRODUCTS

Let x, y and z be real numbers, variables or algebraic expression.

❶ Sum and difference of same terms : $(x + y)(x - y) = x^2 - y^2$
Or Difference of two squares

❷ Square of a binomial : $(x + y)^2 = x^2 + 2xy + y^2$
: $(x - y)^2 = x^2 - 2xy + y^2$

❸ Cube of a binomial : $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
: $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

❹ Difference of two cubes : $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

❺ Sum of two cubes : $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

❻ Square of a trinomial : $(x + y + x)^2 = x^2 + y^2 + z^2 + 2xy + 2xz + 2yz$

PROPERTIES OF PROPORTION

a) If $\frac{a}{y} = \frac{x}{d}$, then a:x = y:d

b) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

c) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$

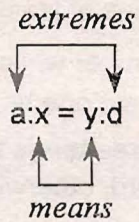
d) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$

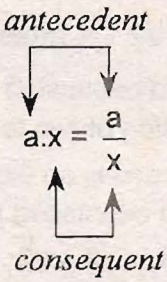
e) If $\frac{a}{b} = \frac{d}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$

f) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

Note in item (a), quantities a & d are called **extremes** and x & y are called **means**. If x = y, this is known as the **mean proportional**. In the ratio x/y, the first term x is called **antecedent** while the second term y is called the **consequent**.

If x = y, the means are known as ***mean proportional***





THE REMAINDER THEOREM & FACTOR THEOREM

If a polynomial in an unknown quantity x is divided by a first degree expression in the same variable, (x – k), where k may be any real or complex number, the **remainder** to be expected will be equal to the sum obtained when the numerical value of k is substituted for x in the polynomial. Thus,

$$\text{Remainder} = f(x)_{x \to k}$$

If the polynomial is divided by (x – k) will result to a remainder of zero, then the value (x – k) is a factor of the polynomial. This is known as the **Factor Theorem**.

Both remainder theorem and factor theorem were suggested by a French Mathematician, **Etienne Bezout** (1730 – 1783).



Tips: 1. **Least Common Denominator (LCD)** – refers to the product of several prime numbers occuring in the denominators, each taken with its greatest multiplicity. For example: LCD of 8, 9, 12 and 15 is 360

What is the LCD of 8, 9, 12 and 15?
Soln: $8 = 2^3$
$9 = 3^2$
$12 = 3 \times 2^2$
$15 = 3 \times 5$
LCD = $2^3 \times 3^2 \times 5 = 360$

2. **Least Common Multiple (LCM)** – refers to the smallest integer that is a multiple of each of the given numbers.

What is the least common multiple of 15 and 18?
Soln: $15 = 3 \times 5$
$18 = 3^2 \times 2$
LCM = $3^2 \times 2 \times 5 = 90$

By principle, the LCD may be regarded as LCM and vice versa.

3. **Greatest Common Factor (GCF)** – refers to the largest integer which is a factor of each of the given numbers.

What is the greatest common factor of 70 and 112?
Soln: $70 = 2 \times 5 \times 7$
$112 = 2^4 \times 7$
Taking the common factors of both 70 and 112 which are 2 and 7, then, GCF = $2 \times 7 = 14$
Note that 2 and 7 are the common to both 70 and 112.

Did you know that... The two long parallel lines (=) as a symbol for equality was introduced by Robert Recorde in 1557.

Proceed to the next page for your second test. GOODLUCK ! ☞

Test 2

Time element: 3 hours & 45 minutes

Problem 51: ECE Board April 1999

If 16 is 4 more than 4x, find 5x – 1.

A. 14
B. 3
C. 12
D. 5

Problem 52: EE Board October 1992

Find the value of x in $\frac{x+1}{3}+\frac{2x}{4}=47-2x$.

A. 16.47
B. 12.87
C. 18.27
D. 20.17

Problem 53: EE Board October 1991

Find the value of x in the equations:

$$10\left[\frac{A}{x}+\frac{A}{y}\right]=A$$

$$2\left[\frac{3A}{x}-\frac{4A}{y}\right]=A$$

A. 50/9
B. 80/9
C. 70/9
D. 60/9

Problem 54: EE Board October 1997

Find the values of x and y from the equations:

$$x - 4y + 2 = 0$$
$$2x + y - 4 = 0$$

A. 11/7, -5/7
B. 14/9, 8/9
C. 4/9, 8/9

D. 3/2, 5/3

Problem 55: ME Board October 1995

Solve for the value of x and y.

$4x + 2y = 5 \qquad 13x - 3y = 2$

A. y = 1/2, x = 3/2
B. y = 3/2, x = 1/2
C. y = 2, x = 1
D. y = 3, x = 1

Problem 56: ME Board October 1996

Solve the simultaneous equations:

$$2x^2 - 3y^2 = 6$$

$$3x^2 + 2y^2 = 35$$

A. x = 3 or –3; y = 2 or –2
B. x = 3 or –3, y = -2 or 1
C. x = 3 or –3, y = -2 or –1
D. x = 3 or –3, y = 2 or –3

Problem 57: CE Board May 1997

Find the value of w in the following equations:

$3x - 2y + w = 11$
$x + 5y - 2w = -9$
$2x + y - 3w = -6$

A. 3
B. 2
C. 4
D. –2

Problem 58: EE Board October 1993

Solve for the value of x.

$2x - y + z = 6$
$x - 3y - 2z = 13$
$2x - 3y - 3z = 16$

A. 4
B. 3
C. 2
D. 1

Problem 59: ME Board October 1996

Solve the simultaneous equations:

$x + y = -4$
$x + z - 1 = 0$
$y + z + 1 = 0$

A. x = -1, y = -5, z = 3
B. x =1, y = 2, z = -3
C. x = -1, y = -3, z = 2
D. x = -2, y = -3, z = -1

Problem 60: EE Board April 1997

Multiply the following: (2x + 5y)(5x – 2y)

A. $10x^2 - 21xy + 10y^2$
B. $-10x^2 + 21xy + 10y^2$
C. $10x^2 + 21xy - 10y^2$
D. $-10x^2 - 21xy - 10y^2$

Problem 61: EE Board March 1998

Determine the sum of the positive valued solution to the simultaneous equations: xy = 15, yz = 35, zx = 21.

A. 15
B. 13
C. 17
D. 19

Problem 62: ECE Board April 1991

Simplify: $\dfrac{\left(x^2y^3z^{-2}\right)^{-3}\left(x^{-3}yz^3\right)^{\frac{1}{2}}}{\left(xyz^{-3}\right)^{\frac{5}{2}}}$

A. $\dfrac{1}{x^2y^7z^5}$
B. $\dfrac{1}{x^2y^7z^3}$
C. $\dfrac{1}{x^2y^5z^7}$
D. $\dfrac{1}{x^5y^7z^2}$

Problem 63: ECE Board November 1993

Simplify the following equation

$$\frac{5x}{2x^2+7x+3} - \frac{x+3}{2x^2-3x-2} + \frac{2x+1}{x^2+x-6}$$

A. $\dfrac{4}{x+3}$
B. $\dfrac{2}{x-3}$

C. $\dfrac{4}{x-3}$

D. $\dfrac{2}{x+3}$

Problem 64: ECE Board April 1991

Simplify: $\left\{x^{\frac{2}{3}}\left[x^{-\frac{1}{3}}y^{-\frac{1}{2}}\left(x^2y^{-2}\right)^{-\frac{2}{3}}\right]^{\frac{1}{2}}\right\}^6$

A. $\dfrac{y^{\frac{5}{2}}}{x}$

B. $\dfrac{y^{\frac{3}{2}}}{x}$

C. $\dfrac{y^{\frac{5}{2}}}{x^2}$

D. $\dfrac{y^{\frac{3}{2}}}{x^2}$

Problem 65: ECE Board April 1991

Simplify: $7^{a+2} - 8(7)^{a+1} + 5(7)^a + 49(7)^{a-2}$

A. –5a
B. –3a
C. –7a
D. –4a

Problem 66:

Solve for x: $x = \dfrac{\left(b^2-4b+16\right)\left(b^2-16\right)}{b^3+64}$

A. b + 4

B. $\dfrac{b-4}{b+2}$

C. $\dfrac{b^2-4}{b+2}$

D. b – 4

Problem 67: ECE Board April 1993

Solve for y: $\frac{x}{(b-c)} = \frac{y}{(a-c)} = \frac{z}{(a-b)}$

A. x - z
B. x + z
C. a + b
D. a - b

Problem 68: ME Board October 1996

Resolve $\frac{x+2}{x^2-7x+12}$ into partial fraction.

A. $\frac{6}{x-4} - \frac{2}{x-3}$
B. $\frac{3}{x-4} - \frac{5}{x-3}$
C. $\frac{6}{x-4} - \frac{5}{x-3}$
D. $\frac{7}{x-4} - \frac{5}{x-3}$

Problem 69: CE Board May 1996

Find the value of A in the equation:

$$\frac{x^2+4x+10}{x^3+2x^2+5x} = \frac{A}{x} + \frac{B(2x+2)}{x^2+2x+5} + \frac{C}{x^2+2x+5}$$

A. –2
B. 1/2
C. –1/2
D. 2

Problem 70: ME Board October 1996

The value of (3 to 2.5 power) square is equal to:

A. 729
B. 140
C. 243
D. 81

Problem 71:

Evaluate: $64^x \cdot 4^y$.

A. 256^{xy}
B. 4^{x+3y}

C. 64^{x+3y}
D. 4^{3x+y}

Problem 72: ECE Board April 1993

Solve for x in the following equations.

$$27^x = 9^y$$

$$81^y\, 3^{-x} = 243$$

A. 1
B. 1.5
C. 2
D. 2.5

Problem 73: ECE Board April 1993

Evaluate: $y = \dfrac{4(5^{2n+1}) - 10(5^{2n-1})}{2(5^{2n})}$

A. $y = 5^n$
B. $y = 9$
C. $y = 5^{2n}$
D. $y = 18$

Problem 74: ECE Board April 1990

Given: $(a^n)(a^m) = 100{,}000$ $\qquad a^{nm} = 1000000$

$$\frac{a^n}{a^m} = 10$$

Find a:

A. 12
B. 9
C. 11
D. 10

Problem 75: ECE Board November 1991

Give the factors of $a^2 - x^2$.

A. $2a - 2x$
B. $(a + x)(a - x)$
C. $(a + x)(a + x)$
D. $2x - 2a$

Problem 76: ME Board April 1996

Factor the expression $x^2 + 6x + 8$ as completely as possible.

A. $(x + 4)(x + 2)$
B. $(x - 4)(x + 2)$

C. $(x-4)(x-2)$
D. $(x+6)(x+2)$

Problem 77: ECE Board November 1990

$(a-b)^3 = ?$

A. $a^3 - 3a^2b + 3ab^2 + b^3$
B. $a^3 - 3a^2b - 3ab^2 - b^3$
C. $a^3 + 3a^2b + 3ab^2 - b^3$
D. $a^3 - 3a^2b + 3ab^2 - b^3$

Problem 78:

Find the value of k so that $4x^2 + 6x + k$ is a perfect square.

A. 36
B. 2.5
C. 9
D. 2.25

Problem 79: ME Board April 1995

Factor the expression $3x^3 - 3x^2 - 18x$

A. $3x(x-3)(x+2)$
B. $3x(x+3)(x+2)$
C. $3x(x+3)(x-2)$
D. $3x(x-3)(x-2)$

Problem 80:

If $p - q = 5$ and $pq = \frac{k}{2}$, then $p^2 + q^2$ equals

A. k
B. 25k
C. 25 + k
D. $\frac{k}{25}$

Problem 81: ME Board April 1995

Simplify $b^{m/n}$

A. $\frac{\sqrt{b^m}}{n}$
B. b^{m+n}

C. $\sqrt[n]{b^m}$

D. $\dfrac{b^m}{n}$

Problem 82: ME Board April 1998

Find the value of x which will satisfy the following expression: $\sqrt{x-2} = \sqrt{x} + 2$.

A. 3/2
B. 9/4
C. 18/6
D. None of these

Problem 83:

Simplify $\sqrt{\dfrac{ab}{\sqrt[3]{ab}}}$

A. $\sqrt[3]{ab}$

B. $\sqrt{ab}$

C. $\dfrac{ab}{\sqrt{ab}}$

D. $\dfrac{ab}{\sqrt[3]{ab}}$

Problem 84: ME Board April 1996

If x to the 3/4 power equals 8, x equals

A. –9
B. 6
C. 9
D. 16

Problem 85:

Solve for x: $\sqrt{x+2\sqrt{2x+3}} - 3 = 0$

A. 3
B. 23
C. 3 and 23
D. 20

Problem 86: CE Board November 1991

Solve for x from the given equation:

$$\sqrt[4]{8\sqrt[3]{2\sqrt{8x}}} = 2$$

A. 4
B. 2
C. 3
D. 5

Problem 87: EE Board October 1997

If $f(x) = 2x^2 + 2x + 4$, what is f(2)?

A. $4x + 2$
B. 16
C. $x^2 + x + 2$
D. 8

Problem 88: EE Board April 1997

If n is any positive integer, when (n-1)(n-2)(n-3)... (3)(2)(1) =

A. $e^{(n-1)}$
B. $(n - 1)!$
C. n!
D. $(n - 1)^n$

Problem 89:

What is the least common multiple of 15 and 18?

A. 3
B. 5
C. 90
D. 270

Problem 90: ECE Board April 1998

What is the lowest common factor of 10 and 32?

A. 320
B. 2
C. 180
D. 90

Problem 91:

The numbers 12 and 16 has the greatest common divisor of

A. 2
B. 4
C. 6
D. 192

Problem 92: EE Board April 1996, EE Board March 1998

The polynomial $x^3 + 4x^2 - 3x + 8$ is divided by $x - 5$, then the remainder is,

A. 175
B. 140
C. 218
D. 200

Problem 93:

Find the quotient of $3x^5 - 4x^3 + 2x^2 + 36x + 48$ divided by $x^3 - 2x^2 + 6$.

A. $3x^2 - 4x - 8$
B. $3x^2 + 4x + 8$
C. $3x^2 - 6x - 8$
D. $3x^2 + 6x + 8$

Problem 94: CE Board November 1997

Find the remainder if we divide $4y^3 + 18y^2 + 8y - 4$ by $(2y + 3)$.

A. 10
B. 11
C. 15
D. 13

Problem 95: ECE Board April 1999

Given: $f(x) = (x + 3)(x - 4) + 4$. When $f(x)$ is divided by $(x - k)$, the remainder is k. Find k.

A. 2
B. 4
C. 6
D. 8

Problem 96:

The expression $x^4 + ax^3 + 5x^2 + bx + 6$ when divided by $(x - 2)$ leaves a remainder of 16 and when divided by $(x + 1)$ leaves a remainder of 10. Find a and b.

A. $a = 5, b = 7$
B. $a = -5, b = 7$
C. $a = -5, b = -7$
D. $a = 5, b = -7$

Problem 97:

The mean of x and y is a, the mean of y and z is b and the mean of x and z is c. What is the mean of x, y and z?

A. $\frac{a + b + c}{3}$

B. $\dfrac{a+b+c}{2}$

C. $\dfrac{a+b+c}{abc}$

D. $\dfrac{abc}{a+b+c}$

Problem 98: ECE Board April 1999

Find the mean proportional of 4 and 36.

A. 72
B. 24
C. 12
D. 20

Problem 99: ECE Board April 1998

The arithmetic mean of 80 numbers is 55. If two numbers namely 250 and 850 are removed, what is the arithmetic mean of the remaining numbers?

A. 42.31
B. 50
C. 38.62
D. 57.12

Problem 100: ECE Board April 1998

The arithmetic mean of 6 numbers is 17. If two numbers are added to the progression, the new set of numbers will have an arithmetic mean of 19. What are the two numbers if their difference is 4?

A. 21, 25
B. 23, 27
C. 8, 12
D. 16,20

ANSWER KEY

51. A	64. A	77. D	90. B
52. A	65. C	78. D	91. B
53. C	66. D	79. A	92. C
54. B	67. B	80. C	93. D
55. B	68. C	81. C	94. B
56. A	69. D	82. D	95. B
57. A	70. C	83. A	96. B
58. C	71. D	84. D	97. A
59. C	72. A	85. A	98. C
60. C	73. B	86. B	99. A
61. A	74. D	87. B	100. B
62. B	75. B	88. B	
63. A	76. A	89. C	

RATING

☐ 43-50 Topnotcher

☐ 33-42 Passer

☐ 25-32 Conditional

☐ 0-24 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 2

51. $16 = 4x + 4$

$x = 3$

$5x - 1 = 5(3) - 1$

$= 14$

52. $\left[\frac{x+1}{3} + \frac{2x}{4} = 47 - 2x\right]12$

$4x + 4 + 6x = 564 - 24x$

$34x = 560$

$x = 16.47$

53. $\left\{10\left[\frac{A}{x} + \frac{A}{y}\right] = A\right\}\frac{1}{10A}$

$\frac{1}{x} + \frac{1}{y} = \frac{1}{10}$

$\frac{1}{y} = \frac{1}{10} - \frac{1}{x}$ ☞ ❶

$\left\{2\left[\frac{3A}{x} - \frac{4A}{y}\right] = A\right\}\frac{1}{A}$

$\frac{6}{x} - \frac{8}{y} = 1$ ☞ ❷

Substitute, (1) in (2):

$\frac{6}{x} - 8\left(\frac{1}{10} - \frac{1}{x}\right) = 1$

$\frac{6}{x} - \frac{8}{10} + \frac{8}{x} = 1$

$\frac{14}{x} = 1 + \frac{8}{10} = \frac{18}{10}$

$x = \frac{140}{18} = \frac{70}{9}$

54. $x - 4y + 2 = 0$

$x = 4y - 2$ ☞ ❶

$2x + y - 4 = 0$ ☞ ❷

Substitute (1) in (2):

$2(4y - 2) + y - 4 = 0$

$8y - 4 + y - 4 = 0$

$$9y = 8$$
$$y = \frac{8}{9}$$

$$x = 4y - 2$$
$$x = 4\left[\frac{8}{9}\right] - 2 = \frac{32}{9} - 2 = \frac{14}{9}$$

55. $4x + 2y = 5$

$$y = \frac{5}{2} - 2x \quad \text{☞ ❶}$$

$13x - 3y = 2$ ☞ ❷

Substitute (1) in (2):

$$13x - 3\left(\frac{5}{2} - 2x\right) = 2$$
$$18x = 2 + \frac{15}{2} = \frac{19}{2}$$
$$x = \frac{19}{2(18)} = \frac{1}{2}$$
$$y = \frac{5}{2} - 2\left(\frac{1}{2}\right) = \frac{3}{2}$$

56. $\left(2x^2 - 3y^2 = 6\right)3$

$6x^2 - 9y^2 = 18$ ☞ ❶

$\left(3x^2 + 2y^2 = 35\right)2$

$6x^2 + 4y^2 = 70$ ☞ ❷

Subtract (1) from (2):

$$6x^2 + 4y^2 - \left(6x^2 - 9y^2\right) = 70 - 18$$
$$13y^2 = 52$$
$$y = \pm 2$$
$$6x^2 - 9(2)^2 = 18$$
$$6x^2 = 54$$
$$x = \pm 3$$

57. $3x - 2y + w = 11$ ☞ ❶

$x + 5y - 2w = -9$ ☞ ❷

$2x + y - 3w = -6$ ☞ ❸

Multiply (2) by 3:

$3x + 15y - 6w = -27$ ☞ ❹

Subtract (4) by (1):

$(3x + 15y - 6w) - (3x - 2y + w) = -27 - 11$

$17y - 7w = -38$

$$y = \frac{7w - 38}{17}$$ ☞ ❺

Multiply (2) by 2:

$2x + 10y - 4w = -18$ ☞ ❻

Subtract (6) by (3):

$(2x + 10y - 4w) - (2x + y - 3w) = -18 - (-6)$

$9y - w = -12$ ☞ ❼

Substitute (5) in (7):

$$9\left[\frac{7w - 38}{17}\right] - w = -12$$

$63w - 342 - 17w = -204$

$w = 3$

58.

$2x - y + z = 6$ ☞ ❶

$x - 3y - 2z = 13$ ☞ ❷

$2x - 3y - 3z = 16$ ☞ ❸

Subtract (2) from (3):

$(2x - 3y - 3z) - (x - 3y - 2z) = 16 - 13$

$x - z = 3$

$x = 3 + z$ ☞ ❹

Multiply (1) by (3):

$6x - 3y + 3z = 18$ ☞ ❺

Subtract (2) from (5):

$(6x - 3y + 3z) - (x - 3y - 2z) = 18 - 13$

$5x + 5z = 5$

$z = 1 - x$ ☞ ❻

Substitute (6) in (4):

$x = 3 + z$

$= 3 + (1 - x)$

$2x = 4$

$x = 2$

59.

$x + y = -4$ ☞ ❶

$x + z - 1 = 0$ ☞ ❷

$y + z + 1 = 0$ ☞ ❸

Subtract (1) from (2):

$x + z - (x + y) = 1 - (-4)$

$$z = 5 + y \qquad \text{☞ (4)}$$

Substitute (4) in (3):

$$y + (5 + y) + 1 = 0$$
$$2y = -6$$
$$y = -3$$
$$z = 5 + (-3) = 2$$
$$x = -4 - (-3) = -1$$

60. $(2x + 5y)(5x - 2y) = 10x^2 - 4xy + 25xy - 10y^2$
$= 10x^2 + 21xy - 10y^2$

61. Multiply the three given equations:

$$(xy)(yz)(zx) = 15(35)(21)$$
$$(xyz)^2 = 11025$$
$$xyz = 105 \qquad \text{☞ (4)}$$

Substitute $xy = 15$, in (4):

$$15z = 105$$
$$z = 7$$

Substitute $yz = 35$, in (4):

$$35x = 105$$
$$x = 3$$

Substitute $zx = 21$, in (4):

$$21y = 105$$
$$y = 5$$

Thus, $x + y + z = 3 + 5 + 7 = 15$

62.
$$\frac{\left(x^2y^3z^{-2}\right)^{-3}\left(x^{-3}yz^3\right)^{-\frac{1}{2}}}{\left(xyz^{-3}\right)^{-\frac{5}{2}}} = \frac{\left(x^{-6}y^{-9}z^6\right)\left(x^{\frac{3}{2}}y^{-\frac{1}{2}}z^{-\frac{3}{2}}\right)}{x^{-\frac{5}{2}}y^{-\frac{5}{2}}z^{\frac{15}{2}}}$$
$$= x^{-6+\frac{3}{2}+\frac{5}{2}}\, y^{-9-\frac{1}{2}+\frac{5}{2}}\, z^{6-\frac{3}{2}-\frac{15}{2}}$$
$$= x^{-2}\, y^{-7}\, z^{-3} = \frac{1}{x^2\, y^7\, z^3}$$

63.
$$\frac{5x}{2x^2+7x+3} - \frac{x+3}{2x^2-3x-2} + \frac{2x+1}{x^2+x-6}$$
$$= \frac{5x}{(2x+1)(x+3)} - \frac{x+3}{(2x+1)(x-2)} + \frac{2x+1}{(x+3)(x-2)}$$
$$= \frac{5x(x-2) - (x+3)(x+3) + (2x+1)(2x+1)}{(2x+1)(x+3)(x-2)}$$
$$= \frac{5x^2 - 10x - x^2 - 6x - 9 + 4x^2 + 4x + 1}{(2x+1)(x+3)(x-2)}$$

$$= \frac{8x^2 - 12x - 8}{(2x+1)(x+3)(x-2)} = \frac{4(2x^2 - 3x - 2)}{(2x+1)(x+3)(x-2)}$$

$$= \frac{4(2x+1)(x-2)}{(2x+1)(x+3)(x-2)}$$

$$= \frac{4}{x+3}$$

64. $$\left\{x^{\frac{2}{3}}\left[x^{-\frac{1}{3}}y^{-\frac{1}{2}}\left(x^2y^{-2}\right)^{-\frac{2}{3}}\right]^{\frac{1}{2}}\right\}^6 = x^4\left[x^{-\frac{1}{3}}y^{-\frac{1}{2}}x^{-\frac{4}{3}}y^{\frac{4}{3}}\right]^3$$

$$= x^4\left[x^{-\frac{5}{3}}y^{\frac{5}{6}}\right]^3 = x^4\left[x^{-5}y^{\frac{5}{2}}\right]$$

$$= x^{-1}y^{\frac{5}{2}} = \frac{y^{\frac{5}{2}}}{x}$$

65. $$7^{a+2} - 8(7)^{a+1} + 5(7)^a + 49(7)^{a-2} = 7^a7^2 - (8)7^a7^1 + (5)7^a + \frac{49(7^a)}{7^2}$$

$$= 7^a(49 - 56 + 5 + 1)$$

$$= -7^a$$

66. $$x = \frac{\left(b^2 - 4b + 16\right)\left(b^2 - 16\right)}{b^3 + 4^3} = \frac{\left(b^2 - 4b + 16\right)(b-4)(b+4)}{(b+4)\left(b^2 - 4b + 4^2\right)} = b - 4$$

67. $$x = y\frac{(b-c)}{a-c} = \frac{by - cy}{a-c}$$

$$z = y\frac{(a-b)}{a-c} = \frac{ay - by}{a-c}$$

$$x + z = \frac{by - cy}{a-c} + \frac{ay - by}{a-c}$$

$$= \frac{ay - cy}{a-c} = \frac{y(a-c)}{a-c}$$

$$x + z = y$$

68. $$\frac{x+2}{x^2 - 7x + 12} = \frac{x+2}{(x-3)(x-4)} = \frac{A}{x-3} + \frac{B}{x-4}$$

$$\frac{x+2}{(x-3)(x-4)} = \frac{A(x-4) + B(x-3)}{(x-3)(x-4)}$$

$$x+2 = A(x-4)+B(x-3)$$
$$x+2 = Ax-A4+Bx-B3$$

Equate coefficients of x:

$1 = A + B$

$A = 1 - B$ ☞ (1)

Equate constants:

$2 = -4A - 3B$ ☞ (2)

Substitute (1) in (2):

$2 = -4(1-B) - 3B$

$2 = -4 + 4B - 3B$

$B = 6$

$A = 1 - 6 = -5$

Thus, $\dfrac{x+2}{x^2-7x+12} = \dfrac{6}{x-4} - \dfrac{5}{x-3}$

69. $\dfrac{x^2+4x+10}{x^3+2x^2+5x} = \dfrac{A}{x} + \dfrac{B(2x+2)}{x^2+2x+5} + \dfrac{C}{x^2+2x+5}$

$$= \frac{A\left(x^2+2x+5\right)+Bx(2x+2)+Cx}{x\left(x^2+2x+5\right)}$$

$$\frac{x^2+4x+10}{x^3+2x^2+5x} = \frac{Ax^2+2Ax+5A+2Bx^2+2Bx+Cx}{x^3+2x^2+5x}$$

By equating constants:

$10 = 5A$

$A = 2$

70. $\left[(3)^{2.5}\right]^2 = 243$

71. $64^x 4^y = (4)^{3x}(4)^y = (4)^{3x+y}$

72. $(81)^y(3)^{-x} = 243$

$(3)^{4y}(3)^{-x} = (3)^5$ ☞ (1)

$(27)^x = (9)^y$

$(3)^{3x} = (3)^{2y}$

Squaring both sides:

$(3)^{6x} = (3)^{4y}$ ☞ (2)

Substitute (2) in (1):

$(3)^{4y}(3)^{-x} = (3)^5$

$(3)^{6x}(3)^{-x} = (3)^5$

$6x - x = 5$

$$5x = 5$$
$$x = 1$$

73. $$y = \frac{4\left(5^{2n+1}\right) - 10\left(5^{2n-1}\right)}{2\left(5^{2n}\right)} = \frac{2\left(5^{2n+1}\right) - 5\left(5^{2n-1}\right)}{5^{2n}}$$

$$= \frac{2\left(5^{2n}.5^{1}\right) - 5\left(5^{2n}.5^{-1}\right)}{5^{2n}}$$

$$y = 2(5^1) - 5(5^{-1}) = 9$$

74. $$\left(a^n\right)\left(a^m\right) = 100{,}000 \quad \text{☞ ❶}$$

$$\frac{a^n}{a^m} = 10\,; \quad a^n = 10\,a^m \quad \text{☞ ❷}$$

Substitute (2) in (1):

$$10\,a^m\,a^m = 100{,}000$$
$$(a^m)^2 = 10{,}000$$
$$a^m = 100 \quad \text{☞ ❸}$$

Substitute $a^m = 100$, in (1):

$$a^n\,(100) = 100{,}000$$
$$a^n = 1000$$

$$a^{nm} = \left(a^n\right)^m = 1000000$$
$$(1000)^m = 100{,}000$$
$$m = 2$$

Substitute m = 2, in (3):

$$a^2 = 100$$
$$a = 10$$

75. $$a^2 - x^2 = (a + x)(a - x)$$

76. $$x^2 + 6x + 8 = (x + 4)(x + 2)$$

77. $$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

78. $$\left[4x^2 + 6x + k = 0\right]\frac{1}{4}$$

$$x^2 + 1.5x + 0.25k = 0$$

$$\left(x + \frac{1.5}{2}\right)^2 = 0$$

Since it is a perfect square, then

$$\left[\frac{1.5}{2}\right]^2 = 0.25k\,, \qquad k = 2.25$$

79. $3x^3 - 3x^2 - 18x = 3x\left(x^2 - x - 6\right)$

$= 3x(x-3)(x+2)$

80. $p - q = 5$

By squaring both sides:

$(p-q)^2 = (5)^2$

$p^2 - 2pq + q^2 = 25$

$p^2 + q^2 = 25 + 2pq$

$= 25 + 2\left(\frac{k}{2}\right)$

$p^2 + q^2 = 25 + k$

81. $b^{m/n} = \left(b^m\right)^{\frac{1}{n}} = \sqrt[n]{b^m}$

82. $\sqrt{x-2} = \sqrt{x} + 2$

By squaring both sides:

$\left(\sqrt{x-2}\right)^2 = \left(\sqrt{x}+2\right)^2$

$x - 2 = x + 4\sqrt{x} + 4$

$4\sqrt{x} = -6$

$\sqrt{x} = -\frac{3}{2}$

$x = \frac{9}{4}$

Note: Since $x = 9/4$ will not satisfy to the given general equation when substituted, this equation is classified as defective and thus, the answer is "None of these".

83. $\sqrt{\frac{ab}{\sqrt[3]{ab}}} = \sqrt{\frac{ab}{(ab)^{\frac{1}{3}}}} = \sqrt{(ab)^{\frac{2}{3}}} = \left[(ab)^{\frac{2}{3}}\right]^{\frac{1}{2}} = (ab)^{\frac{1}{3}} = \sqrt[3]{ab}$

84. $\left[x^{\frac{3}{4}} = 8\right]^{\frac{4}{3}}$

$x = (8)^{\frac{4}{3}} = 16$

85. $\sqrt{x + 2\sqrt{2x+3}} = 3$

By squaring both sides:

$x + 2\sqrt{2x+3} = 9$

$2\sqrt{2x+3} = 9 - x$

By squaring both sides:

$$4(2x + 3) = (9 - x)^2$$
$$8x + 12 = 81 - 18x + x^2$$
$$x^2 - 26x = -69$$

By completing the square:

$$(x - 13)^2 = -69 + (13)^2$$
$$(x - 13)^2 = 100$$
$$x - 13 = \pm 10$$
$$x = 23 \text{ (absurd)}$$
$$x = 3$$

86. $\sqrt[4]{8\sqrt[3]{2\sqrt{8x}}} = 2$

By raising both sides to exponent 4:

$$8\sqrt[3]{2\sqrt{8x}} = (2)^4 = 16$$
$$\sqrt[3]{2\sqrt{8x}} = 2$$

By raising both sides to exponent 3:

$$2\sqrt{8x} = (2)^3 = 8$$
$$\sqrt{8x} = 4$$

By raising both sides to exponent 2:

$$8x = (4)^2 = 16$$
$$x = 2$$

87. $f(x) = 2x^2 + 2x + 4$

$f(2) = 2(2)^2 + 2(2) + 4 = 16$

88.
$$n! = n(n-1)(n-2)(n-3) \ldots\ldots (3)(2)(1)$$
$$(n-1)! = (n-1)(n-2)(n-3)\ldots\ldots\ldots (3)(2)(1)$$
} *Formulas*

89. $15 = 5 \cdot 3$

$18 = 6 \cdot 3$

Least common multiple $= 5 \cdot 6 \cdot 3 = 90$

90. $10 = 5 \cdot 2$

$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Lowest common factor = 2

91. $12 = 4.3 = 4 \cdot 3$

$16 = 4.4 = 4 \cdot 4$

Greatest common divisor = 4

92. $f(x) = x^3 + 4x^2 - 3x + 8$; divisor $= x - 5$

Note: Using remainder theorem, remainder = f (5).

Remainder $= (5)^3 + 4(5)^2 - 3(5) + 8 = 218$

93.

$$x^3 + 2x^2 + 6 \overline{)\, 3x^5 - 4x^3 + 2x^2 + 36x + 48}$$

$$3x^2 + 6x + 8$$

$$(-)\ 3x^5 - 6x^4 + 18x^2$$

$$6x^4 - 4x^3 - 16x^2 + 36x$$

$$(-)\ 6x^4 - 12x^3 + 36x$$

$$8x^3 - 16x^2 + 48$$

$$(-)\ 8x^3 - 16x^2 + 48$$

$$0$$

94. $f(y) = 4y^3 + 18y^2 + 8y - 4$

divisor $= 2y + 3 = y - \left[-\frac{3}{2}\right]$

Note: Using remainder theorem, remainder $= f\left[-\frac{3}{2}\right]$

$$f\left[-\frac{3}{2}\right] = 4\left[-\frac{3}{2}\right]^3 + 18\left[-\frac{3}{2}\right]^2 + 8\left[-\frac{3}{2}\right] - 4 = 11$$

95. $f(x) = (x+3)(x-4) + 4$

$= x^2 - 4x + 3x - 12 + 4$

$f(x) = x^2 - x - 8$

Remainder $= f(k)$

$f(k) = k^2 - k - 8$ ☞ ❶

Substitute the given remainder = k in (1):

$k = k^2 - k - 8$

$k^2 - 2k - 8 = 0$

$(k - 4)(k + 2) = 0$

$k_1 = 4$

$k_2 = -2$

96. $f(x) = x^4 + ax^3 + 5x^2 + bx + 6$

Note: Remainder $= f(r)$.

When divisor is $(x - 2)$, $r = 2$ & $f(r) = 16$

$f(r) = (2)^4 + a(2)^3 + 5(2)^2 + b(2) + 6$

$= 8a + 2b + 42$

$16 = 8a + 2b + 42$

$-26 = 8a + 2b$

$b = -13 - 4a$ ☞ ❶

When divisor is $(x + 1)$, $r = -1$ & $f(r) = 10$

$f(r) = (-1)^4 + a(-1)^3 + 5(-1)^2 + b(-1) + 6$

$= -a - b + 12$

$10 = -a - b + 12$

$2 = a + b$ ☞ ❷

Substitute (1) in (2):

$2 = a + (-13 - 4a)$

$-3a = 15$

$a = -5$

$b = -13 - 4(-5) = 7$

97. $\frac{x+y}{2} = a$; $\frac{y+z}{2} = b$; $\frac{x+z}{2} = c$

By adding a, b and c:

$$a + b + c = \frac{x+y}{2} + \frac{y+z}{2} + \frac{x+z}{2}$$

$$= \frac{1}{2}[2x + 2y + 2z]$$

$$a + b + c = x + y + z$$

$$\text{Mean} = \frac{x+y+z}{3} = \frac{a+b+c}{3}$$

98. Let: x = the mean proportion of 4 and 36

$$\frac{4}{x} = \frac{x}{36}$$

$$x^2 = 144$$

$$x = 12$$

99. Let: x = the arithmetic sum of 80 numbers,

$$\text{Arithmetic mean} = \frac{x}{80} = 55$$

$$x = 80(55) = 4400$$

y = new Arithmetic mean

$$= \frac{x - (250 + 850)}{80 - 2}$$

$$y = 42.31$$

100. Let: x = the first number

x + 4 = the second number

y = sum of the original 6 numbers.

$$\text{Arithmetic mean} = \frac{y}{6} = 17$$

$$y = 17(6) = 102$$

$$\frac{y + x + (x + 4)}{6 + 2} = 19$$

$$\frac{102 + 2x + 4}{8} = 19$$

$$106 + 2x = 19(8)$$

$$2x = 46$$

$$x = 23$$

$$x + 4 = 27$$

DAY 3

QUADRATIC EQUATION, BINOMIAL THEOREM, LOGARITHM

QUADRATIC EQUATION

The general quadratic equation is expressed as:

$$Ax^2 + Bx + C = 0,$$

where a, b and c are real numbers and with $a \neq 0$. A quadratic equation in x is also known as a **second-degree polynomial equation.**

Quadratic formula: $$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Nature of roots:

The discriminant, $B^2 - 4AC$ determines the nature of the roots of a quadratic equation.

When $B^2 - 4AC = 0$, roots are real and equal (one root only)
When $B^2 - 4AC > 0$, roots are real and unequal
When $B^2 - 4AC < 0$, roots are imaginary and unequal

Properties of roots:

Let r_1 and r_2 be the roots of a quadratic equation.

When the two roots are added: $$r_1 + r_2 = \frac{-B}{A}$$

When the two roots are multiplied: $$r_1 r_2 = \frac{C}{A}$$

Pure quadratic equation – is when B = 0. In such case, $r_1 = -r_2$

BINOMIAL THEOREM

Binomial is a polynomial or expression of two terms. When a binomial is raised to a certain power, the expansions are as follows:

$$(x+y)^0 = 1$$
$$(x+y)^1 = x+y$$
$$(x+y)^2 = x^2+2xy+y^2$$
$$(x+y)^3 = x^3+3x^2y+3xy^2+y^3$$
$$(x+y)^4 = x^4+4x^3y+6x^2y^2+4xy^3+y^4$$
$$\vdots \qquad\qquad \vdots$$

PROPERTIES OF BINOMIAL EXPANSION:

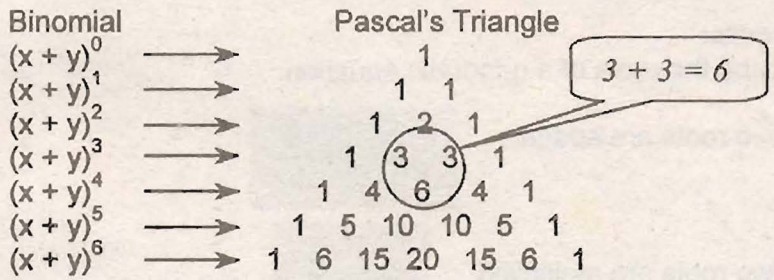
In the expansion of $(x+y)^n$, and as observed in the given expansions above, the following are the properties of a binomial expansion:

1. The number of terms in a resulting expansion = n + 1.
2. The powers of x decrease by 1 in successive terms, while the power of y increase by 1 in successive terms.
3. The sum of the powers of each term = n.
4. The first term is x^n and the last term is y^n and each of the terms has a coefficient of 1.
5. The coefficients increase and then decrease in a symmetrical pattern.

To express everything in general terms, the so-called **Binomial Theorem** was established and expressed as follows:

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \cdots + nxy^{n-1} + y^n$$

Also, the coefficients of a binomial expansion can be conveniently obtained by arranging them in a triangular array or pattern. This is known as **Pascal's Triangle**, named after the famous French Mathematician **Blaise Pascal** (1623 –1662).



Note: Any number in the Pascal's triangle is obtained by adding the two adjacent numbers above it. For example, the number 6 is obtained by adding 3 and 3.

Another way of determining the coefficient of any term in a binomial expansion is to use the following formula:

$$\text{Coeff. of any term} = \frac{(\text{Coeff. of P.T.})(\text{Power of x of P.T.})}{\text{Power of y of P.T.} + 1}$$

where: P.T. = preceding term

The rth term of the binomial expansion of $(x + y)^n$ may be calculated using the following formulas:

$$r^{th}\text{term} = \frac{(n)(n-1)(n-2)...(n-r+2)}{(r-1)!}x^{(n-r+1)}y^{r-1}$$

$$r^{th}\text{ term} = {}_nC_{r-1}\, x^{(n-r+1)}\, y^{r-1}$$

A term involving a variable with a specific exponent is obtained by using the following formula:

$$y^r = \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}x^{n-r}y^r$$

Sum of the coefficients of the expansion of $(x + y)^n$:

$$\text{Sum} = (\text{Coeff. of } x + \text{coeff. of } y)^n$$

Sum of exponents of the expansion $(x + y)^n$:

$$\text{Sum} = n(n + 1)$$

LOGARITHMS

The term **logarithm** was derived from Greek words, "*logus*" meaning "ratio" and "*aritmus*" meaning "number". **John Napier** (1550 – 1617) of Scotland invented logarithm 1614. Napier used e = 2.718... for its base. In 1616, it was improved by a professor of geometry at Gresham College in London, **Henry Briggs** (1561 – 1630) using 10 as base.

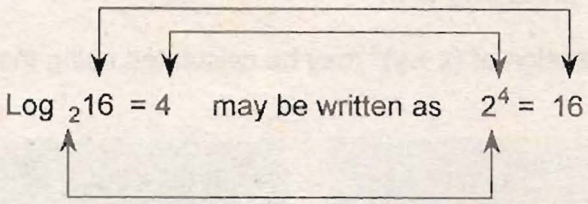
Common logarithm ($\log_{10}$ or simply log) is a logarithm using the decimal base 10. This is also known as **Brigg's or Briggsian logarithm**.

Natural logarithm (ln) is a logarithm using the base e. This is also known as **Napier's or Napierian logarithm** in honor of Napier. The number, e is known as **Euler's Number,**named after the Swiss mathematician, Leonhard Euler (1707 – 1783) and is defined as,

$$e = \lim_{n \to \infty}\left(1+\frac{1}{n}\right)^n$$

Binary logarithm (denoted as lb) is a logarithm with a base value of 2.

Since logarithm is an exponent, this illustrates that $\log_b x$ is the exponent to which b must be raised to obtain **x**. Therefore,



PROPERTIES OF LOGARITHMS:

❶ $\log (xy) = \log x + \log y$

❷ $\log \frac{x}{y} = \log x - \log y$

❸ $\log x^n = n \log x$

❹ $\log_b x = \frac{\log x}{\log b}$

❺ $\log_a x = \frac{\log_b x}{\log_b a}$

❻ $\log_a a = 1$

The natural logarithm can be converted into a common logarithm and vice versa. To obtain this, a factor known as the **modulus of logarithms** is necessary, such as

$\log x = 0.4343 \ln x$ also, $\ln x = 2.3026 \log x$

modulus



Tip: **Degree of a polynomial or equation** – refers to the maximum sum of the exponents of the variables in any term of the polynomial.

What is the degree $3x^4y - 2x^3z^4 + 7yz^5$?
Ans. 7, which is the sum of 3 and 4 of the second term.

Did you know that... Newton while a student at age 22, invented differential and integral calculus, discovered the law of universal gravitation, formulated the three laws of motion, developed the new theory of light in just 18 months and set a record of the most productive periods of achievement by an individual in the history of science.

Proceed to the next page for your third test. GOODLUCK ! ☞

Test 3

Time element: 3.0 hours

Problem 101: ECE Board March 1996

The equation of whose roots are the reciprocal of the roots of $2x^2 - 3x - 5 = 0$ is,

A. $5x^2 + 3x - 2 = 0$
B. $2x^2 + 3x - 5 = 0$
C. $3x^2 - 3x + 2 = 0$
D. $2x^2 + 5x - 3 = 0$

Problem 102: EE Board October 1993

In the equation $x^2 + x = 0$, one root is x equal to

A. 1
B. 5
C. 1/4
D. none of these

Problem 103: ECE Board April 1990

Solve for the value of "a" in the equation $a^8 - 17a^4 + 16 = 0$.

A. ± 2
B. ± 3
C. ± 4
D. ± 5

Problem 104: ME Board October 1996

Solve for x that satisfies the equation $6x^2 - 7x - 5 = 0$.

A. $\frac{5}{3}$ or $\frac{-1}{2}$
B. $\frac{3}{2}$ or $\frac{3}{8}$
C. $\frac{7}{5}$ or $\frac{-7}{15}$
D. $\frac{3}{5}$ or $\frac{3}{4}$

Problem 105: EE Board October 1997

Find the values of x in the equation $24x^2 + 5x - 1 = 0$.

A. $(\frac{1}{6}, 1)$

B. $(\frac{1}{6}, \frac{1}{5})$

C. $(\frac{1}{2}, \frac{1}{5})$

D. $(\frac{1}{8}, -\frac{1}{3})$

Problem 106: EE Board October 1990

Determine k so that the equation $4x^2 + kx + 1 = 0$ will have just one real solution.

A. 3
B. 4
C. 5
D. 6

Problem 107: ME Board April 1996

Solve for x: $10x^2 + 10x + 1 = 0$

A. – 0.113, - 0.887
B. – 0.331, - 0.788
C. – 0.113, - 0.788
D. – 0.311, - 0.887

Problem 108:

If 1/3 and –3/2 are the roots of a quadratic equation, then the equation is

A. $6x^2 + 7x - 3 = 0$
B. $6x^2 - 7x + 3 = 0$
C. $6x^2 - 7x - 3 = 0$
D. $6x^2 - 7x + 1 = 0$

Problem 109:

Which of the following is a root of this quadratic equation, $30x^2 + 49x + 20 = 0$?

A. 0.6
B. –0.6
C. –0.8
D. 0.75

Problem 110:

What is the discriminant of the equation $4x^2 = 8x - 5$?

A. 8
B. –16
C. 16
D. –8

Problem 111:

Given the equation $3x^2 + Bx + 12 = 0$. What is the value of B so that the roots of the equation are equal?

A. 4
B. 8
C. 10
D. –12

Problem 112:

Find the term involving y^5 in the expansion of $(2x^2 + y)^{10}$.

A. $8064\ x^{10}\ y^5$
B. $8046\ x^5\ y^5$
C. $8046\ x^{10}\ y^5$
D. $4680\ x^5\ y^5$

Problem 113:

Find the 5th term of the expansion of $(x^2 + \frac{1}{x})^{10}$.

A. $260\ x^8$
B. $5040\ x^8$
C. $210\ x^8$
D. $420\ x^8$

Problem 114: ECE Board April 1998

In the expression of $(x + 4y)^{12}$, the numerical coefficient of the 5th term is,

A. 63,360
B. 126,720
C. 506,880
D. 253,440

Problem 115:

What is the fourth term of the expansion of $(x + x^2)^{100}$?

A. $1650\ x^{103}$
B. $161700\ x^{103}$
C. $167100\ x^{103}$
D. $167100\ x^{100}$

Problem 116:
What is the numerical coefficient of the term next to $495x^8 y^4$?

A. 660
B. 792
C. 990
D. 1100

Problem 117: CE Board November 1996
Find the 6th term of the expansion of $\left(\frac{1}{2a} - 3\right)^{16}$.

A. $-\frac{66939}{256a^{11}}$
B. $-\frac{66339}{128a^{11}}$
C. $-\frac{33669}{256a^{11}}$
D. $-\frac{39396}{128a^{11}}$

Problem 118:
What is the coefficient of the term free of x of the expansion of $(2x - 5y)^4$?

A. 256
B. 526
C. 265
D. 625

Problem 119:
Find the 6th term of $(3x - 4y)^8$

A. $-148{,}288\ x^3y^5$
B. $-548\ x^2y^5$
C. $-154{,}288\ x^3y^5$
D. $-1{,}548{,}288\ x^3y^5$

Problem 120: ECE Board November 1995
What is the sum of the coefficients of the expansion of $(2x - 1)^{20}$?

A. 0
B. 1
C. 2
D. 3

Problem 121: ECE Board April 1995

What is the sum of the coefficients in the expansion of $(x + y - z)^8$?

A. 0
B. 1
C. 2
D. 3

Problem 122: CE Board November 1993, ECE Board Nov. 1993

Find the value of $\log_8 48$.

A. 1.86
B. 1.68
C. 1.78
D. 1.98

Problem 123: CE Board November 1997

Evaluate the $\log_6 845 = x$.

A. 3.76
B. 5.84
C. 4.48
D. 2.98

Problem 124: ME Board April 1997

What is the value of log to base 10 of $1000^{3.3}$?

A. ¡0.9
B. 99.9
C. 9.9
D. 9.5

Problem 125: ECE Board April 1998

What is the value of (log 5 to the base 2) + (log 5 to the base 3)?

A. 7.39
B. 3.79
C. 3.97
D. 9.37

Problem 126:

Find the value of $\log_4 (\log_3 5)$.

A. 1.460
B. 0.275
C. 1.273
D. 0.165

Problem 127:

Given: $\log_4 7 = n$. Find $\log_4 \frac{1}{7}$.

A. 1/n
B. n
C. –1/n
D. – n

Problem 128: CE Board November 1992, CE Board May 1994

If $\log_a 10 = 0.25$, what is the value of $\log_{10} a$?

A. 2
B. 4
C. 6
D. 8

Problem 129: ECE Board November 1995

Given: $\log_b y = 2x + \log_b x$. Which of the following is true?

A. $y = b^{2x}$
B. $y = 2xb$
C. $y = \frac{2x}{b}$
D. $y = xb^{2x}$

Problem 130: ME Board October 1996

Which value is equal to log to the base e of e to the –7x power?

A. – 7x
B. 10 to the –7x power
C. 7
E. –7 log to the base 10

Problem 131: ME Board April 1996

Log of the nth root of x equals log of x to 1/n power and also equal to

A. $\frac{\log x}{n}$
B. n log x
C. $\frac{\log(x \text{ to the } 1/n \text{ power})}{n}$
D. (n – 1) log x

Problem 132: ECE Board November 1990

Log (MN) is equal to:

A. Log M – N
B. Log M + N
C. N log M
D. Log M + Log N

Problem 133: ME Board April 1997

What expression is equivalent to log (x) – log (y + z)?

A. log x + log y + log z
B. log [x / (y + z)]
C. log x – log y – log z
D. log y + log (x + z)

Problem 134: ECE Board November 1991

Given: $\log_b 1024 = \frac{5}{2}$. Find b.

A. 2560
B. 16
C. 4
D. 2

Problem 135:

Given: $\log_3 (x^2 - 8x) = 2$. Find x.

A. –1
B. 9
C. –1 and 9
D. 1 and – 9

Problem 136: ECE Board April 1993

Solve for the value of x in the following equation: $x^{3\log x} = 100x$

A. 12
B. 8
C. 30
D. 10

Problem 137: EE Board October 1992

Given: log 6 + x log 4 = log 4 + log $(32 + 4^x)$. Find x.

A. 2
B. 3
C. 4
D. 6

Problem 138: ECE November 1998

If log of 2 to the base 2 plus log of x to the base 2 is equal to 2, then the value of x is,

A. 4
B. –2
C. 2
D. –1

Problem 139: ME Board October 1997

Find the value of x if $\log_{12} x = 2$.

A. 144
B. 414
C. 524
D. 425

Problem 140:

Solve for the value of x:

$$\log 2x^3 + \log\frac{6}{x} = 6.278$$

A. 379.65
B. 365.97
C. 397.56
D. 356.79

ANSWER KEY

101. A	111. D	121. B	131. A
102. D	112. A	122. A	132. D
103. A	113. C	123. A	133. B
104. A	114. B	124. C	134. B
105. D	115. B	125. B	135. C
106. B	116. B	126. B	136. D
107. A	117. B	127. D	137. B
108. A	118. D	128. B	138. C
109. C	119. D	129. D	139. A
110. B	120. A	130. A	140. C

RATING

☐ 34-40 Topnotcher
☐ 26-33 Passer
☐ 20-25 Conditional
☐ 0-19 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 3

101. $2x^2 - 3x - 5 = 0$

$(2x + 2)(x - 2.5) = 0$

$x_1 = 2.5$
$x_2 = -1$ } *Roots of the given equation*

Thus, the roots of the second equation are:

$$x_1 = \frac{1}{2.5} = 0.4 \quad \text{and} \quad x_2 = \frac{1}{-1} = -1$$

$$(x - 0.4)(x + 1) = 0$$

$$x^2 + x - 0.4x - 0.4 = 0$$

$$\left(x^2 + 0.6x - 0.4 = 0\right)5$$

$$5x^2 + 3x - 2 = 0$$

102. $x(x + 1) = 0$

$x = 0 \qquad x = -1$

103. $a^8 - 17a^4 + 16 = 0$

Let: $x = a^4$ and $x^2 = a^8$

$x^2 - 17x + 16 = 0$

Using the quadratic formula;

$$x = \frac{17 \pm \sqrt{(17)^2 - 4(1)(16)}}{2}$$

$$x = \frac{17 \pm 15}{2}$$

Thus,

$x_1 = 1 \qquad x_2 = 16$
$a^4 = 1 \qquad a^4 = 16$
$a = \pm 1 \qquad a = \pm 2$

104. $6x^2 - 7x - 5 = 0$

Using the quadratic formula;

$$x = \frac{7 \pm \sqrt{(7)^2 - 4(6)(-5)}}{2(6)}$$

$$x = \frac{7 \pm 13}{12}$$

Thus,

$$x_1 = \frac{5}{3} \quad \text{and} \quad x_2 = -\frac{1}{2}$$

105. $24x^2 + 5x - 1 = 0$

Using the quadratic formula;

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(24)(-1)}}{2(24)}$$

$$x = \frac{-5 \pm 11}{48}$$

Thus, $x_1 = \frac{1}{8}$ and $x_2 = -\frac{1}{3}$

106. Note: There is only one solution to the equation ($4x^2 + kx + 1 = 0$), if the the discriminant ($B^2 - 4AC$) is equal to zero.

where: A = 4; B = k & C = 1

$$B^2 - 4AC = 0$$
$$k^2 - 4(4)(1) = 0$$
$$k^2 = 16$$
$$k = \pm 4$$

107. $10x^2 + 10x + 1 = 0$

Using the quadratic formula:

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(10)(1)}}{2(10)}$$

$$x = \frac{-10 \pm 7.746}{20}$$

Thus, $x_1 = -0.887$ and $x_2 = -0.113$

108. $x_1 = \frac{1}{3}$ and $x_2 = -\frac{3}{2}$

$$x_1 + x_2 = \frac{-B}{A} \qquad x_1 x_2 = \frac{C}{A}$$

$$\frac{1}{3} + \left(-\frac{3}{2}\right) = \frac{-B}{A} \qquad \frac{1}{3}\left(-\frac{3}{2}\right) = \frac{C}{A}$$

$$B = \frac{7}{6}A \qquad C = \frac{-1}{2}A$$

Substitute to the general quadratic equation:

$$Ax^2 + Bx + C = 0$$

$$Ax^2 + \left(\frac{7}{6}A\right)x + \left(\frac{-1}{2}A\right) = 0$$

$$\left[Ax^2 + \frac{7}{6}Ax - \frac{1}{2}A = 0\right]\frac{6}{A}$$

$$6x^2 + 7x - 3 = 0$$

109. $30x^2 + 49x + 20 = 0$

Using the quadratic formula;

$$x = \frac{-49 \pm \sqrt{(49)^2 - 4(30)(20)}}{2(30)}$$

$$x = \frac{-49 \pm 1}{60}$$

Thus, $x_1 = \frac{-49 + 1}{60} = -0.8$ and $x_2 = \frac{-49 - 1}{60} = -0.833$

110. $4x^2 - 8x + 5 = 0$

where: A = 4; B = - 8 & C = 5

Discriminant = $B^2 - 4AC$

$= (-8)^2 - 4(4)(5)$

$= -16$

111. Note: The roots of the equation ($3x^2 + Bx + 12 = 0$) are equal, if the discriminant ($B^2 - 4AC$) is equal to zero.

where: A = 4; B = - 8 & C = 5

Substitute:

$B^2 - 4AC = 0$

$B^2 - 4(3)(12) = 0$

$B^2 = 144$

$B = \pm 12$

112. Note: The term involving y^5 is the 6th term of the expansion $(2x^2 + y)^{10}$

6th term = ${}_nC_{r-1}\,(2x^2)^{n-r+1}\,(y)^{r-1}$

where: n = 10; r = 6

$$6^{th} \text{ term} = {}_{10}C_5\,(2x^2)^5\,y^5 = \frac{10!}{(10-5)!5!}\left(32x^{10}\right)y^5$$

6th term = $8064\,x^{10}y^5$

113. $\left(x^2 + \frac{1}{x}\right)^{10}$

5th term = ${}_nC_{r-1}\,(x^2)^{n-r+1}\left(\frac{1}{x}\right)^{r-1}$

where: n = 10; r = 5

$$5^{th} \text{ term} = {}_{10}C_4\,(x^2)^6\left(\frac{1}{x}\right)^4 = \frac{10!}{(10-4)!4!}x^{12}\left(\frac{1}{x^4}\right)$$

5th term = $210\,x^8$

114. $(x+4y)^{12}$

5th term $= {}_nC_{r-1}\,(x)^{n-r+1}(4y)^{r-1}$

where: $n = 12$; $r = 5$

$$5^{th}\text{ term} = {}_{12}C_4\,(x)^8(4y)^4 = \frac{12!}{(12-4)!4!}(x^8)(256y^4)$$

$$5^{th}\text{ term} = 126{,}720\,x^8y^4$$

115. $(x+x^2)^{100}$

4th term $= {}_nC_{r-1}\,(x^2)^{n-r+1}(x^2)^{r-1}$

where: $n = 100$; $r = 4$

$$4^{th}\text{ term} = {}_{100}C_3\,(x)^{97}(x^2)^3 = \frac{10!}{(100-3)!3!}x^{97}(x^6)$$

$$4^{th}\text{ term} = 161{,}700\,x^{103}$$

116. $$\text{Coefficient of next term} = \frac{(\text{Coefficient of PT})(\text{Exponent of x})}{(\text{Exponent of y})+1}$$

$$\text{Coefficient of next term} = \frac{495\,(8)}{4+1} = 792$$

117. $\left(\frac{1}{2a}-3\right)^{16}$

$$6^{th}\text{ term} = {}_nC_{r-1}\left(\frac{1}{2a}\right)^{n-r+1}(-3)^{r-1}$$

where: $n = 16$; $r = 6$

$$6^{th}\text{ term} = {}_{16}C_5\left(\frac{1}{2a}\right)^{11}(-3)^5 = \frac{16!}{(16-5)!5!}\left(\frac{1}{2048\,a^{11}}\right)(-243)$$

$$= \frac{4368(-243)}{2048\,a^{11}} \div \frac{16}{16}$$

$$6^{th}\text{ term} = \frac{66339}{128\,a^{11}}$$

118. Note: The term in the expansion $(2x-5y)^4$ which is free of x is the last term or the 5th term.

5th term $= {}_nC_{r-1}\,(2x)^{n-r+1}(-5y)^{r-1}$

where: $n = 4$; $r = 5$

5th term $= {}_4C_4\ (2x)^0\ (-5y)^4 = \dfrac{4!}{(4-4)!4!}(1)(625\ y^4)$

5th term $= 625\ y^4$

119. $(3x\text{-}4y)^8$

6th term $= {}_nC_{r-1}\ (3x)^{n-r+1}(-4y)^{r-1}$

where: n = 8; r = 6

6th term $= {}_8C_5\ (3x)^3(-4y)^5 = \dfrac{8!}{(8-5)!5!}(27x^3)(-1024y^5)$

6th term $= -\ 1{,}548{,}288\ x^3y^5$

120. Note: To solve the sum of the coefficients of $(2x-1)^{20}$, substitute one (1) to x, calculate, then subtract a value of $(-1)^{20}$ from the result.

Sum of coefficients $= [(2)(1)-1]^{20} - (-1)^{20} = 0$

121. Note: To solve the sum of the coefficients of $(x + y - z)^8$, substitute one(1) to all the variables and calculate.

Sum of coefficients $= (\ 1 + 1 - 1\ )^8 = 1$

122. $\log_8 48 = \dfrac{\log_{10} 48}{\log_{10} 8} = 1.86$

123. $\log_6 845 = \dfrac{\log_{10} 845}{\log_{10} 6} = 3.76$

124. $\log_{10} 1000^{3.3} = 3.3 \log_{10} 1000 = 9.9$

125. $\log_2 5 + \log_3 5 = \dfrac{\log_{10} 5}{\log_{10} 2} + \dfrac{\log_{10} 5}{\log_{10} 3} = 3.79$

126. $\log_4(\log_3 5) = \dfrac{\log_{10}(\log_3 5)}{\log_{10} 4} = \dfrac{\log_{10}\left(\dfrac{\log_{10} 5}{\log_{10} 3}\right)}{\log_{10} 4} = 0.275$

127. $\log_4 \dfrac{1}{7} = \log_4 1 - \log_4 7$

Note: Logarithm of 1 to any base is equal to zero.

$\log_4 \dfrac{1}{7} = 0 - \log_4 7 = -\ n$

128. $\log_a 10 = 0.25$

$$\frac{\log_{10} 10}{\log_{10} a} = 0.25$$

$$\log_{10} a = \frac{\log_{10} 10}{0.25} = 4$$

129. $\log_b y = 2x + \log_b x$

$$\log_b y - \log_b x = 2x$$

$$\frac{\log_{10} y}{\log_{10} b} - \frac{\log_{10} x}{\log_{10} b} = 2x$$

$$\log_{10} y - \log_{10} x = 2x \log_{10} b$$

$$\log_{10} \frac{y}{x} = \log_{10} b^{2x}$$

$$y = xb^{2x}$$

130. $\log_e e^{-7x} = (-7x)\log_e e = (-7x)(1) = -7x$

131. $\log \sqrt[n]{x} = \log(x)^{\frac{1}{n}} = \frac{1}{n}\log(x)$

132. $\log MN = \log M + \log N$

133. $\log (x) - \log (y + z) = \log\left(\frac{x}{y+z}\right)$

134. $\log_b 1024 = \frac{5}{2}$

$$\frac{\log_{10} 1024}{\log_{10} b} = \frac{5}{2}$$

$$\log_{10} b = \frac{\log_{10} 1024}{2.5} = 1.204$$

$$b = \text{anti}\log (1.204) = 16$$

135. $\log_3 (x^2-8x) = 2$

$$\frac{\log_{10}(x^2 - 8x)}{\log_{10} 3} = 2$$

$$\log_{10}(x^2 - 8x) = 2\log_{10} 3 = \log_{10}(3)^2$$

$$\log_{10}(x^2 - 8x) = \log_{10} 9$$

$$x^2 - 8x = 9$$

$$x^2 - 8x - 9 = 0$$

$$(x + 1)(x - 9) = 0$$

$x_1 = -1$
$x_2 = 9$

136. $x^{3\log x} = 100x$

Take logarithm on both sides:

$$\log x^{3\log x} = \log 100x$$
$$3(\log x)(\log x) = \log 100 + \log x$$
$$3(\log x)^2 - \log x - 2 = 0$$
$$(3(\log x) + 2)(\log x - 1) = 0$$
$$\log x_1 = 1$$
$$x_1 = \text{anti}\log(1) = 10$$
$$\log x_2 = -\frac{2}{3}$$
$$x_2 = \text{anti}\log\left(-\frac{2}{3}\right) = 0.215 \text{ (absurd)}$$

137. $\log 6 + x \log 4 = \log 4 + \log (32 + 4^x)$
$\log 6 + \log 4^x = \log 4 + \log (32 + 4^x)$
$\log (6)(4^x) = \log 4(32 + 4^x)$
$6(4^x) = 4(32) + 4(4^x)$
$2(4^x) = 128$
$4^x = 64$

Take logarithm on both sides:
$\log (4)^x = \log 64$
$x \log 4 = \log 64$
$x = 3$

138. $\log_2 2 + \log_2 x = 2$

$$\frac{\log_{10} 2}{\log_{10} 2} + \frac{\log_{10} x}{\log_{10} 2} = 2$$
$$1 + \frac{\log_{10} x}{\log_{10} 2} = 2$$
$$\frac{\log_{10} x}{\log_{10} 2} = 1$$
$$\log_{10} x = \log_{10} 2$$
$$x = 2$$

139. $\log_{12} x = 2$
$x = (12)^2 = 144$

140. $\log 2x^3 + \log \dfrac{6}{x} = 6.278$

$\log 2x^3 + (\log 6 - \log x) = 6.278$

$\log 2x^3 - \log x = 6.278 - \log 6$

$\log \dfrac{2x^3}{x} = 5.49984$

$2x^2 = \text{antilog } (5.49984) = 316111.2849$

$x^2 = 158055.6425$

$x = \pm 397.56$

DAY 4

AGE, WORK, MIXTURE, DIGIT, MOTION PROBLEMS

AGE PROBLEMS

One of the most common problems in Algebra is the age problem. This type of problem must be solve meticulously by giving more emphasis to the tenses (i.e. past, present or future) of the statements.

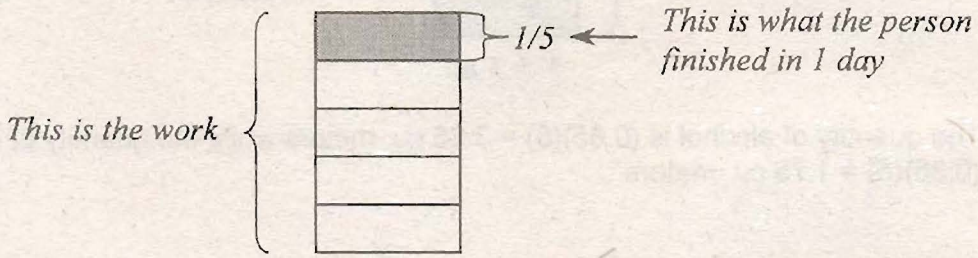
Example: The ages of a certain person in the past, present and future in terms of x are as follows:

6 years ago	Present	5 years hence
x - 6	x	x + 5

WORK PROBLEMS

Suppose that a person can do a certain work in 5 days. This means that the said person can finish 1/5 of the work in one day. Thus, his rate is 1/5 of the work per day.

Illustration:



For a complete job,

$$rt = 1 \quad \text{or} \quad r = \frac{1}{t}$$

where: r = rate of doing the work
t = time to finish the work

When there is a specific work and specific time and manpower, the rate of doing the work may be computed using the number of **man-hour.**

For example:

If 20 bakers can bake 40 pizzas in 8 hours, how many bakers can bake 10 pizzas in 2 hours?

The solution is to get the rate (in man-hour) of baking a pizza:

$$\text{Rate} = \frac{20\ \text{bakers}\,(8\ \text{hours})}{40\text{pizzas}}$$

$$\text{Rate} = \frac{4\text{baker}-\text{hr}}{\text{pizza}}$$

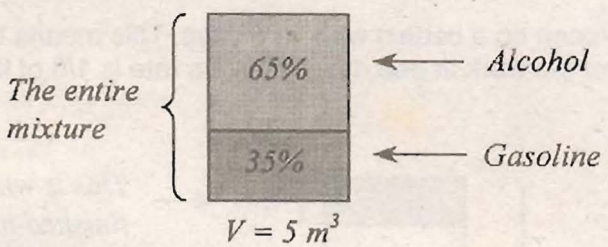
This means that to bake a pizza, you need either 4 bakers in 1 hour or 1 baker in 4 hours.

$$\text{No. of bakers} = \left(\frac{4\ \text{baker}-\text{hour}}{\text{pizza}}\right)\left(\frac{10\ \text{pizzas}}{2\ \text{hours}}\right) = 20\ \text{bakers}$$

MIXTURE PROBLEMS

The easiest way to solve a mixture problem is to draw a rectangle or square which will illustrate the content of the mixture as shown in the following illustration.

Consider a 5 cubic meter mixture containing 65% alcohol and 35% gasoline:



The quantity of alcohol is (0.65)(5) = 3.25 cu. meters while the quantity of gasoline is (0.35)(5) = 1.75 cu. meters.

DIGIT PROBLEMS

Let, h, t and u be the hundreds', tens' and units' digit, respectively. A three-digit number must be represented in the following manner:

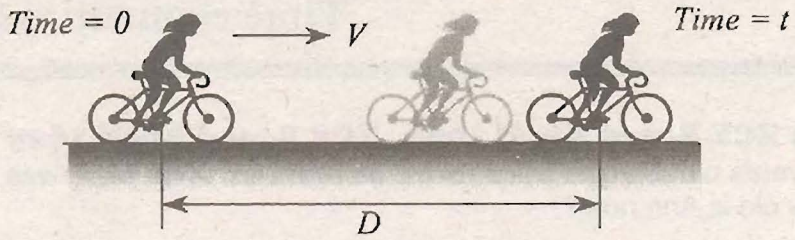
$$\text{Number} = h(100) + t(10) + u$$

A two-digit number is represented by:

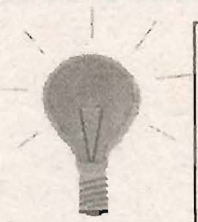
$$\text{Number} = t(10) + u$$

MOTION PROBLEMS

In algebra, the problems pertaining to motion deals only with a uniform velocity, i.e, no acceleration nor deceleration in the process. The following is the relationship between the distance, time and velocity.



$D = Vt$ $\quad V = \frac{D}{t}$ $\quad t = \frac{D}{V}$



Tip: **For problems involving COINS:**
Under American denomination, US Dollar, the following are the coins and their corresponding equivalent:

Penny	= 1 cent
Nickel	= 5 cents
Dime	= 10 cents
Quarter	= 25 cents
Half	= 50 cents

Did you know that... 16th century Italian mathematician and physician Gerolamo Cardano, was the first to introduce the concepts of probability and defined it as the number of favorable outcomes divided by the number of possible outcomes. Because of this he is regarded as "The Father of the Theory of Probability".

Proceed to the next page for your fourth test. GOODLUCK ! ☞

Test 4

Time element: 4 hours

Problem 141: ECE Board April 1995, ECE Board April 1999

Mary is 24 years old. Mary is twice as old as Ann was when Mary was as old as Ann is now. How old is Ann now?

A. 16
B. 18
C. 12
D. 15

Problem 142: EE Board April 1997

The sum of Kim's and Kevin's ages is 18. In 3 years, Kim will be twice as old as Kevin. What are their ages now?

A. 4, 14
B. 5, 13
C. 7, 11
D. 6, 12

Problem 143: GE Board February 1994

Robert is 15 years older than his brother Stan. However "y" years ago, Robert was twice as old as Stan. If Stan is now "b" years old and b>y, find the value of (b – y).

A. 15
B. 16
C. 17
D. 18

Problem 144:

JJ is three times as old as Jan-Jan. Three years ago, JJ was four times as old as Jan-Jan. The sum of their ages is

A. 20
B. 24
C. 28
D. 36

Problem 145:

A girl is one-third as old as her brother and 8 years younger than her sister. The sum of their ages is 38 years. How old is the girl?

A. 4
B. 5
C. 6
D. 7

Problem 146:

Paula is now 18 years old and his colleague Monica is 14 years old. How many years ago was Paula twice as old as Monica?

A. 5
B. 7
C. 8
D. 10

Problem 147:

A father tells his son, "I was your age now when you were born." If the father is now 38 years old, how old was his son 2 years ago?

A. 15
B. 17
C. 19
D. 21

Problem 148:

Six years ago, Nilda was five times as old as Riza. In five years, Nilda will be three times as old as Riza. What is the present age of Riza?

A. 17
B. 16
C. 15
D. 14

Problem 149:

At present, the sum of the parents' ages is twice the sum of the children's ages. Five years ago, the sum of the parents' ages was 4 times the sum of the children's ages. Fifteen years hence, the sum of the parents' ages will be equal to the sum of the children's ages. How many children are there?

A. 3
B. 4
C. 5
D. 6

Problem 150:

Debbie is now twice as old as Jerry. Four years ago, Debbie was three times as old as Jerry then. How old is Debbie?

A. 14
B. 16
C. 18
D. 24

Problem 151: ME Board April 1998

A pump can pump out water from a tank in 11 hours. Another pump can pump out water from the same tank in 20 hours. How long will it take both pumps to pump out the water in the tank?

A. 7 hours
B. 6 hours
C. 7 1/2 hours
D. 6 1/2 hours

Problem 152: CE Board November 1993

A 400-mm ∅ pipe can fill the tank alone in 5 hours and another 600-mm ∅ pipe can fill the tank alone in 4 hours. A drain pipe 300-mm ∅ can empty the tank in 20 hours. With all the three pipes open, how long will it take to fill the tank?

A. 2.00 hours
B. 2.50 hours
C. 2.25 hours
D. 2.75 hours

Problem 153:

A tank is filled with an intake pipe in 2 hours and emptied by an outlet pipe in 4 hours. If both pipes are opened, how long will it take to fill the empty tank?

A. 3 hours
B. 4 hours
C. 5 hours
D. 6 hours

Problem 154:

A tank can be filled in 9 hours by one pipe, 12 hours by a second pipe and can be drained when full by a third pipe in 15 hours. How long will it take to fill an empty tank with all pipes in operation?

A. 7 hours and 12 minutes
B. 7 hours and 32 minutes
C. 7 hours and 42 minutes
D. 7 hours and 50 minutes

Problem 155: ME Board April 1995

If A can do the work in "x" days and B in "y" days, how long will they finish the job working together?

A. $\frac{x+y}{xy}$

B. $\frac{x+y}{2}$

C. $\frac{xy}{x+y}$

D. $\sqrt{xy}$

Problem 156: ECE Board November 1995

Pedro can paint a fence 50% faster than Juan and 20% faster than Pilar, and together they can paint a given fence in 4 hours. How long will it take Pedro to paint the same fence if he had to work alone?

A. 6
B. 8
C. 10
D. 12

Problem 157:

Glenn can paint a house in 9 hours while Stewart can paint the same house in 16 hours. They work together for 4 hours. After 4 hours, Stewart left and Glenn finished the job alone. How many more days did it take Glenn to finish the job?

A. 2.75 hours
B. 2.50 hours
C. 2.25 hours
D. 3.00 hours

Problem 158: CE Board November 1993

It takes Butch twice as long as it takes Dan to do a certain piece of work. Working together they can do the work in 6 days. How long would it take Dan to do it alone?

A. 9 days
B. 10 days
C. 11 days
D. 12 days

Problem 159: ME Board April 1995

A and B working together can finish painting a house in 6 days. A working alone can finish it in 5 days less than B. How long will it take each of them to finish the work alone?

A. 8, 13
B. 10, 15
C. 6, 11
D. 7, 12

Problem 160: EE Board April 1996

A and B can do a piece of work in 42 days, B and C in 31 days and C and A in 20 days. In how many days can all of them do the work together?

A. 19
B. 17
C. 21
D. 15

Problem 161:

It takes Myline twice as long as Jeana to do a certain piece of work. Working together, they can finish the work in 6 hours. How long would it take Jeana to do it alone?

A. 9 hours
B. 18 hours
C. 12 hours
D. 14 hours

Problem 162: ECE Board April 1999

Mike, Loui and Joy can mow the lawn in 4, 6 and 7 hours respectively. What fraction of the yard can they mow in 1 hour if they work together?

A. 47/84
B. 45/84
C. 84/47
D. 39/60

Problem 163:

A farmer can plow the field in 8 days. After working for 3 days, his son joins him and together they plow the field in 3 more days. How many days will it require for the son to plow the field alone?

A. 10
B. 11
C. 12
D. 13

Problem 164: ECE Board November 1991

Crew No. 1 can finish installation of an antenna tower in 200 man-hour while Crew No. 2 can finish the same job in 300 man-hour. How long will it take both crews to finish the same job, working together?

A. 100 man-hour
B. 120 man-hour
C. 140 man-hour
D. 160 man-hour

Problem 165: ME Board October 1994

On one job, two power shovels excavate 20,000 cubic meters of earth, the larger shovel working 40 hours and the smaller for 35 hours. On another job, they removed 40,000 cubic meters with the larger shovel working 70 hours and the smaller working 90 hours, How much earth can each remove in 1 hour working alone?

A. 169.2, 287.3
B. 178.3, 294.1
C. 173.9, 347.8
D. 200.1, 312.4

Problem 166: EE Board October 1997

Ten liters of 25 % salt solution and 15 liters of 35 % salt solution are poured into a drum originally containing 30 liters of 10% salt solution. What is the per cent concentration of salt in the mixture?

A. 19.55 %
B. 22.15 %
C. 27.05 %
D. 25.72 %

Problem 167: ME Board October 1992

A Chemist of a distillery experimented on two alcohol solutions of different strength, 35% alcohol and 50% alcohol, respectively. How many cubic meters of each strength must he use in order to produce a mixture of 60 cubic meters that contain 40% alcohol?

A. 20 m^3 of solution with 35% alcohol, 40 m^3 of solution with 50% alcohol
B. 50 m^3 of solution with 35% alcohol, 20 m^3 of solution with 50% alcohol
C. 20 m^3 of solution with 35% alcohol, 50 m^3 of solution with 50% alcohol
D. 40 m^3 of solution with 35% alcohol, 20 m^3 of solution with 50% alcohol

Problem 168:

A goldsmith has two alloys of gold, the first being 70% pure and the second being 60% pure. How many ounces of the 60% pure gold must be used to make 100 ounces of an alloy which will be 66% gold?

A. 40
B. 35

C. 45
D. 38

Problem 169: ME Board October 1994

Two thousand (2000) kg of steel containing 8% nickel is to be made by mixing a steel containing 14% nickel with another containing 6% nickel. How much of each is needed?

A. 1500 kg of steel with 14% nickel, 500 k g of steel with 6% nickel
B. 750 kg of steel with 14% nickel, 1250 k g of steel with 6% nickel
C. 500 kg of steel with 14% nickel, 1500 k g of steel with 6% nickel
D. 1250 kg of steel with 14% nickel, 750 k g of steel with 6% nickel

Problem 170:

How much water must be evaporated from 10 kg solution which has 4% salt to make a solution of 10% salt?

A. 4 kg
B. 5 kg
C. 6 kg
D. 7 kg

Problem 171: EE Board October 1994

If a two digit number has x for its unit's digit and y for its ten's digit, represent the number.

A. $10x + y$
B. $10y + x$
C. yx
D. xy

Problem 172: EE Board October 1994

One number is 5 less than the other. If their sum is 135, what are the numbers?

A. 85, 50
B. 80, 55
C. 70, 65
D. 75, 60

Problem 173: ECE Board March 1996

Ten less than four times a certain number is 14. Determine the number.

A. 6
B. 7
C. 8
D. 9

Problem 174: ECE Board March 1996

The sum of two numbers is 21 and one number is twice the other. Find the numbers.

A. 6, 15
B. 7,14
C. 8,13
D. 9,12

Problem 175: EE Board April 1993

If eight is added to the product of nine and the numerical number, the sum is seventy-one. Find the unknown number.

A. 5
B. 6
C. 7
D. 8

Problem 176:

Find the fraction such that if 2 is subtracted from its terms its becomes 1/4, but if 4 is added to its terms it becomes 1/2.

A. 3/5
B. 5/12
C. 5/14
D. 6/13

Problem 177: GE Board February 1992

The product of $\frac{1}{4}$ and $\frac{1}{5}$ of a number is 500. What is the number?

A. 50
B. 75
C. 100
D. 125

Problem 178:

If 3 is subtracted from the numerator of a certain fraction, the value of the fraction becomes 3/5. If 1 is subtracted from the denominator of the same fraction, it becomes 2/3. Find the original fraction.

A. 35/55
B. 36/55
C. 3/7
D. 32/41

Problem 179: ECE Board November 1997

The denominator of a certain fraction is three more than twice the numerator. If 7 is added to both terms of the fraction, the resulting fraction is 3/5. Find the original fraction.

A. 8/5
B. 13/5
C. 5/13
D. 3/5

Problem 180:

Find the product of two numbers such that twice the first added to the second equals 19 and three times the first is 21 more than the second.

A. 24
B. 32
C. 18
D. 20

Problem 181:

The tens' digit of a number is 3 less than the units' digit. If the number is divided by the sum of the digits, the quotient is 4 and the remainder is 3. What is the original number?

A. 36
B. 47
C. 58
D. 69

Problem 182:

The second of the four numbers is three less than the first, the third is four more than the first and the fourth is two more than the third. Find the fourth number if their sum is 35.

A. 10
B. 11
C. 12
D. 13

Problem 183: EE Board April 1997

A jogger starts a course at a steady rate of 8 kph. Five minutes later, a second jogger starts the same course at 10 kph. How long will it take the second jogger to catch the first?

A. 20 min
B. 21 min
C. 22 min
D. 18 min

Problem 184: EE Board April 1997

A boat man rows to a place 4.8 miles with the stream and back in 14 hours, but finds that he can row 14 miles with the stream in the same time as 3 miles against the stream. Find the rate of the stream.

A. 1.5 miles per hour
B. 1 mile per hour
C. 0.8 mile per hour
D. 0.6 mile per hour

Problem 185: ECE Board November 1998

A man rows downstream at the rate of 5 mph and upstream at the rate of 2 mph. How far downstream should he go if he is to return in 7/4 hours after leaving?

A. 2.5 miles
B. 3.3 miles
C. 3.1 miles
D. 2.7 miles

Problem 186: CE Board November 1994

An airplane flying with the wind, took 2 hours to travel 1000 km and 2.5 hours in flying back. What was the wind velocity in kph?

A. 50
B. 60
C. 70
D. 40

Problem 187: CE Board May 1998

A boat travels downstream in 2/3 of the time as it goes going upstream. If the velocity of the river's current is 8 kph, determine the velocity of the boat in still water.

A. 40 kph
B. 50 kph
C. 30 kph
D. 60 kph

Problem 188:

Two planes leave Manila for a southern city, a distance of 900 km. Plane A travels at a ground speed of 90 kph faster than the plane B. Plane A arrives in their destination 2 hours and 15 minutes ahead of Plane B. What is the ground speed of plane A?

A. 205 kph
B. 315 kph
C. 240 kph
D. 287 kph

Problem 189: EE Board April 1997

A train, an hour after starting, meets with an accident which detains it an hour, after which it proceeds at 3/5 of its former rate and arrives three hour after time; but had the accident happened 50 miles farther on the line, it would have arrived one and one-half hour sooner. Find the length of the journey.

A. 910/9 miles
B. 800/9 miles
C. 920/9 miles
D. 850/9 miles

Problem 190:

On a certain trip, Edgar drive 231 km in exactly the same time as Erwin drive 308 km. If Erwin's rate exceeded that of Edgar by 13 kph, determine the rate of Erwin.

A. 39 kph
B. 44 kph
C. 48 kph
D. 52 kph

ANSWER KEY

141. B	154. D	167. D	180. A
142. B	155. C	168. A	181. B
143. A	156. C	169. C	182. D
144. D	157. A	170. C	183. A
145. C	158. A	171. B	184. C
146. D	159. B	172. C	185. A
147. B	160. A	173. A	186. A
148. A	161. A	174. B	187. A
149. C	162. A	175. C	188. C
150. B	163. C	176. C	189. B
151. A	164. B	177. C	190. D
152. B	165. C	178. B	
153. B	166. A	179. C	

RATING

☐ 43-50 Topnotcher
☐ 33-42 Passer
☐ 25-32 Conditional
☐ 0-24 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 4

141. $24 - x = x - 12$

$36 = 2x$

$x = 18$ years old

	Past	*Present*
Mary	*x*	*24*
Ann	*12*	*x*

142. $x + y = 18$

$y = 18 - x$ ☞ ❶

$(y + 3) = 2(x + 3)$ ☞ ❷

Substitute y in equation (2):

$(18 - x) + 3 = 2x + 6$

$21 - x = 2x + 6$

$x = 5$ years old

$y = 18 - 5$

$y = 13$ years old

	Present	*Future*
Kevin	*x*	*x +3*
Kim	*y*	*y + 3*

143. $(b + 15 - y) = 2(b - y)$

$b + 15 - y = 2b - 2y$

$2b - b - 2y + y = 15$

$b - y = 15$

	Past	*Present*
Robert	*b+ 15 - y*	*b + 15*
Stan	*b - y*	*b*

144. $3x - 3 = 4(x - 3)$

$3x - 3 = 4x - 12$

$x - 12 = - 3$

$x = 9$ years old.

$3x = 3(9) = 27$ years old

Sum of ages = 9 + 27 = 36 years

	Past	*Present*
JJ	*3x -3*	*3x*
Jan-jan	*x - 3*	*x*

145. Let: x = age of the girl

y = age of her brother

z = age of her sister

$y = 3x$ ☞ ❶

$z = x + 8$ ☞ ❷

$x + y + z = 38$ ☞ ❸

Substitute (1) and (2) in (3):

$x + (3x) + (x + 8) = 38$

$x = 6$ years old

146. $(18 - x) = 2(14 - x)$

$18 - x = 28 - 2x$

$x = 10$ years ago

	Past	*Present*
Paula	*18 – x*	*18*
Monica	*14 - x*	*14*

147. $38 - x = x - 0$

$x = 19$ years old

Two years ago, the son was $(19 - 2) = 17$ years old

	Past	Present
Father	x	38
Son	0	x

148. $3(x + 5) - 5(x - 6) = x + 5 - (x - 6)$

$3x + 15 - 5x + 30 = x + 5 - x + 6$

$-2x + 45 = 11$

$x = 17$ years old

	Past	Present	Future
Nilda	$5(x-6)$	?	$3(x+5)$
Riza	$x-6$	x	$x+5$

149. $2x - 10 = 4(x - 5n)$

$2x - 10 = 4x - 20n$

$2x = 20n - 10$

$x = 10n - 5$ ☞ ❶

$2x + 30 = x + 15n$

$x = 15n - 30$ ☞ ❷

Equate (1) and (2):

$15n - 30 = 10n - 5$

$n = 5$ children

	Past	Present	Future
Parents	$2x-10$	$2x$	$2x+30$
Children	$x-5n$	x	$x+15n$

150. $2x - 3(x - 4) = x - (x - 4)$

$2x - 3x + 12 = x - x + 4$

$-x + 12 = 4$

$x = 8$ years old

$2x = 16$ years old

Thus, Debbie is now 16 years old.

	Past	Present
Debbie	$3(x-4)$	$2x$
Jerry	$x-4$	x

151. Let: x = time needed to complete the work

$$\frac{1}{20} + \frac{1}{11} = \frac{1}{x}$$

$x = 7.096$ hours

152. $$\frac{1}{5} + \frac{1}{4} - \frac{1}{20} = \frac{1}{x}$$

$x = 2.5$ hours

153. $$\frac{1}{2} - \frac{1}{4} = \frac{1}{x}$$

$x = 4$ hours

154. $$\frac{1}{9} + \frac{1}{12} - \frac{1}{15} = \frac{1}{x}$$

$x = 7.826$ hours = 7 hours & 0.826(60) min

$x = 7$ hours and 50 minutes

155. Let: n = number of days needed to complete the work

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{n}$$

$$\frac{1}{n}=\frac{x+y}{xy}$$

$$n=\frac{xy}{x+y}$$

156. Let: A = number of hours, Pedro can paint the house
B = number of hours, Juan can paint the house
C = number of hours, Pilar can paint the house

$$\frac{1}{A}+\frac{1}{B}+\frac{1}{C}=\frac{1}{4} \quad \text{☞ ❶}$$

$$\frac{1}{A}=1.5\left(\frac{1}{B}\right);\ \frac{1}{B}=0.666\left(\frac{1}{A}\right) \quad \text{☞ ❷}$$

$$\frac{1}{A}=1.2\left(\frac{1}{C}\right);\ \frac{1}{C}=0.833\left(\frac{1}{A}\right) \quad \text{☞ ❸}$$

Substitute (2) and (3) in (1):

$$\frac{1}{A}+0.666\left(\frac{1}{A}\right)+0.833\left(\frac{1}{A}\right)=\frac{1}{4}$$

A = 10 hours

157. Note: (rate)(time) = 1(complete job)

$$\left(\frac{1}{9}+\frac{1}{16}\right)4+\frac{1}{9}(x)=1$$

$$0.6944 + 0.111\,x = 1$$

$$x = 2.75 \text{ hours}$$

158. Let: x = time for Butch to finish a certain job working alone
y = time for Dan to finish a certain job working alone

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{6} \quad \text{☞ ❶}$$

$$x = 2y \quad \text{☞ ❷}$$

Substitute (2) in (1):

$$\frac{1}{2y}+\frac{1}{y}=\frac{1}{6}$$

$$\frac{1+2}{2y}=\frac{1}{6}$$

y = 9 days

159. $\frac{1}{A}+\frac{1}{B}=\frac{1}{6}$ ☞ ❶

$A = B - 5$ ☞ ❷

Substitute (2) in (1):

$$\frac{1}{B-5}+\frac{1}{B}=\frac{1}{6}$$

$$\frac{B+(B-5)}{B(B-5)}=\frac{1}{6}$$

$$\frac{2B-5}{B^2-5B}=\frac{1}{6}$$

$$12B - 30 = B^2 - 5B$$

$$B^2 - 17B + 30 = 0$$

$$(B - 15)(B - 2) = 0$$

$B = 15$ days

$B = 2$ (absurd)

Substitute $B = 15$ in (2):

$A = 15 - 5 = 10$ days

160. Let: x = number of days needed by A, B and C to finished the work working together.

$$\frac{1}{A}+\frac{1}{B}+\frac{1}{C}=\frac{1}{x}$$

$\frac{1}{A}+\frac{1}{B}=\frac{1}{42}$ ☞ ❶

$\frac{1}{B}+\frac{1}{C}=\frac{1}{31}$ ☞ ❷

$\frac{1}{A}+\frac{1}{C}=\frac{1}{20}$ ☞ ❸

Add the three equations:

$$\left[\frac{1}{A}+\frac{1}{B}\right]+\left[\frac{1}{B}+\frac{1}{C}\right]+\left[\frac{1}{A}+\frac{1}{C}\right]=\frac{1}{42}+\frac{1}{31}+\frac{1}{20}=0.106$$

$$\frac{2}{A}+\frac{2}{B}+\frac{2}{C}=0.106$$

$$\frac{1}{A}+\frac{1}{B}+\frac{1}{C}=0.053=\frac{1}{x}$$

$x = 18.87$ days (approximately 19 days)

161. Let: x = time for Myline to finish the job

y = time for Jeana to finish the job

$\frac{1}{x}+\frac{1}{y}=\frac{1}{6}$ ☞ ❶

$x = 2y$ ☞ ❷

Substitute (2) in (1):

$$\frac{1}{2y}+\frac{1}{y}=\frac{1}{6}$$

$$\frac{3}{2y}=\frac{1}{6}$$

$$y = 9 \text{ hours}$$

162. Let: x = fraction of the lawn that can mowed after one hour

$$x=\left(\frac{1}{4}+\frac{1}{6}+\frac{1}{7}\right)(1)=\frac{42+28+24}{168}=\frac{94}{168}=\frac{47}{84}$$

163. Let: x = time for the farmer to flow the field
y = time for the son to flow the field,

$$\frac{1}{x}(3)+\left[\frac{1}{x}+\frac{1}{y}\right](3)=1 \quad \text{☞ ❶}$$

Substitute x = 8 in (1):

$$\left[\frac{1}{8}(3)+\left[\frac{1}{8}+\frac{1}{y}\right](3)=1\right]\frac{1}{3}$$

$$\frac{1}{8}+\frac{1}{8}+\frac{1}{y}=\frac{1}{3}$$

$$y = 12 \text{ days}$$

164. Let: x = number of man-hours needed by crew number 1 and number 2 to finish the job.

$$\frac{1}{200}+\frac{1}{300}=\frac{1}{x}$$

$$x = 120 \text{ man-hours}$$

165. Let: x = capacity of the larger shovel in m^3/hr
y = capacity of the smaller shovel in m^3/hr

$$40x + 35y = 20{,}000$$

$$x = 500 - 0.875y \quad \text{☞ ❶}$$

$$70x + 90y = 40{,}000 \quad \text{☞ ❷}$$

Substitute (1) in (2);

$$70(500 - 0.875y) + 90y = 40{,}000$$

$$35{,}000 - 61.25y + 90y = 40{,}000$$

$$y = 173.9 \text{ m}^3/\text{hr}$$

$$x = 500 - 0.875(173.9) = 347.8 \text{ m}^3/\text{hr}$$

166.

25%	+	35%	+	10%	=	x%
10		15		30		55

$$0.25(10) + 0.35(15) + 0.10(30) = x(55)$$
$$x = 19.55\ \%$$

167.

35%	+	50%	=	40%
x		$60-x$		60

$$0.35(x) + 0.50(60 - x) = 0.40(60)$$
$$0.35x + 30 - 0.5x = 24$$
$$x = 40\ m^3 \quad \text{☞ for the 35\% solution}$$
$$60 - x = 20\ m^3 \quad \text{☞ for the 50\% solution}$$

168.

70%	+	60%	=	66%
$100-x$		x		100

$$0.70(100 - x) + 0.60(x) = 0.66(100)$$
$$70 - 0.7x + 0.6x = 66$$
$$x = 40 \text{ ounces}$$

169.

14%	+	6%	=	8%
x		$2000-x$		2000

$$0.14(x) + 0.06(2000 - x) = 0.08(2000)$$
$$0.14x + 120 - 0.06x = 160$$
$$x = 500\ kg \quad \text{☞ for the 14\% substance}$$
$$2000 - x = 1500\ kg \quad \text{☞ for the 6\% substance}$$

170.

4%	−	0%	=	10%
10		x		$10-x$

$$0.04(10) - 0(x) = 0.10(10 - x)$$
$$0.4 = 1 - 0.1x$$
$$x = 6\ kg$$

171. Let: y = tens' digit of the number
x = units' digit of the number
The two digit number is represented by: $10y + x$

172. Let: x = the first number
$x - 5$ = the second number

$$x + (x - 5) = 135$$
$$2x = 140$$
$$x = 70$$
$$x - 5 = 65$$

Thus, the numbers are 70 and 65.

173. Let: x = the number

$4x - 10 = 14$

$x = 6$

174. Let: x = the first number

2x = the second number

$x + 2x = 21$

$x = 7$

$2x = 14$

Thus, the numbers are 7 and 14.

175. Let: x = the number

$9x + 8 = 71$

$x = 7$

176. Let: $\frac{x}{y}$ = the fraction

$$\frac{x-2}{y-2} = \frac{1}{4}$$

$4x - 8 = y - 2$

$y = 4x - 6$ ☞ ❶

$$\frac{x+4}{y+4} = \frac{1}{2}$$

$2x + 8 = y + 4$ ☞ ❷

Substitute (1) in (2):

$2x + 8 = (4x - 6) + 4$

$10 = 2x$

$x = 5$

$y = 4(5) - 6 = 14$

Thus, the fraction is $\frac{5}{14}$.

177. Let: x = the number

$$\left[\frac{1}{4}(x)\right]\left[\frac{1}{5}(x)\right] = 500$$

$$\frac{x^2}{20} = 500$$

$x^2 = 10{,}000$

$x = 100$

178. Let: $\frac{x}{y}$ = the fraction

$$\frac{x-3}{y}=\frac{3}{5}$$

$$3y = 5x - 15$$

$$y = \frac{5}{3}x - 5 \qquad \text{☞ ❶}$$

$$\frac{x}{y-1}=\frac{2}{3}$$

$$3x = 2y - 2 \qquad \text{☞ ❷}$$

Substitute (1) in (2):

$$3x = 2\left[\frac{5}{3}x - 5\right] - 2$$

$$3x = \frac{10}{3}x - 10 - 2$$

$$12 = 0.333\,x$$

$$x = 36$$

$$y = \frac{5}{3}(36) - 5 = 55$$

Thus, the fraction is $\frac{36}{55}$

179. Let: x = numerator of the faction
y = denominator of the fraction

$$y = 2x + 3 \qquad \text{☞ ❶}$$

$$\frac{x+7}{y+7}=\frac{3}{5}$$

$$5x + 35 = 3y + 21 \qquad \text{☞ ❷}$$

Substitute (1) in (2):

$$5x + 35 = 3(2x + 3) + 21$$

$$5x + 35 = 6x + 9 + 21$$

$$x = 5$$

$$y = 2(5) + 3 = 13$$

Thus, the original fraction is $\frac{5}{13}$

180. Let: x = the first number
y = the second number

$$2x + y = 19$$

$$y = 19 - 2x \qquad \text{☞ ❶}$$

$$3x = y + 21 \qquad \text{☞ ❷}$$

Substitute (1) in (2):

$3x = (19 - 2x) + 21$

$5x = 40$

$x = 8$

$y = 19 - 2(8) = 3$

Thus, the product of the numbers is $8(3) = 24$.

181. Let: t = ten's digit of the number

u = unit's digit of the number

$10t + u$ = the number

$$t = u - 3 \qquad ☞ ❶$$

$$\frac{10t+u}{t+u} = 4 + \frac{3}{t+u}$$

$$\frac{10t+u-3}{t+u} = 4$$

$$10t + u - 3 = 4t + 4u$$

$$6t - 3 = 3u \qquad ☞ ❷$$

Substitute (1) in (2):

$6(u - 3) - 3 = 3u$

$6u - 18 - 3 = 3u$

$3u = 21$

$u = 7$

$t = 7 - 3 = 4$

Thus the number is $10t + u = 10(4) + 7 = 47$

182. Let: w = first number

x = second number

y = third number

z = fourth number

$x = w - 3 \qquad ☞ ❶$

$y = w + 4 \qquad ☞ ❷$

$z = y + 2$

$= (w + 4) + 2$

$z = w + 6 \qquad ☞ ❸$

$w + x + y + z = 35 \qquad ☞ ❹$

Substitute (1),(2) and (3) in (4):

$w + (w - 3) + (w + 4) + (w + 6) = 35$

$4w = 28$

$w = 7$

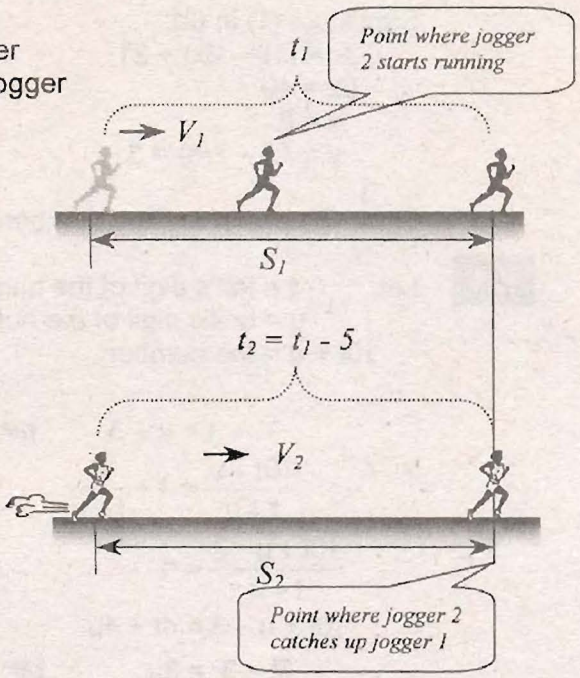
Substitute $w = 7$ in (3):

$z = 7 + 6 = 13$

183. Let: V_1 = velocity of the first jogger
V_2 = velocity of the second jogger

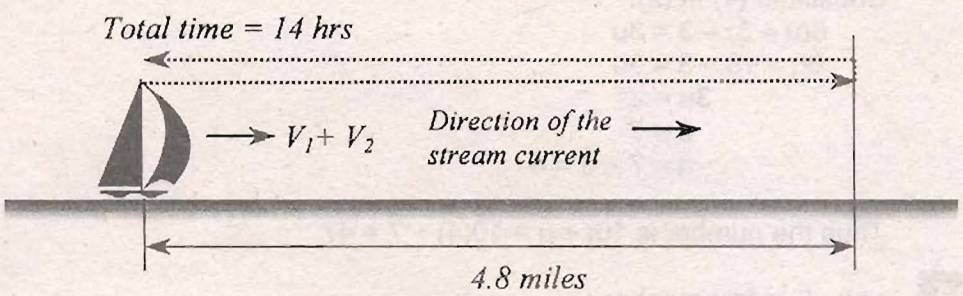
$$V_1 = \frac{8\text{km}}{\text{hr}} \times \frac{1\text{hr}}{60\,\text{min}} = \frac{8}{60}\,\text{km/min}$$

$$V_2 = \frac{10}{60}\,\text{km/min}$$



$$S_1 = S_2$$
$$V_1 t_1 = V_2 t_2$$
$$\frac{8}{60}(t_1) = \frac{10}{60}(t_1 - 5)$$
$$8t_1 = 10t_1 - 50$$
$$t_1 = 25 \text{ minutes}$$
$$t_2 = 25 - 5 = 20 \text{ minutes}$$

184. Let: V_1 = velocity of boatman
V_2 = velocity of stream



$$\frac{4.8}{V_1 + V_2} + \frac{4.8}{V_1 - V_2} = 14$$

$$\frac{4.8(V_1 + V_2) + 4.8(V_1 + V_2)}{(V_1 + V_2)(V_1 - V_2)} = 14$$

$$\frac{4.8V_1 - 4.8V_2 + 4.8V_1 + 4.8V_2}{V_1^2 - V_1V_2 + V_1V_2 - V_2^2} = 14$$

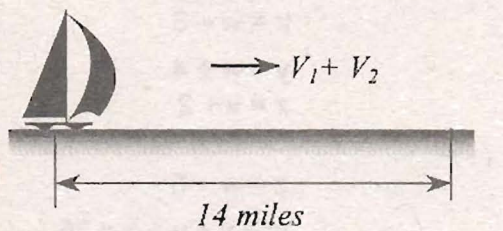
$$9.6V_1 = 14V_1^2 - 14V_2^2 \quad \text{☞ ❶}$$

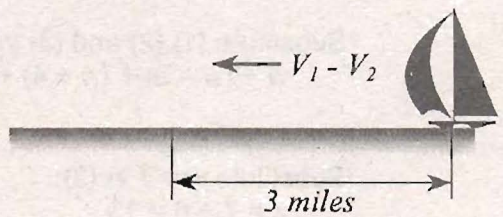
$$\frac{14}{V_1 + V_2} = \frac{3}{V_1 - V_2}$$

$$14V_1 - 14V_2 = 3V_1 + 3V_2$$
$$11V_1 = 17V_2$$
$$\therefore V_1 = 1.545V_2 \quad \text{☞ ❷}$$

Substitute (2) in (1):

The two figures below have the same time, t:



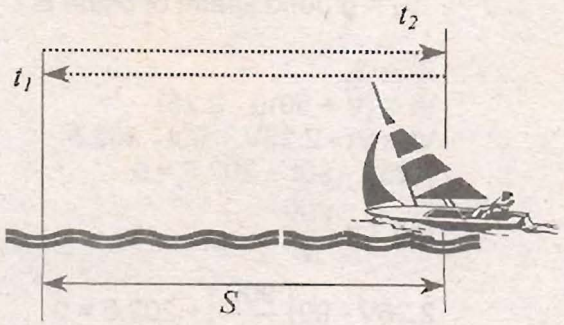


$9.6(1.545V_2) = 14(1.545V_2)^2 - 14V_2^2$

$14.832V_2 = 19.418V_2^2$

$V_2 = 0.76$ mph

185. Note: $\text{time} = \dfrac{\text{distance}}{\text{velocity}}$



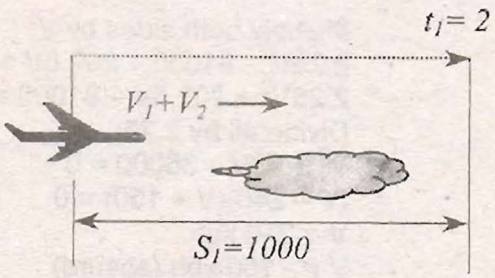
$t_1 + t_2 = t_{total}$

$\dfrac{S}{5} + \dfrac{S}{2} = \dfrac{7}{4}$

$0.7(S) = 1.75$

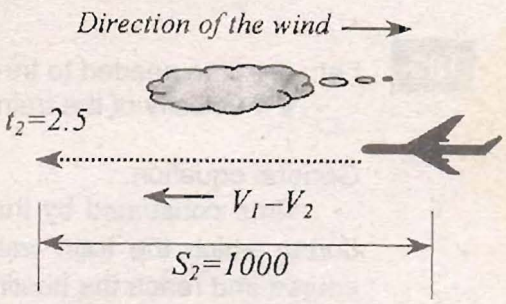
$S = 2.5$ miles

186. Let: V_1 = velocity of airplane
V_2 = velocity of wind



$V_1 + V_2 = \dfrac{1000}{2} = 500$ ☞ ❶

$V_1 - V_2 = \dfrac{1000}{2.5} = 400$ ☞ ❷



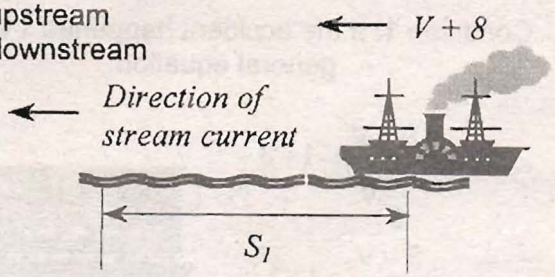
Subtract (2) form (1):

$(V_1 + V_2) - (V_1 - V_2) = 500 - 400$

$2V_2 = 100$

$V_2 = 50$ kph

187. Let: V = velocity of the boat in still water
S_1 = distance traveled upstream
S_2 = distance traveled downstream

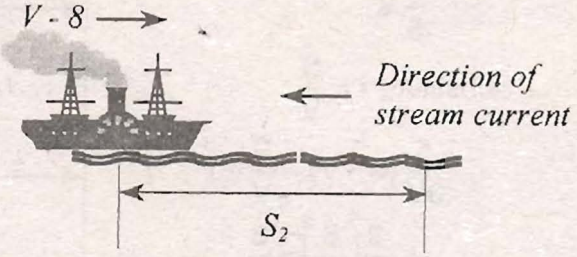


$S_1 = S_2$

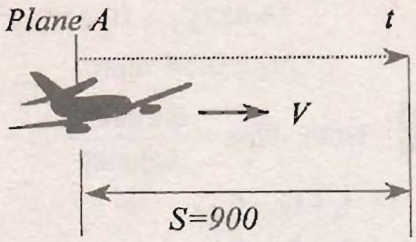
$(V - 8)(t) = (V + 8)\left[\dfrac{2}{3}t\right]$

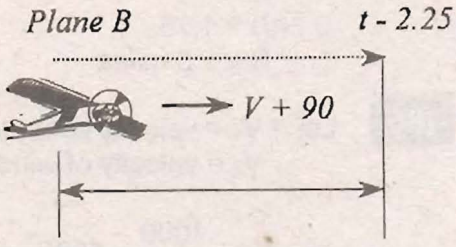
$V - 8 = \dfrac{2}{3}V + \dfrac{16}{3}$

$V = 40$ kph



188. Let: V_1 = ground speed of plane A
V_2 = ground speed of plane B





$S_1 = S_2$
$Vt = (V + 90)(t - 2.25)$
$Vt = Vt - 2.25V + 90t - 202.5$
$2.25V - 90t + 202.5 = 0$

But $t = \dfrac{900}{V}$

$$2.25V - 90\left(\frac{900}{V}\right) + 202.5 = 0$$

Multiply both sides by V:
$2.25V^2 - 81000 + 202.5V = 0$
$2.25V^2 + 202.5V - 81000 = 0$
Divide all by 2.25:
$V^2 + 90V - 36000 = 0$
$(V - 240)(V + 150) = 0$
$V = 240$ kph
$V = -150$ kph (absurd)

189. Let: t = time needed to travel and reach destination without any delay
V = velocity of the train

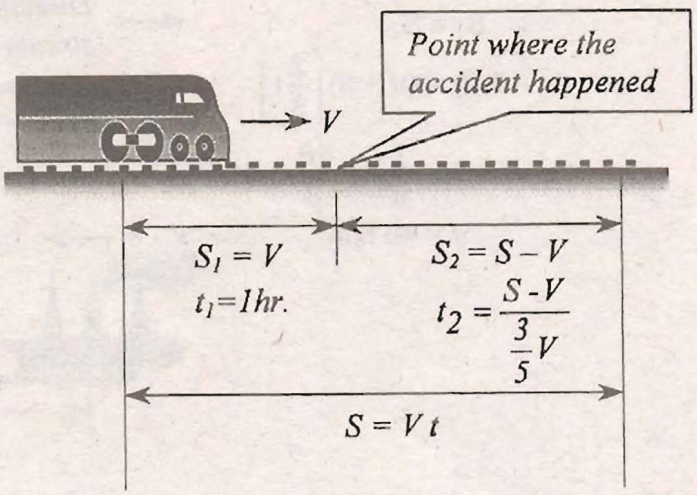
General equation:

Time consumed by the train traveling, before the accident + Time during which the train was detained + Time needed to continue the course and reach the destination = Time needed to travel and reach the destination without any delay + Time of delay

Condition 1: If the accident happened 1 hour after, substitute values to the general equation:

$$1 + 1 + \frac{S - V}{\frac{3}{5}V} = t + 3$$

$$\frac{S - V}{\frac{3}{5}V} = t + 1$$



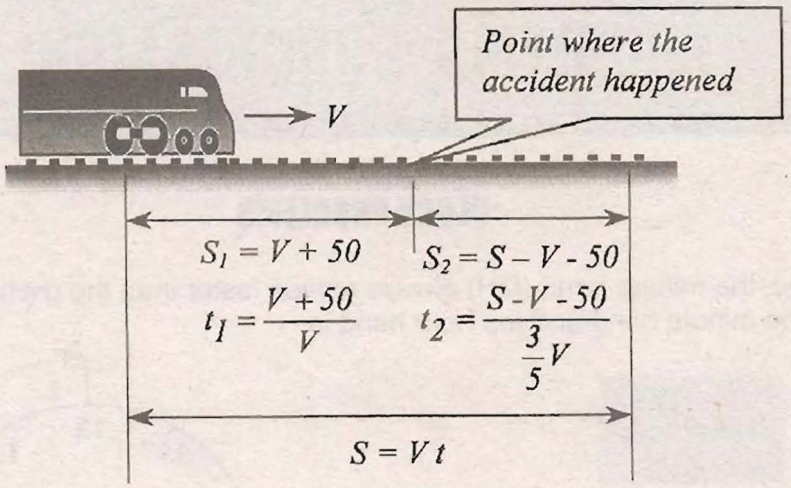
Substitute $t = \dfrac{S}{V}$:

$$\frac{5}{3V}(S - V) = \frac{S}{V} + 1$$

$$\frac{2}{3}\left(\frac{S}{V}\right) = \frac{8}{3}$$

$$V = \frac{S}{4}$$ ☞ ❶

Condition 2: If the accident happened 50 miles farther, substitute values to the general equation:



$$\frac{50+V}{V} + 1 + \frac{S-(50+V)}{\frac{3}{5}V} = t + \frac{3}{2}$$

$$\left[\frac{50+V}{V} + \frac{5}{3V}(S-50-V) = \frac{S}{V} + \frac{1}{2}\right]V$$

$$50 + V + \frac{5}{3}S - \frac{250}{3} - \frac{5}{3}V = S + \frac{V}{2}$$ ☞ ❷

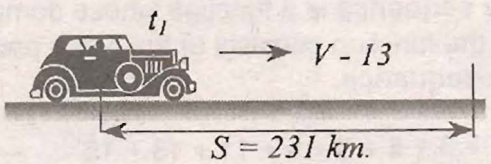
Substitute (1) in (2):

$$50 + \frac{S}{4} + \frac{5}{3}S - \frac{250}{3} - \frac{5}{3}\left(\frac{S}{4}\right) = S + \frac{1}{2}\left(\frac{S}{4}\right)$$

$$-\frac{100}{3} = -\frac{3}{8}S$$

$$S = \frac{800}{9} \text{ miles}$$

190. Let: V = rated of Erwin
V – 13 = rate of Edgar

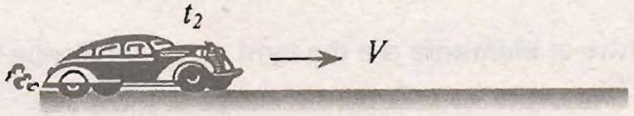


$t_1 = t_2$

$$\frac{231}{V-13} = \frac{308}{V}$$

$231V = 308V - 4004$

$V = 52$ kph



DAY 5

CLOCK, VARIATION & MISCELLANEOUS PROBLEMS , PROGRESSION

CLOCK PROBLEMS

By principle, the minute hand (MH) always moves faster than the (HH). The relation between the minute hand and the hour hand is

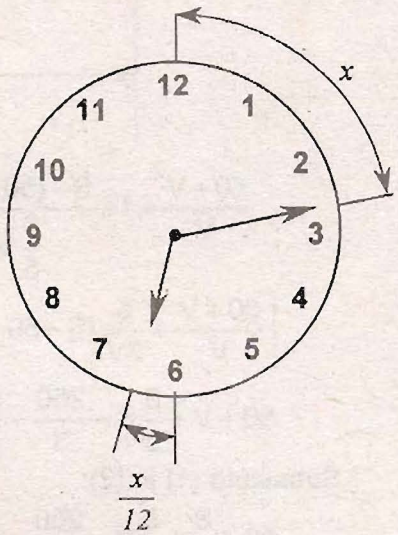
$$HH = \frac{MH}{12}$$

where : MH is in number of minutes

Also, the hour hand in terms of second hand is expressed as

$$HH = \frac{SH}{720}$$

where: SH is in number of seconds



PROGRESSION / SEQUENCE & SERIES

A **sequence** or **progression** is a set or collection of numbers arranged in an orderly manner such that the preceding and the following numbers are completely specified.

An **infinite sequence** is a function whose domain is the set of positive integer. If the domain of the function consists of the first n positive integers only, then it is said to be a **finite sequence.**

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 \longrightarrow$ Finite sequence

$1 + 3 + 5 + 7 + 9 + 11 + 13 + \cdots \longrightarrow$ Infinite sequence

Terms or **elements** are the term used to describe the numbers in a given sequence.

Series is the sum of the terms in a sequence. An **alternating series** has positive and negative terms arranged alternately. If an infinite series has a finite sum, it is referred to as **convergent series** and **divergent series** if it has no sum at all.

The most common types of sequence are Arithmetic, Harmonic and Geometric Progression.

ARITHMETIC PROGRESSION (A.P.):

A sequence is said to be in **arithmetic progression** if its succeeding terms have a common difference. The corresponding sum of all the terms in arithmetic progression is called as **arithmetic series**.

There are only two formulas (i.e. last term and sum) to remember and used in solving a problem in arithmetic sequence.

Last term (n^{th} term):

$$a_n = a_1 + (n-1)\,d$$

Sum of all terms:

$$S = \frac{n}{2}(a_1 + a_n) \quad \text{or} \quad S = \frac{n}{2}[2a_1 + (n-1)d]$$

where: a_1 = first term
a_n = last term (n^{th} term)
n = number of terms
d = common difference = $a_2 - a_1 = a_3 - a_2 = \cdots$

GEOMETRIC PROGRESSION (G.P.):

A sequence is said to be a **geometric progression** if its succeeding terms have a common ratio. The corresponding sum of all the terms in geometric progression is called as **geometric series**.

Also, there are only two formulas (i.e. last term and sum) to remember and used in solving a problem in geometric sequence.

Last term (n^{th} term):

$$a_n = a_1 r^{n-1}$$

Sum of all terms:

$$S = \frac{a_1(r^n - 1)}{r - 1} \quad \text{or} \quad S = \frac{a_1(1 - r^n)}{1 - r}$$

where: a_1 = first term
a_n = last term (n^{th} term)
n = number of terms

$$r = \text{common ratio} = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \ldots$$

INFINITE GEOMETRIC PROGRESSION:

This type of progression is a geometric progression only that the number of terms (n) is extremely large or infinity.

If r > 1, sum of all terms is infinite
If r < 1, the sum of all terms is

$$S = \frac{a_1}{1 - r}$$

where: a_1 = first term
r = common ratio

HARMONIC PROGRESSION:

A sequence of numbers whose reciprocals form an arithmetic progression is known as **harmonic progression**. In solving a problem, it would be wise to convert all given terms into arithmetic sequence by getting its reciprocals. Use the formulas in arithmetic sequence and take the reciprocal of resulting value to obtain the equivalent harmonic term for an answer.

OTHER RELATED SEQUENCES:

1. **Fibonacci Numbers** – Named after the Italian merchant and mathematician, **Leonardo di Pisa** or **Fibonacci** (Figlio dei Bonacci, "Son of the Bonnaccis").

   1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144...

   Each number is equal to the sum of the two preceding numbers.

2. **Lucas Sequence** – Named after Edouard Lucas (1841 – 1891). Like the Fibonacci numbers, every term of the Lucas sequence is the sum of the two preceding numbers.

   1, 3, 4, 7, 11, 18, 29, 47, 76, 123...

3. **Figurate Numbers:**

   A. **Triangular numbers**: Numbers which can be drawn as dots and arranged in triangular shape.

      1, 3, 6, 10, 15, 21,...

   B. **Square numbers:** Numbers which can be drawn as dots and arranged in square shape.

      1, 4, 9, 16, 25, 36, ...

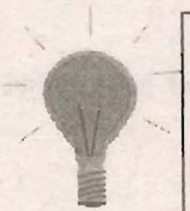
   C. **Gnomons**: Numbers which can be drawn as dots on equally long legs of a right angle.

      1, 3, 5, 7, 9, 11, ...

   D. **Oblong numbers**: Numbers which can be drawn as dots and arranged in a rectangle shape.

      2, 6, 12, 20, 30, ...

   E. **Pentagonal numbers**: 1, 5, 12, 22, 35, ...
   F. **Cubic numbers**: 1, 8, 27, 64, ...
   G. **Tetrahedral numbers**: 1, 4, 10, 20, 35, 56, ...
   H. **Square pyramidal numbers**: 1, 5, 14, 30, ...
   I. **Supertetrahedral numbers**: 1, 5, 15, 35, 70,...



Tips: 1. Diophantine Equations – refers to systems of equations where the number of equations is one less the number of unknowns. These equations yield whole number for its answers. This was named after a Greek mathematician, **Diophantus** (c. 250 AD).

2. Variation Problems: Variations are expressed in the following:

☞ x varies directly with y. $x = ky$

x varies inversely with y $x = k\frac{1}{y}$

where: k = proportionality constant

Did you know that...the eminent German mathematician, Carl Freidrich Gauss's father is an Accountant and young Carl corrected his father's spreadsheet at the age of 3!

Proceed to the next page for your fifth test. GOODLUCK ! ☞

Test 5

Time element: 4 hours and 30 minutes

Problem 191: CE Board May 1995

In how many minutes after 2 o'clock will the hands of the clock extend in opposite directions for the first time?

A. 42.4 minutes
B. 42.8 minutes
C. 43.2 minutes
D. 43.6 minutes

Problem 192: CE Board November 1995

In how many minutes after 7 o'clock will the hands be directly opposite each other for the first time?

A. 5.22 minutes
B. 5.33 minutes
C. 5.46 minutes
D. 5.54 minutes

Problem 193: CE Board May 1997

What time after 3 o'clock will the hands of the clock be together for the first time?

A. 3:02.30
B. 3:17.37
C. 3:14.32
D. 3:16.36

Problem 194: GE Board February 1997

At what time after 12:00 noon will the hour hand and minute hand of the clock first form an angle of 120°?

A. 12:18.818
B. 12:21.818
C. 12:22.818
D. 12:24.818

Problem 195:

At what time between 8 and 9 o'clock will the minute hand coincide with the hour hand?

A. 8:42.5
B. 8:43.2
C. 8:43.6
D. 8:43.9

Problem 196: EE Board October 1990

A man left his home at past 3:00 o'clock PM as indicated in his wall clock, between 2 to 3 hours after, he returns home and noticed the hands of the clock interchanged. At what time did the man leave his home?

A. 3:31.47
B. 3:21.45
C. 3:46.10
D. 3:36.50

Problem 197: GE Board February 1994

From the time 6:15 PM to the time 7:45 PM of the same day, the minute hand of a standard clock describe an arc of

A. 60°
B. 90°
C. 180°
D. 540°

Problem 198: EE Board April 1990

A storage battery discharges at a rate which is proportional to the charge. If the charge is reduced by 50% of its original value at the end of 2 days, how long will it take to reduce the charge to 25% of its original charge?

A. 3
B. 4
C. 5
D. 6

Problem 199: ECE Board April 1990

The resistance of a wire varies directly with its length and inversely with its area. If a certain piece of wire 10 m long and 0.10 cm in diameter has a resistance of 100 ohms, what will its resistance be if it is uniformly stretched so that its length becomes 12 m?

A. 80
B. 90
C. 144
D. 120

Problem 200: CE Board May 1993

Given that "w" varies directly as the product of "x" and "y" and inversely as the square of "z" and that w = 4 when x = 2, y = 6 and z = 3. Find the value of "w" when x = 1, y = 4 and z = 2.

A. 3
B. 4
C. 5
D. 6

Problem 201: ECE Board November 1993

If x varies directly as y and inversely as z, and x = 14 when y = 7 and z = 2, find the value of x when y = 16 and z = 4.

A. 14
B. 4
C. 16
D. 8

Problem 202: EE Board March 1998

The electric power which a transmission line can transmit is proportional to the product of its design voltage and current capacity, and inversely to the transmission distance. A 115-kilovolt line rated at 100 amperes can transmit 150 megawatts over 150 km. How much power, in megawatts can a 230 kilovolt line rated at 150 amperes transmit over 100 km?

A. 785
B. 485
C. 675
D. 595

Problem 203: ME Board October 1992

The time required for an elevator to lift a weight varies directly with the weight and the distance through which it is to be lifted and inversely as the power of the motor. If it takes 30 seconds for a 10 hp motor to lift 100 lbs through 50 feet, what size of motor is required to lift 800 lbs in 40 seconds through 40 feet?

A. 42
B. 44
C. 46
D. 48

Problem 204:

The selling price of a TV set is double that of its cost. If the TV set was sold to a customer at a profit of 25% of the net cost, how much discount was given to the customer?

A. 33.7 %
B. 35.7 %
C. 37.5 %

D. 34.7 %

Problem 205:

A group of EE examinees decided to hire a mathematics tutor from Excel Review Center and planned to contribute equal amount for the tutor's fee. If there were 10 more examinees, each would have paid P 2 less. However, if there were 5 less examinees, each would have paid P 2 more. How many examinees are there in the group?

A. 14
B. 16
C. 18
D. 20

Problem 206: EE Board March 1998

A bookstore purchased a best selling price book at P 200.00 per copy. At what price should this book be sold so that, giving a 20% discount, the profit is 30%?

A. P 450
B. P 500
C. P 357
D. P 400

Problem 207: ECE Board November 1993

Jojo bought a second hand Betamax VCR and then sold it to Rudy at a profit of 40%. Rudy then sold the VCR to Noel at a profit of 20%. If Noel paid P 2,856 more than it cost to Jojo, how much did Jojo paid for the unit?

A. P 4,000
B. P 4,100
C. P 4,200
D. P 4,300

Problem 208: EE Board March 1998

In a certain community of 1,200 people, 60% are literate. Of the males, 50% are literate and of the females 70% are literate. What is the female population?

A. 850
B. 500
C. 550
D. 600

Problem 209: ECE Board March 1996

A merchant has three items on sale, namely a radio for P 50, a clock for P 30 and a flashlight for P 1. At the end of the day, he sold a total of 100 of the three items and has taken exactly P 1,000 on the total sales. How many radios did he sale?

A. 16
B. 20
C. 18

D. 24

Problem 210: ME Board October 1996

The arithmetic mean of a and b is

A. $\frac{a+b}{2}$

B. $\sqrt{ab}$

C. $\frac{ab}{2}$

D. $\frac{a-b}{2}$

Problem 211:

The sum of three arithmetic means between 34 and 42 is

A. 114
B. 124
C. 134
D. 144

Problem 212: EE Board March 1998

Gravity causes a body to fall 16.1 ft in the first second, 48.3 in the 2nd second, 80.5 in the 3rd second. How far did the body fall during the 10th second?

A. 248.7 ft
B. 308.1 ft
C. 241.5 ft
D. 305.9 ft

Problem 213:

If the first term of an arithmetic progression is 25 and the fourth term is 13, what is the third term?

A. 17
B. 18
C. 19
D. 20

Problem 214: ECE Board November 1998

Find the 30th term of the arithmetic progression 4, 7, 10,...

A. 75
B. 88
C. 90
D. 91

Problem 215: CE Board May 1993, CE Board May 1994, CE Board November 1994

How many terms of the progression 3, 5, 7, ... must be taken in order that their sum will be 2600?

A. 48
B. 49
C. 50
D. 51

Problem 216: ME Board April 1995

In a pile of logs, each layer contains one more log than the layer above and the top contains just one log. If there are 105 logs in the pile, how many layers are there?

A. 11
B. 12
C. 13
D. 14

Problem 217: CE Board May 1995

What is the sum of the progression 4, 9, 14, 19 ... up to the 20th term?

A. 1030
B. 1035
C. 1040
D. 1045

Problem 218: EE Board April 1997

A stack of bricks has 61 bricks in the bottom layer, 58 bricks in the second layer, 55 bricks in the third layer, and so on until there are 10 bricks in the last layer. How many bricks are there all together?

A. 638
B. 637
C. 639
D. 640

Problem 219: CE Board May 1998

Determine the sum of the progression if there are 7 arithmetic mean between 3 and 35.

A. 171
B. 182
C. 232
D. 216

Problem 220: ECE Board April 1995

A besiege fortress is held by 5700 men who have provisions for 66 days. If the garisson losses 20 men each day, for how many days can the provision hold out?

A. 72
B. 74
C. 76
D. 78

Problem 221: CE Board May 1991

In the recent "Gulf War" in the Middle East, the allied forces captured 6400 of Saddam's soldiers and with provisions on hand it will last for 216 meals while feeding 3 meals a day. The provision lasted 9 more days because of daily deaths. At an average, how many died per day?

A. 15
B. 16
C. 17
D. 18

Problem 222: GE Board July 1993

A Geodetic Engineering student got a score of 30% on Test 1 of the five number test in Surveying. On the last number he got 90% in which a constant difference more on each number that he had on the immediately preceding one. What was his average score in Surveying?

A. 50
B. 55
C. 60
D. 65

Problem 223: ME Board April 1999

If the sum is 220 and the first term is 10, find the common difference if the last term is 30.

A. 2
B. 5
C. 3
D. 2/3

Problem 224: EE Board April 1997

Once a month, a man puts some money into the cookie jar. Each month he puts 50 centavos more into the jar than the month before. After 12 years, he counted his money, he had P 5,436. How much money did he put in the jar in the last month?

A. P 73.50
B. P 75.50
C. P 74.50
D. P 72.50

Problem 225: EE Board April 1997

A girl on a bicycle coasts downhill covering 4 feet the first second, 12 feet the second second, and in general, 8 feet more each second than the previous second. If she reaches the bottom at the end of 14 seconds, how far did she coasts?

A. 782 feet
B. 780 feet
C. 784 feet
D. 786 feet

Problem 226:

When all odd numbers from 1 to 101 are added, the result is

A. 2500
B. 2601
C. 2501
D. 3500

Problem 227:

How many times will a grandfather's clock strikes in one day if it strikes only at the hours and strike once at 1 o'clock, twice at 2 o'clock , thrice at 3 o'clock and so on?

A. 210
B. 24
C. 156
D. 300

Problem 228: CE Board May 1992

To conserve energy due to the present energy crisis, the Meralco tried to re-adjust their charges to electrical energy users who consume more than 2000 kw-hrs. For the first 100 kw-hr, they charged 40 centavos and increasing at a constant rate more than the preceding one until the fifth 100 kw-hr, the charge is 76 centavos. How much is the average charge for the electrical energy per 100 kw-hr?

A. 58 centavos
B. 60 centavos
C. 62 centavos
D. 64 centavos

Problem 229: CE Board November 1993

The 3rd term of a harmonic progression is 15 and the 9th term is 6. Find the 11th term.

A. 4
B. 5
C. 6
D. 7

Problem 230: ECE Board November 1995

Find the fourth term of the progression 1/2 , 0.2, 0.125, ...

A. 1/10
B. 1/11
C. 0.102
D. 0.099

Problem 231:

Find the 9th term of the harmonic progression 3, 2, 3/2.....

A. 3/5
B. 3/8
C. 4/5
D. 4/9

Problem 232:

Find the sum of 4 geometric means between 160 and 5.

A. 130
B. 140
C. 150
D. 160

Problem 233: EE Board October 1991

The fourth term of a G. P. is 216 and the 6th term is 1944. Find the 8th term.

A. 17649
B. 17496
C. 16749
D. 17964

Problem 234: ECE Board April 1999

Determine x so that: x, 2x + 7, 10x – 7 will be a geometric progression.

A. 7, -7/12
B. 7, -5/6
C. 7, -14/5
D. 7, - 7/6

Problem 235: ECE Board April 1999

If one third of the air in a tank is removed by each stroke of an air pump, what fractional part of the total air is removed in 6 strokes?

A. 0.7122
B. 0.9122
C. 0.6122
D. 0.8122

Problem 236: ME Board October 1996

A product has a current selling of P 325.00. If its selling price is expected to decline at the rate of 10% per annum because of obsolescence, what will be its selling price four years hence?

A. P 213.23
B. P 202.75
C. P 302.75
D. P 156.00

Problem 237: CE Board May 1995

The numbers 28, x + 2, 112 form a G. P. What is the 10th term?

A. 14336
B. 13463
C. 16433
D. 16344

Problem 238: ECE Board April 1998

The sum of the first 10 terms of a geometric progression 2, 4, 8,... is

A. 1023
B. 2046
C. 225
D. 1596

Problem 239:

If the first term of a G.P. is 9 and the common ratio is –2/3, find the fifth term.

A. 8/5
B. 16/9
C. 15/7
D. 13/4

Problem 240: EE Board April 1997

The seventh term is 56 and the twelfth term is –1792 of a geometric progression. Find the common ratio and the first term. Assume the ratios are equal.

A. –2, 5/8
B. –1, 5/8
C. –1, 7/8
D. –2, 7/8

Problem 241:

A person has 2 parents, 4 grandparents, 8 great grandparents and so on. How many ancestors during the 15 generations preceding his own, assuming no duplication?

A. 131070
B. 65534

C. 32766
D. 16383

Problem 242:

In the PBA three-point shootout contest, the committee decided to give a prize in the following manner: A prize of P1 for the first basket made, P 2 for the second, P 4 for the third, P8 for the fourth and so on. If the contestant wants to win a prize of no less than a million pesos, what is the minimum number of baskets to be converted?

A. 20
B. 19
C. 18
D. 21

Problem 243: CE Board November 1994

In a benefit show, a number of wealthy men agreed that the first one to arrive would pay 10 centavos to enter and each later arrive would pay twice as much as the preceding man. The total amount collected from all of them was P 104,857.50. How many wealthy men paid?

A. 18
B. 19
C. 20
D. 21

Problem 244:

A man mailed 10 chain letters to ten of his friends with a request to continue by sending a similar letter to each of their ten friends. If this continue for 6 sets of letters and if all responded, how much will the Phil. Postal office earn if minimum postage costs P 4 per letter?

A. P 6,000,000
B. P 60,000
C. P 2,222,220
D. P 4,444,440

Problem 245: EE Board March 1998

Determine the sum of the infinite series: $S = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots + \left(\frac{1}{3}\right)^n$

A. 4/5
B. 3/4
C. 2/3
D. 1/2

Problem 246:

Under favorable condition, a single cell bacteria divided into two about every 20 minutes. If the same rate of division is maintained for 10 hours, how many organisms is produced from a single cell?

A. 1,073,741
B. 1,730,74
C. 1,073,741,823
D. 1,037,417

Problem 247: EE Board October 1994

A rubber ball is made to fall from a height of 50 feet and is observed to rebound 2/3 of the distance it falls. How far will the ball travel before coming to rest if the ball continues to fall in this manner?

A. 200 feet
B. 225 feet
C. 250 feet
D. 275 feet

Problem 248: EE Board April 1990

What is the fraction in lowest term equivalent to 0.133133133?

A. $\frac{133}{666}$
B. $\frac{133}{777}$
C. $\frac{133}{888}$
D. $\frac{133}{999}$

Problem 249: ECE Board April 1998

Find the sum of the infinite geometric progression 6, -2, 2/3, ...

A. 9/2
B. 5/2
C. 7/2
D. 11/2

Problem 250: CE Board May 1998

Find the sum of $1, -\frac{1}{5}, \frac{1}{25}, \ldots$

A. 5/6
B. 2/3
C. 0.84
D. 0.72

Problem 251: ECE Board November 1998

Find the ratio of an infinite geometric progression if the sum is 2 and the first term is 1/2.

A. 1/3
B. 1/2
C. 3/4
D. 1/4

Problem 252: EE Board April 1997

If equal spheres are piled in the form of a complete pyramid with an equilateral triangle as base, find the total number of spheres in the pile if each side of the base contains 4 spheres.

A. 15
B. 20
C. 18
D. 21

Problem 253:

Find the 6th term of the sequence 55, 40, 28, 19, 13,...

A. 10
B. 9
C. 8
D. 11

Problem 254: EE Board October 1997

In the series 1, 1, 1/2, 1/6, 1/24,..., determine the 6th term.

A. 1/80
B. 1/74
C. 1/100
D. 1/120

Problem 255: ECE Board April 1998

Find the 1987th digit in the decimal equivalent to $\frac{1785}{9999}$ starting from the decimal point.

A. 8
B. 1
C. 7
D. 5

ANSWER KEY

191. D	208. D	225. C	242. A
192. C	209. A	226. B	243. C
193. D	210. A	227. C	244. D
194. B	211. A	228. A	245. D
195. C	212. D	229. B	246. C
196. A	213. A	230. B	247. C
197. D	214. D	231. A	248. D
198. A	215. C	232. C	249. A
199. C	216. D	233. B	250. A
200. A	217. A	234. D	251. C
201. C	218. C	235. B	252. B
202. C	219. A	236. A	253. A
203. D	220. C	237. A	254. D
204. C	221. D	238. B	255. A
205. D	222. C	239. B	
206. C	223. A	240. D	
207. C	224. A	241. B	

RATING

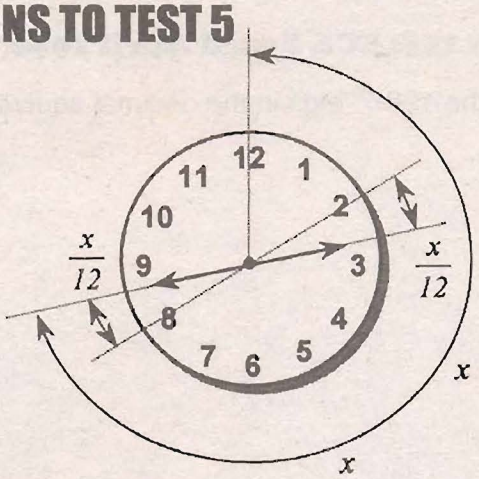
- ☐ 55-65 Topnotcher
- ☐ 42-54 Passer
- ☐ 32-42 Conditional
- ☐ 0-31 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 5

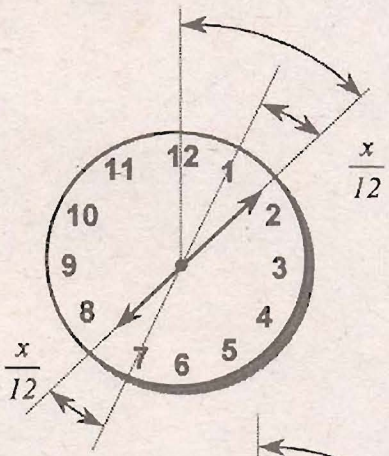
191. $x = 40 + \frac{x}{12}$

$x = 43.6$ minutes



192. $x = 5 + \frac{x}{12}$

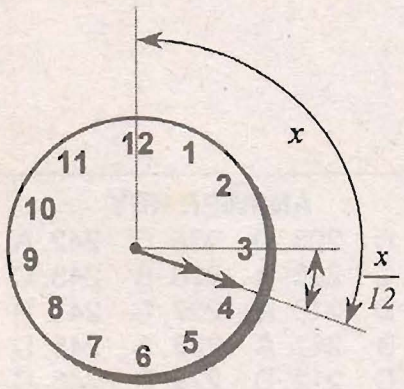
$x = 5.454$ minutes



193. $x = 15 + \frac{x}{12}$

$x = 16.36$ minutes

Thus, the time is 3:16.36'

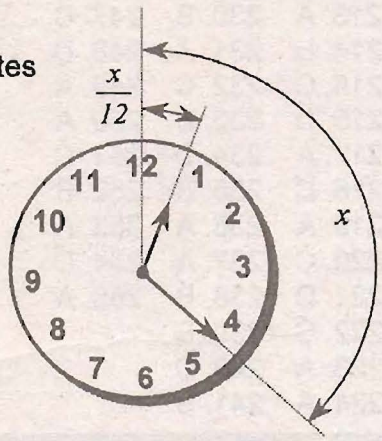


194. Note: $120°\left(\frac{30 \text{ minutes}}{180°}\right) = 20$ minutes

$x = \frac{x}{12} + 20$

$x = 21.818$ minutes

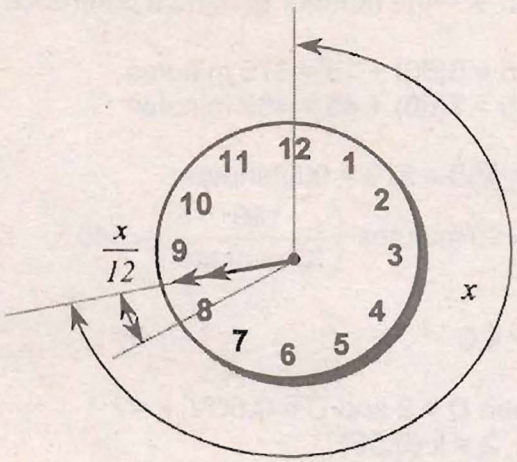
Thus, the time is 12:21.818'



195. $x = 40 + \frac{x}{12}$

$x = 43.6$ minutes

Thus, the time is 8:43.6'



196. $x = 30 + \frac{y}{12}$ ☞ ❶

$y = 15 + \frac{x}{12}$ ☞ ❷

Substitute (2) in (1):

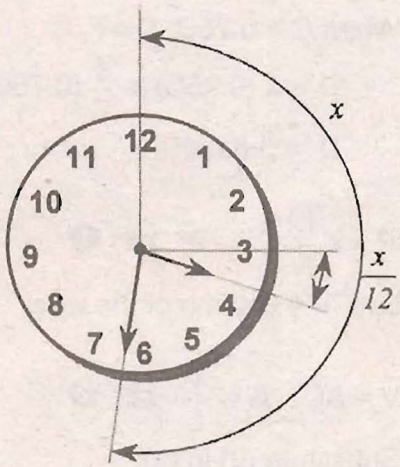
$$x = 30 + \frac{15 + \frac{x}{12}}{12}$$

$$\left[x = 30 + \frac{15 + \frac{x}{12}}{12} \right] 12$$

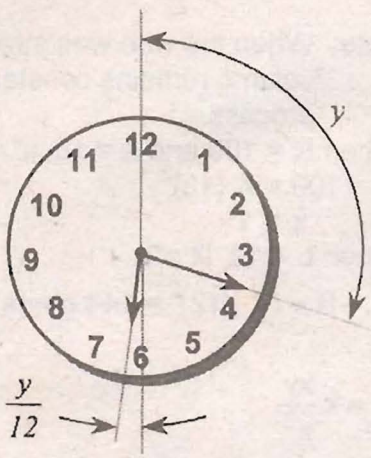
$$12x = 360 + 15 + \frac{x}{12}$$

$x = 31.47$ minutes

Thus, the time when he left home was 3:31.47'.



Time when he left his home



Time when he returned home

197. Let: x = the number of minute difference between 6:15 and 7:45.

6:15 = 6(60) + 15 = 375 minutes
7.45 = 7(60) + 45 = 465 minutes

x = 465 – 375 = 90 minutes

$$x = 90 \text{ minutes} \left(\frac{180^\circ}{30 \text{ minutes}}\right) = 540^\circ$$

198. D = k C

When D = 2 and C = 0.5CV, k =?

$$2 = k\,(0.5C)$$

$$k = \frac{4}{C}$$

When C = 0.75C, D =?

$$D = k\,(0.75C) = \frac{4}{C}(0.75C)$$

D = 3 days

199. $R = k\frac{L}{A}$ ☞ ❶

Let: V = volume of the wire

$V = AL$; $A = \frac{V}{L}$ ☞ ❷

Substitute (2) in (1):

$$R = k\frac{L}{\frac{V}{L}} = k'(L)^2 \; ; \; k' = \frac{k}{V}$$

Note: When the wire was stretched, the diameter was changed but the volume remains constant assuming there was no losses in the process.

When R = 100 and L = 10, k' =?

$$100 = k'\,(10)^2$$

$$k' = 1$$

When L = 12, R =?

$$R = (1)(12)^2 = 144 \text{ ohms}$$

200. $w = k\frac{xy}{z^2}$

When w = 4, x = 2, y = 6 and z = 3, k =?

$$4 = k\frac{(2)(6)}{(3)^2}$$

$$k = 3$$

When x = 1, y = 4 and z = 2, w =?

$$w = 3\left[\frac{(1)(4)}{(2)^2}\right] = 3$$

201. $x = k\frac{y}{z}$

When x = 14, y = 7 and z = 2, k =?

$$14 = k\frac{7}{2}$$

$$k = 4$$

When y = 16 and z = 4, x =?

$$x = (4)\left[\frac{16}{4}\right] = 16$$

202. $P = k\frac{VI}{d}$

When P = 150, V = 115, I = 100 and d = 150, k =?

$$150 = k\frac{115(100)}{150}$$

$$k = 1.956$$

When V = 230, I = 150 and d = 100, P =?

$$P = 1.956\frac{230(150)}{100} = 675 \text{ megawatts}$$

203. $t = k\frac{WS}{P}$

When t = 30, W = 100, S = 50 and P =10, k =?

$$30 = k\left[\frac{(100)(50)}{10}\right]$$

$$k = 0.06$$

When t = 40, W = 800 and S = 40, P =?

$$40 = (0.06)\left[\frac{(800)(40)}{P}\right]$$

P = 48 horsepower

204. Let: x = net cost
2x = selling price
d = discount
2x (1 – d) = new selling price

New selling price = Net cost + Gain

$$2x(1 - d) = x + 0.25x$$

$$2x - 2xd = 1.25x$$

$$2xd = 0.75x$$
$$d = 0.375 \text{ or } 37.5\ \%$$

205. Let: x = number of examinees
y = tutor's fee
$\frac{x}{y}$ = original fee shared per examinee

If there were 10 more examines who will join,

$$y = (x+10)\left(\frac{y}{x} - 2\right)$$
$$y = y - 2x + \frac{10y}{x} - 20$$
$$10y = 2x^2 + 20x$$
$$y = 0.2x^2 + 2x \qquad \text{☞ ❶}$$

If there were 5 examinees who will back-out,

$$y = (x-5)\left(\frac{y}{x} + 2\right)$$
$$y = y + 2x - \frac{5y}{x} - 10$$
$$5y = 2x^2 - 10x$$
$$y = 0.4x^2 - 2x \qquad \text{☞ ❷}$$

Equate (1) and (2):
$$0.2x^2 + 2x = 0.4x^2 - 2x$$
$$4x = 0.2x^2$$
$$x = 20 \text{ examinees}$$

206. Let: x = selling price without discount
0.8x = new selling price (with discount)

Profit = Income – Expenses
$0.3\ (0.8x) = 0.8x - 200$
$0.24\ x = 0.8x - 200$
$x = P\ 357.14$

207. Let: x = price Jojo paid for the VCR
1.4x = price Rudy paid for the VCR
1.2(1.4x) = price Noel paid for the VCR

$1.2(1.4x) = x + 2856$
$1.68\ x = x + 285$
$x = P\ 4{,}200$

208. Let: x = number of men in the population
y = number of female in the population

$x + y = 1200$

$x = 1200 - y$ ☞ ❶

$0.5x + 0.7y = 0.6(1200)$ ☞ ❷

Substitute (1) in (2):

$0.5(1200 - y) - 0.7y = 720$

$600 - 0.5y + 0.7y = 720$

$y = 600$ females

209. Let: x = number of radios sold out
y = number of clocks sold out
z = number of flashlight sold out

$x + y + z = 100$ ☞ ❶

$50x + 30y + z = 1000$ ☞ ❷

Subtract (2) by (1):

$(50x + 30y + z) - (x + y + z) = 1000 - 100$

$49x + 29y = 900$

Assume x = 16:

$49(16) + 29y = 900$

$y = 4$

Substitute x = 20 and y = 4 in (1):

$16 + 4 + z = 100$

$z = 80$

Substitute x = 20, y = 4 and z = 80 in (2):

$50(16) + 30(4) + 80 = 1000$

$1000 = 1000$ ☞ *Check!*

Thus, x = 16 radios.

210. Note: Arithmetic mean is the same term as average.

Thus, the arithmetic mean of a and b is, $\frac{a+b}{2}$

211. 34, a_2, a_3, a_4, 42

$a_5 = a_1 + 4d$

$42 = 34 + 4d$

$d = 2$

Thus, $a_2 = 36$, $a_3 = 38$ and $a_4 = 40$

Sum = 36 + 38 + 40 = 114

212. $a_1 = 16.1;\ a_2 = 48.3;\ a_3 = 80.5$

$d = a_2 - a_1 = 48.3 - 16.1 = 32.2$
$a_{10} = a_1 + 9d$
$a_{10} = 16.1 + 9(32.2) = 305.9$ feet

213. $a_1 = 25;\ a_4 = 13$

$a_4 = a_1 + 3d$
$13 = 25 + 3d$
$d = -4$

$a_3 = a_1 + 2d$
$a_3 = 25 + 2(-4) = 17$

214. $a_1 = 4;\ a_2 = 7;\ a_3 = 10$

By inspection, $d = 3$
$a_{30} = a_1 + 29d$
$a_{30} = 4 + 29(3) = 91$

215. $a_1 = 3;\ a_2 = 5;\ a_3 = 7$

By inspection, $d = 2$

$$S = \frac{n}{2}[2a_1 + (n-1)d]$$

$$2600 = \frac{n}{2}[2(3) + (n-1)2] = \frac{n}{2}[6 + 2n - 2] = \frac{n}{2}[4 + 2n]$$

$2600 = 2n + n^2$
$0 = n^2 + 2n - 2600$
$0 = (n + 51)(n - 50)$
$n = -51$ (absurd)
$n = 50$ numbers

216. $a_1 = 1;\ d = 1;\ S = 105$

$$S = \frac{n}{2}[2a_1 + (n-1)d]$$

$$105 = \frac{n}{2}[2(1) + (n-1)1] = \frac{n}{2}[2 + n - 1] = \frac{n}{2}[1 + n]$$

$210 = n + n^2$
$n^2 + n - 210 = 0$
$(n - 14)(n + 15) = 0$
$n = -15$ (absurd)
$n = 14$ layers of log

217. $a_1 = 4;\ a_2 = 9;\ a_3 = 14;\ a_4 = 19;\ n = 20$

By inspection, $d = 5$

$$S = \frac{n}{2}[2a_1 + (n-1)d] = \frac{20}{2}[2(4) + 19(5)] = 1030$$

218. $a_1 = 61$; $a_2 = 58$; $a_3 = 55$; $a_n = 10$

By inspection, $d = -3$

$a_n = a_1 + (n-1)d$

$10 = 61 + (n-1)(-3)$

$10 = 61 - 3n + 3$

$n = 18$

$$S = \frac{n}{2}[2a_1 + (n-1)d] = \frac{18}{2}[2(61) + (18-1)(-3)]$$

$S = 639$ logs

219. 3, a_2, a_3, a_4, a_5, a_6, a_7, a_8, 35

By inspection: $a_1 = 3$; $a_n = 35$; $n = 7 + 2 = 9$

$$S = \frac{n}{2}(a_1 + a_n)$$

$$S = \frac{9}{2}(3 + 35) = 171$$

220. Total provision = 5700 (66) = 376,200

Note: $a_1 = 5700$; $d = -20$

$$S = \frac{n}{2}[2a_1 + (n-1)d]$$

$$376{,}200 = \frac{n}{2}[2(5700) + (n-1)(-20)] = \frac{n}{2}[11400 - 20n + 20]$$

$376200 = 5710n - 10n^2$

$37620 = 571n - n^2$

$n^2 - 571n + 37620 = 0$

$(n - 76)(n - 495) = 0$

$n = 495$ (absurd)

$n = 76$ days

221. Let: x = number of days, the meal can last
S = total number of provisions
n = number of days the total provisions can last
d = number of soldiers died per day

$$x = \frac{216}{3} = 72 \text{ days}$$

$S = 6400(72) = 460{,}800$ meals

$n = 72 + 9 = 81$ days

$$S = \frac{n}{2}[2a_1 + (n-1)d]$$

Note: $a_1 = 6400$
Substitute:

$$460,800 = \frac{81}{2}[2(6400) + 80d]$$

$$d = (-)18 \text{ soldiers died per day}$$

222. $a_1 = 30$; $a_5 = 90$

$$a_5 = a_1 + 4d$$
$$90 = 30 + 4d$$
$$d = 15$$
$$S = \frac{n}{2}[2a_1 + (n-1)d] = \frac{5}{2}[2(30) + 4(15)]$$
$$S = 300$$

$$\text{Average score} = \frac{300}{5} = 60\%$$

223. $S = 220$; $a_1 = 10$; $a_n = 30$

$$S = \frac{n}{2}(a_1 + a_n)$$
$$220 = \frac{n}{2}(10 + 30)$$
$$n = 11$$

$$a_{11} = a_1 + 10d$$
$$30 = 10 + 10d$$
$$d = 2$$

224. $d = 0.50$; $n = 12(12) = 144$

$$S = \frac{n}{2}[2a_1 + (n-1)d]$$
$$5436 = \frac{144}{2}[2a_1 + 143(0.50)]$$
$$5436 = 144a_1 + 5148$$
$$a_1 = 2$$

$$a_{144} = a_1 + 143d$$
$$= 2 + 143(2)$$
$$a_{144} = P\ 73.50$$

225. $a_1 = 4$; $d = 8$; $n = 14$

$$S = \frac{n}{2}[2a_1 + (n-1)d] = \frac{14}{2}[2(4) + 13(8)] = 784 \text{ feet}$$

226. $a_1 = 1$; $a_n = 101$; $d = 2$

$$a_n = a_1 + (n-1)d$$
$$101 = 1 + (n-1)(2)$$
$$101 = 1 + 2n - 2$$

$n = 51$

$$S = \frac{n}{2}(a_1 + a_n) = \frac{51}{2}(1+101) = 2601$$

227. $a_1 = 1; a_2 = 2; a_3 = 3; \ldots\ldots\ldots\ldots a_{12} = 12$

$$S = \frac{n}{2}(a_1 + a_n) = \frac{12}{2}(1+12)$$

$S = 78$

Note: One day is equivalent to 24 hours.
Thus, total = 2(78) = 156 times

228. $a_1 = 40; a_5 = 76$

$a_5 = a_1 + 4d$
$76 = 40 + 4d$
$d = 9$

Thus,
$a_2 = 40 + 9 = 49$
$a_3 = 49 + 9 = 58$
$a_4 = 58 + 9 = 67$

$$\text{Average} = \frac{40+49+58+67+76}{5} = 58 \text{ centavos}$$

229. $h_3 = 15; a_3 = \frac{1}{15}$ $h_9 = 6; a_9 = \frac{1}{6}$

$a_3 = a_1 + 2d$

$$\frac{1}{15} = a_1 + 2d$$

$$a_1 = \frac{1}{15} - 2d \quad \text{☞ ❶}$$

$a_9 = a_1 + 8d$

$$\frac{1}{6} = a_1 + 8d \quad \text{☞ ❷}$$

Substitute (1) in (2):

$$\frac{1}{6} = \frac{1}{15} - 2d + 8d$$

$$d = \frac{1}{60}$$

Substitute d in (1):

$$a_1 = \frac{1}{15} - 2\left(\frac{1}{60}\right) = \frac{1}{30}$$

$a_{11} = a_1 + 10d$

$$a_{11} = \frac{1}{30} + 10\left(\frac{1}{60}\right) = \frac{1}{5}$$

Thus, $h_{11} = \dfrac{1}{a_{11}} = \dfrac{1}{\frac{1}{5}} = 5$

230. $h_1 = \frac{1}{2}$; $a_1 = 2$ $h_2 = 0.2 = \frac{1}{5}$; $a_2 = 5$ $h_3 = 0.125 = \frac{1}{8}$; $a_3 = 8$

By inspection, d = 3

$a_4 = a_1 + 3d$

$a_4 = 2 + 3(3) = 11$

Thus, $h_4 = \dfrac{1}{a_4} = \dfrac{1}{11}$

231. $h_1 = 3$; $a_1 = \frac{1}{3}$ $h_2 = 2$; $a_2 = \frac{1}{2}$

$$d = a_2 - a_1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$a_9 = a_1 + 8d$

$$a_9 = \frac{1}{3} + 8\left(\frac{1}{6}\right) = \frac{5}{3}$$

$$h_9 = \frac{1}{a_9} = \frac{1}{\frac{5}{3}} = \frac{3}{5}$$

232. $a_1 = 160$; $a_6 = 5$

$a_6 = a_1 r^5$

$5 = 160\, r^5$

$r = 0.5$

Thus,

$a_2 = 160(0.5) = 80$

$a_3 = 80(0.5) = 40$

$a_4 = 40(0.5) = 20$

$a_5 = 20(0.5) = 10$

Sum = 80 + 40 + 20 + 10 = 150

233. $a_4 = 216$; $a_6 = 1994$

$a_4 = a_1 r^3$

$216 = a_1 r^3$ ☞ ❶

$a_6 = a_1 r^5$

$1994 = a_1 r^5$ ☞ ❷

Divide (2) by (1):

$$\frac{a_1 r^5}{a_1 r^3} = \frac{1944}{216}$$

$r^2 = 9$

$r = 3$

Substitute r in (1):

$216 = a_1(3)^3$

$a_1 = 8$

$a_8 = a_1r^7$

$a_8 = 8(3)^7 = 17496$

234. $a_1 = x$; $a_2 = 2x + 7$; $a_3 = 10x - 7$

$$r = \frac{a_2}{a_1} = \frac{a_3}{a_2}$$

$$\frac{2x+7}{x} = \frac{10x-7}{2x+7}$$

$$(2x + 7)^2 = x\,(10x - 7)$$

$$4x^2 + 28x + 49 = 10x^2 - 7x$$

$$6x^2 - 35x - 49 = 0$$

Using the quadratic formula:

$$x = \frac{35 \pm \sqrt{(-35)^2 - 4(6)(-49)}}{2(6)} = \frac{35 \pm 49}{12}$$

$$x_1 = \frac{35 + 49}{12} = 7$$

$$x_2 = \frac{35 - 49}{12} = -\frac{7}{6}$$

235. Let: x = total volume of air in the tank

y = total volume removed from the tank after the 6th stroke

Volume left after 1st stroke = $x - \frac{1}{3}x = \frac{2}{3}x$

Volume left after 2nd stroke = $\frac{2}{3}x - \frac{1}{3}\left(\frac{2}{3}x\right) = \frac{4}{9}x$

Volume left after 3rd stroke = $\frac{4}{9}x - \frac{1}{3}\left(\frac{4}{9}x\right) = \frac{8}{27}x$

Note: By inspection, the volume left after each stroke forms a GP whose common ratio $r = \frac{4/9}{2/3} = \frac{2}{3}$

Solving for the volume left after the 6th stroke:

$$a_6 = a_1r^5 = \left[\frac{2}{3}x\right]\left[\frac{2}{3}\right]^5 = 0.08779x$$

Thus, the total volume removed after the 6th stroke:

$$y = x - 0.08779x = 0.9122x$$

236. $r = 0.9$

$a_1 = 325(0.9) = 292.5$

$a_4 = a_1r^3 = (292.5)(0.9)^3 = P\ 213.23$

237. $a_1 = 28;\ a_2 = x + 2;\ a_3 = 112$

$$r = \frac{a_2}{a_1} = \frac{a_3}{a_2}$$

$$\frac{x+2}{28} = \frac{112}{x+2}$$

$$(x + 2)^2 = 112(28) = 3136$$

$$x + 2 = 56$$

$$x = 54$$

Solving for r:

$$r = \frac{a_2}{a_1} = \frac{54+2}{28} = 2$$

$$a_{10} = a_1r^9 = 28(2)^9 = 14336$$

238. $a_1 = 2;\ a_2 = 4;\ a_3 = 8;\ n = 10$

By inspection, $r = 2$

$$S = \frac{a_1(r^n - 1)}{r - 1} = \frac{2(2^{10} - 1)}{2 - 1} = 2046$$

239. $a_1 = 9;\ r = -\frac{2}{3}$

$$a_5 = a_1r^4 = (9)\left(-\frac{2}{3}\right)^4 = (9)\left(\frac{16}{81}\right) = \frac{16}{9}$$

240. $a_7 = 56;\ a_{12} = -1792$

$a_7 = a_1r^6$ $\qquad$ $a_{12} = a_1r^{11}$

$56 = a_1r^6$ ☞ ❶ $\qquad$ $-1792 = a_1r^{11}$ ☞ ❷

Divide (2) by (1):

$$\frac{a_1r^{11}}{a_1r^6} = \frac{-1792}{56}$$

$$r^5 = -32$$

$$r = -2$$

Substitute r in (1):

$$56 = a_1(-2)^6$$

$$a_1 = \frac{7}{8}$$

241. $a_1 = 2$; $r = 2$; $n = 15$

$$S = \frac{a_1(r^n - 1)}{r - 1} = \frac{2(2^{15} - 1)}{2 - 1} = 65{,}534 \text{ ancestors}$$

242. $a_1 = 1$; $r = 2$

$$S = \frac{a_1(r^n - 1)}{r - 1}$$

$$1{,}000{,}000 = \frac{1(2^n - 1)}{2 - 1}$$

$$2^n - 1 = 1{,}000{,}000$$

$$2^n = 1{,}000{,}001$$

Take ln on both sides:

$$\ln 2^n = \ln 1{,}000{,}001$$

$$n \ln 2 = \ln 1{,}000{,}001$$

$$n = \frac{\ln 1{,}000{,}001}{\ln 2} = 19.93$$

n = approximately 20 baskets

243. $a_1 = 0.1$; $r = 2$

$$S = \frac{a_1(r^n - 1)}{r - 1}$$

$$104{,}857.5 = \frac{0.1(2^n - 1)}{2 - 1}$$

$$2^n - 1 = 1048575$$

$$2^n = 1048586$$

Take in on both sides:

$$\ln 2^n = \ln 1048576$$

$$n \ln 2 = \ln 1048576$$

$$n = \frac{\ln 1048576}{\ln 2} = 20 \text{ wealthy men}$$

244. $a_1 = 10$; $r = 10$; $n = 6$

$$S = \frac{a_1(r^n - 1)}{r - 1} = \frac{10(10^6 - 1)}{10 - 1} = 1{,}111{,}110$$

Total cost = 4(1,111,110) = P 4,444,440

245. $a_1 = \frac{1}{3}$; $a_2 = \frac{1}{9}$; $a_3 = \frac{1}{27}$

By inspection, $r = \frac{1}{3}$

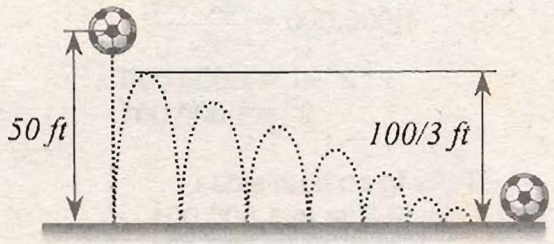
$$S = \frac{a_1}{1-r} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$$

246. $a_1 = 1; r = 2$

$$n = \frac{60}{20}(10) = 30$$

$$S = \frac{a_1(r^n - 1)}{r-1} = \frac{1(2^{30} - 1)}{2-1} = 1{,}073{,}741{,}823 \text{ organisms}$$

247. $a_1 = 50\left[\frac{2}{3}\right] = \frac{100}{3}$; $n = \infty$



$$S = \frac{a_1}{1-r} = \frac{\frac{100}{3}}{1-\frac{2}{3}} = 100$$

Let: D = total distance traveled by ball

D = 50 + 2S

D = 50 + 2(100) = 250 feet

248. 0.133133133133 = 0.133 + 0.000133 + 0.000000133 + ...

Note: The numbers being added are in a GP and 0.133133133133... is the sum of an infinite GP.

Solving for the common ratio:

$$r = \frac{0.000133}{0.133} = \frac{1}{1000}$$

$$S = \frac{a_1}{1-r} = \frac{0.133}{1-\frac{1}{1000}} = \frac{0.133}{\frac{999}{1000}} = \frac{133}{999}$$

249. $a_1 = 6; a_2 = -2$

$$r = \frac{a_2}{a_1} = \frac{-2}{6} = -\frac{1}{3}$$

$$S = \frac{a_1}{1-r} = \frac{6}{1-\left(-\frac{1}{3}\right)} = \frac{9}{2}$$

250. $a_1 = 1, a_2 = -\frac{1}{5}, a_3 = \frac{1}{25}$

By inspection, $r = -\frac{1}{5}$;

$$S = \frac{a_1}{1-r} = \frac{1}{1-\left[-\frac{1}{5}\right]} = \frac{5}{6}$$

251. $S = 2;\ a_1 = \dfrac{1}{2}$

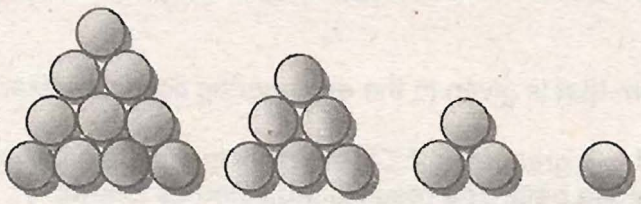
$$S = \frac{a_1}{1-r} = \frac{\frac{1}{2}}{1-r}$$

$$2 = \frac{\frac{1}{2}}{1-r} = \frac{1}{2(1-r)}$$

$$4 = \frac{1}{1-r}$$

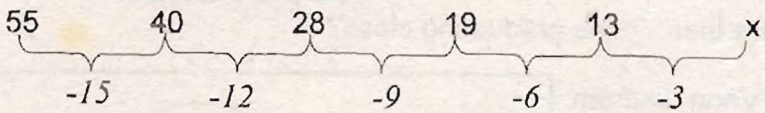
$$r = \frac{3}{4}$$

252. Total spheres = 10 + 6 + 3 + 1 = 20 spheres



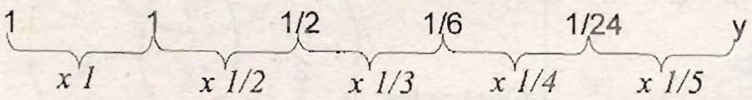
Layer I *Layer II* *Layer III* *Layer IV*

253. Let: x = the 6th term of the number series



Thus, x = 13 + (– 3) = 10

254. Let: y = the 6th term of the number series



Thus, $y = \dfrac{1}{24}\left[\dfrac{1}{5}\right] = \dfrac{1}{120}$

255.

$\dfrac{1785}{9999} = 0.178517851\ldots$ ☞ *This is a repeating digit decimal number*

Note: The four repeated digits are 1,7,8 & 5.

$\dfrac{1987}{4} = 496.75$; 496(4) = 1984

Thus, 1984th digit = 5 1986th digit = 7
1985th digit = 1 1987th digit = 8

DAY 6

VENN DIAGRAM, PERMUTATION, COMBINATION & PROBABILITY

VENN DIAGRAM

Venn diagram is a rectangle (the universal set) that includes circles depicting the subsets. This diagram is named after the English logician **John Venn** (1834 – 1923) in 1880.

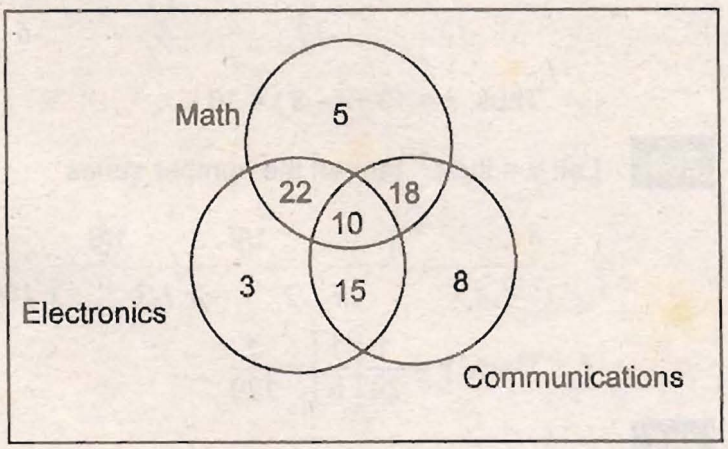
Below is a typical problem that is given in the engineering licensure examinations.

A survey was conducted in a graduating ECE students in a certain university on which board subject they like best. The result is tabulated as follows:

Mathematics	55	Math & Electronics	32
Electronics	50	Math & Communications	28
Communications	51	Electronics & Comm	25
		All three subjects	10

How many were there in the graduating class?

Solution: Use Venn diagram:



$N = 5 + 22 + 10 + 18 + 3 + 15 + 8$

$N = 81$ students

PERMUTATION / COMBINATION / PROBABILITY

Fundamental principle of counting:

" If a thing can be done in m different ways and another thing can be done in n different ways, then the two things can be done in m times n different ways."

$$N = m \cdot n$$

PERMUTATION: (Arrangement with specific order)
"The number of permutations of n different things taken r at a time is

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

If taken all, $ {}_nP_n = n! $

COMBINATION:(Arrangement regardless of the order)
"The number of combinations of n different things taken r at a time is

$$ {}_nC_r = \frac{n!}{(n-r)!r!} $$

If taken all, $ {}_nC_r = 1 $

Relation between permutation and combination:

$$ {}_nC_r = \frac{{}_nP_r}{r!} $$

PROBABILITY:

The probability (chance) of occurrence of a certain event in the following topics are based on the idea that all possible outcomes are equally likely to occur. This means that if a die is thrown once, the probability of getting a six is 1/6. The same probability with all the other numbers, i.e for 1 is 1/6, for 2 is 1/6 and so on.

I. Probability that an event E will happen:

$$ P_E = \frac{S}{T} $$

where: S = number of successful outcomes
T = number of outcomes

II. Probability that it is not E:

$$ P_{notE} = 1 - P_E $$

III. Mutually exclusive event:

$$ P_{E\,or\,F} = P_E + P_F $$

IV. Conditional and Independent probability:

$$P_{E\,\&\,F} = P_E \times P_F$$

V. Binomial or repeated trial probability:

$$P = {}_nC_r\, p^r\, q^{n-r}$$

where: p = probability of success
q = probability of failure = 1 – p
n = number of trials
r = number of successful trials



Tips: 1. CARDS. When dealing with playing cards, **pack of cards** or **deck of cards** is understood to be 52 cards.

The playing cards were used to describe a calendar year. The 52 cards represent the 52 weeks in a year. The 4 figures (Heart, Spade, Club and Diamond) represent the 4 seasons of the year and the 12 persons (Kings, Queens, Jacks) are the 12 months of the year.

2. **DICES.** Dices were first used by the Chinese. The sum of the opposite faces of a die is always equal to **7**. And the sum of all the vertical faces of a die, no matter how it rolls is always equal to **14**.

Did you know that...the number 1 followed by 100 zeros is called "google" and the term "google" was coined in the 1930s by the nine-year old nephew of the American mathematician Edward Kasner when he was asked to come up with a name for a very large number.

Proceed to the next page for your sixth test. GOODLUCK ! ☞

Test 6

Time element: 4 hours

Problem 256: EE Board October 1993

In a class of 40 students, 27 like Calculus and 25 like Chemistry. How many like both Calculus and Chemistry?

A. 10
B. 11
C. 12
D. 13

Problem 257: ECE Board November 1998

A club of 40 executives, 33 like to smoke Marlboro and 20 like to smoke Philip Morris. How many like both?

A. 10
B. 11
C. 12
D. 13

Problem 258: GE Board February 1994

A survey of 100 persons revealed that 72 of them had eaten at restaurant P and that 52 of them had eaten at restaurant Q. Which of the following could not be the number of persons in the surveyed group who had eaten at both P and Q?

A. 20
B. 22
C. 24
D. 26

Problem 259: ECE Board November 1992

The probability for the ECE board examinees from a certain school to pass the subject Mathematics is 3/7 and for the subject Communications is 5/7. If none of the examinees fails both subject and there are 4 examinees who pass both subjects, find the number of examinees from that school who took the examinations.

A. 20
B. 25
C. 30
D. 28

Problem 260: EE Board March 1998

In a commercial survey involving 1000 persons on brand preference, 120 were found to prefer brand x only, 200 prefer brand y only, 150 prefer brand z only, 370 prefer either brand x or y but not z, 450 prefer brand y or z but not x and 370 prefer either brand z or x but not y. How many persons have no brand preference, satisfied with any of the three brands?

A. 280
B. 230
C. 180
D. 130

Problem 261: EE Board April 1997

A toothpaste firm claims that in a survey of 54 people, they were using either Colgate, Hapee or Close-up brand. The following statistics were found: 6 people used all three brands, 5 used only Hapee and Close-up, 18 used Hapee or Close-up, 2 used Hapee, 2 used only Hapee and Colgate, 1 used Close-up and Colgate, and 20 used only Colgate. Is the survey worth paying for?

A. Neither yes nor no
B. Yes
C. No
D. Either yes or no

Problem 262:

How many four-letter words beginning and ending with a vowel without any letter repeated can be formed from the word "personnel"?

A. 40
B. 480
C. 20
D. 312

Problem 263:

Five different mathematics books, 4 different electronics books and 2 different communications books are to be placed in a shelf with the books of the same subject together. Find the number of ways in which the books can be placed.

A. 292
B. 5760
C. 34560
D. 12870

Problem 264:

The number of ways can 3 nurses and 4 engineers be seated on a bench with the nurses seated together is

A. 144
B. 258
C. 720

D. 450

Problem 265: ECE Board November 1998

If 15 people won prizes in the state lottery (assuming that there are no ties), how many ways can these 15 people win first, second, third, fourth and fifth prizes?

A. 4,845
B. 116,260
C. 360,360
D. 3,003

Problem 266: CE Board November 1996

How many 4 digit numbers can be formed without repeating any digit from the following digits: 1, 2, 3, 4 and 6?

A. 120
B. 130
C. 140
D. 150

Problem 267: EE Board June 1990

How many permutations are there if the letters PNRCSE are taken six at a time?

A. 1440
B. 480
C. 720
D. 360

Problem 268: EE Board April 1996

In how many ways can 6 distinct books be arranged in a bookshelf?

A. 720
B. 120
C. 360
D. 180

Problem 269: EE Board April 1997

What is the number of permutations of the letters in the word BANANA?

A. 36
B. 60
C. 52
D. 42

Problem 270: ME Board April 1994

A PSME unit has 10 ME's, 8 PME's and 6 CPM's. If a committee of 3 members, one from each group is to be formed, how many such committees can be formed?

A. 2,024
B. 12,144
C. 480

D. 360

Problem 271: ME Board October 1992

In how many ways can a PSME Chapter with 15 directors choose a President, a Vice President, a Secretary, a Treasurer and an Auditor, if no member can hold more than one position?

A. 360,360
B. 32,760
C. 3,003
D. 3,603,600

Problem 272: EE Board October 1997

Four different colored flags can be hung in a row to make coded signal. How many signals can be made if a signal consists of the display of one or more flags?

A. 64
B. 66
C. 68
D. 62

Problem 273: EE Board June 1990, EE Board April 1993, CHE Board May 1994

In how many ways can 4 boys and 4 girls be seated alternately in a row of 8 seats?

A. 1152
B. 2304
C. 576
D. 2204

Problem 274: EE Board October 1997

There are four balls of four different colors. Two balls are taken at a time and arranged in a definite order. For example, if a white and a red balls are taken, one definite arrangement is white first, red second, and another arrangement is red first, white second. How many such arrangements are possible?

A. 24
B. 6
C. 12
D. 36

Problem 275:

How many different ways can 5 boys and 5 girls form a circle with boys and girls alternate?

A. 28,800
B. 2,880
C. 5,600
D. 14,400

Problem 276: EE Board October 1997

There are four balls of different colors. Two balls at a time are taken and arranged any way. How many such combinations are possible?

A. 36
B. 3
C. 6
D. 12

Problem 277: EE Board March 1998

How many 6-number combinations can be generated from the numbers from 1 to 42 inclusive, without repetition and with no regards to the order of the numbers?

A. 850,668
B. 5,245,786
C. 188,848,296
D. 31,474,716

Problem 278:

Find the total number of combinations of three letters, J, R, T taken 1, 2, 3 at a time.

A. 7
B. 8
C. 9
D. 10

Problem 279: ME Board October 1997

In how many ways can you invite one or more of your five friends in a party?

A. 15
B. 31
C. 36
D. 25

Problem 280: CHE November 1996

In how many ways can a committee of three consisting of two chemical engineers and one mechanical engineer can be formed from four chemical engineers and three mechanical engineers?

A. 18
B. 64
C. 32
D. None of these

Problem 281: EE Board April 1995

In Mathematics examination, a student may select 7 problems from a set of 10 problems. In how many ways can he make his choice?

A. 120

B. 530
C. 720
D. 320

Problem 282: EE Board April 1997

How many committees can be formed by choosing 4 men from an organization of a membership of 15 men?

A. 1390
B. 1240
C. 1435
D. 1365

Problem 283: ECE Board April 1998

A semiconductor company will hire 7 men and 4 women. In how many ways can the company choose from 9 men and 6 women who qualified for the position?

A. 680
B. 540
C. 480
D. 840

Problem 284: ECE Board April 1994

There are 13 teams in a tournament. Each team is to play with each other only once. What is the minimum number of days can they all play without any team playing more than one game in any day?

A. 11
B. 12
C. 13
D. 14

Problem 285: EE Board October 1996

There are five main roads between the cities A and B, and four between B and C. In how many ways can a person drive from A to C and return, going through B on both trips without driving on the same road twice?

A. 260
B. 240
C. 120
D. 160

Problem 286: EE Board April 1991

There are 50 tickets in a lottery in which there is a first and second prize. What is the probability of a man drawing a prize if he owns 5 tickets?

A. 50%
B. 25%
C. 20%
D. 40%

Problem 287:

Roll a pair of dice. What is the probability that the sum of two numbers is 11?

A. 1/36
B. 1/9
C. 1/18
D. 1/20

Problem 288:

Roll two dice once. What is the probability that the sum is 7?

A. 1/6
B. 1/8
C. 1/4
D. 1/7

Problem 289:

In a throw of two dice, the probability of obtaining a total of 10 or 12 is

A. 1/6
B. 1/9
C. 1/12
D. 1/18

Problem 290:

Determine the probability of drawing either a king or a diamond in a single draw from a pack of 52 playing cards.

A. 2/13
B. 3/13
C. 4/13
D. 1/13

Problem 291:

A card is drawn from a deck of 52 playing cards. Find the probability of drawing a king or a red card.

A. 0.5835
B. 0.5385
C. 0.3585
D. 0.8535

Problem 292: CE Board November 1998

A coin is tossed 3 times. What is the probability of getting 3 tails up?

A. 1/8
B. 1/16
C. 1/4
D. 7/8

Problem 293: EE Board April 1996

The probability of getting at least 2 heads when a coin is tossed four times is,

A. 11/16
B. 13/16
C. 1/4
D. 3/8

Problem 294:

A fair coin is tossed three times. What is the probability of getting either 3 heads or 3 tail?

A. 1/8
B. 3/8
C. 1/4
D. 1/2

Problem 295: ECE Board March 1996

The probability of getting a credit in an examination is 1/3. If three students are selected at random, what is the probability that at least one of them got a credit?

A. 19/27
B. 8/27
C. 2/3
D. 1/3

Problem 296:

There are 3 questions in a test. For each question 1 point is awarded for a correct answer and none for a wrong answer. If the probability that Janine correctly answers a question in the test is 2/3, determine the probability that she gets zero in the test.

A. 8/27
B. 4/9
C. 1/30
D. 1/27

Problem 297: EE Board April 1991

In the ECE Board Examinations, the probability that an examinee will pass each subject is 0.8. What is the probability that an examinee will pass at least two subjects out of the three board subjects?

A. 70.9 %
B. 80.9 %
C. 85.9 %
D. 89.6 %

Problem 298:

In a multiple choice test, each question is to be answered by selecting 1 out of 5 choices, of which only 1 is right. If there are 10 questions in a test, what is the probability of getting 6 right of pure guesswork?

A. 10 %
B. 6 %
C. 0.44 %
D. 0.55 %

Problem 299: ME Board April 1994

From a box containing 6 red balls, 8 white balls and 10 blue balls, one ball is drawn at random. Determine the probability that it is red or white.

A. 1/3
B. 7/12
C. 5/12
D. 1/4

Problem 300: EE Board October 1990

From a bag containing 4 black balls and 5 white balls, two balls are drawn one at a time. Find the probability that both balls are white. Assume that the first ball is returned before the second ball is drawn.

A. 25/81
B. 16/81
C. 5/18
D. 40/81

Problem 301: CE Board May 1996

A bag contains 3 white and 5 black balls. If two balls are drawn in succession without replacement, what is the probability that both balls are black?

A. 5/16
B. 5/28
C. 5/32
D. 5/14

Problem 302: ME Board April 1996

An urn contains 4 black balls and 6 white balls. What is the probability of getting 1 black and 1 white ball in two consecutive draws from the urn?

A. 0.24
B. 0.27
C. 0.53
D. 0.04

Problem 303: EE Board October 1990

From a bag containing 4 black balls and 5 white balls, two balls are drawn one at a time. Find the probability that one ball is white and one ball is black. Assume that the first ball is returned before the second ball is drawn.

A. 16/81
B. 25/81
C. 20/81
D. 40/81

Problem 304: EE Board October 1997

A group of 3 people enter a theater after the lights had dimmed. They are shown to the correct group of 3 seats by the usher. Each person holds a number stub. What is the probability that each is in the correct seat according to the numbers on seat and stub?

A. 1/6
B. 1/4
C. 1/2
D. 1/8

Problem 305:

From 20 tickets marked with the first 20 numerals, one is drawn at random. What is the chance that it will be a multiple of 3 or of 7?

A. 1/2
B. 8/15
C. 3/10
D. 2/5

ANSWER KEY

256. C	269. B	282. D	295. A
257. D	270. C	283. B	296. D
258. C	271. A	284. C	297. D
259. D	272. A	285. B	298. D
260. A	273. A	286. C	299. B
261. C	274. C	287. C	300. A
262. A	275. B	288. A	301. D
263. C	276. C	289. B	302. C
264. C	277. B	290. C	303. D
265. C	278. A	291. B	304. A
266. A	279. B	292. A	305. D
267. C	280. A	293. A	
268. A	281. A	294. C	

RATING

- ☐ 43-50 Topnotcher
- ☐ 33-42 Passer
- ☐ 25-32 Conditional
- ☐ 0-24 Failed

If FAILED, repeat the test.

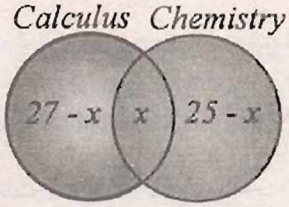
SOLUTIONS TO TEST 6

256. Let: x = number of students who like both subjects

$(27 - x) + x + (25 - x) = 40$

$27 + 25 - x = 40$

$x = 12$ students

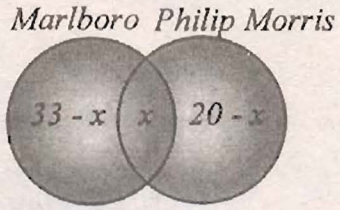


257. Let: x = number of executives who smoke both brand of cigarettes

$(33 - x) + x + (20 - x) = 40$

$33 + 20 - x = 40$

$x = 13$ executives

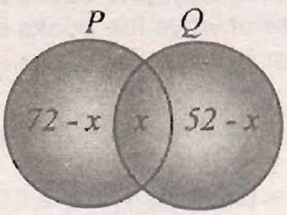


258. Let: x = number of persons who have eaten in both restaurants

$(72 - x) + x + (52 - x) = 100$

$72 + 52 - x = 100$

$x = 24$ persons

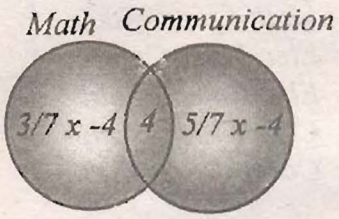


259. Let: x = number of examinees who took the examination

$$x = \left[\frac{3}{7}x - 4\right] + 4 + \left[\frac{5}{7}x - 4\right]$$

$$x = \frac{8}{7}x - 4$$

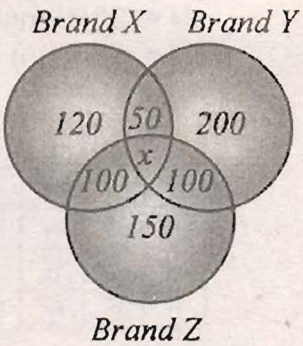
$x = 28$ examinees



260. Let: x = number of persons who have no brand preference

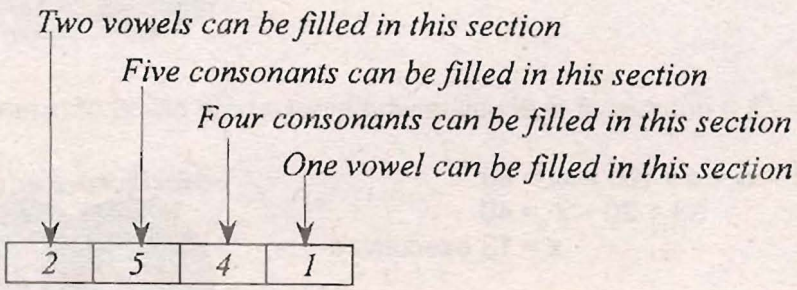
$1000 = x + 120 + 50 + 200 + 100 + 150 + 100$

$x = 280$ persons



261. Note: The survey is *not worth* paying for. The error is that according to the said survey, there are *6 people* who used *all three brands* but only *5 people* used the brands *Hapee and Close-up*.

262. Note: " PERSONNEL"
Number of vowels = 2 (E & O)
Number of constants = 5 (P, R, S, N & L)

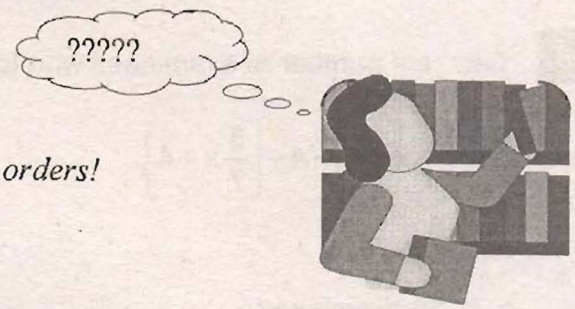


Let: N = number of words
N = 2(5)(4)(1) = 40 words

263. Number of ways the books in "Math" can be arranged = 5!
Number of ways the books in "Elec." can be arranged = 4!
Number of ways the books in "Comm." can be arranged = 2!

Let: N = total number of ways
N = (5!)(4!)(2!)(number of sequence or order)
N = (5!)(4!)(2!)(6) = 34,560 ways

Math-Elec-Comm
Math-Comm-Elec
Elec-Math-Comm
Elec-Comm-Math
Comm-Elec-Math
Comm-Math-Elec
} *6 orders!*



264. Number of ways the 3 nurses can be arranged = 3!
Number of ways the 4 engineers can be arranged = 4!

N = (3!)(4!)(number of sequence or order)
N = (3!)(4!)(5) = 720 ways

n	*n*	*n*	E	E	E	E
E	*n*	*n*	*n*	E	E	E
E	E	*n*	*n*	*n*	E	E
E	E	E	*n*	*n*	*n*	E
E	E	E	E	*n*	*n*	*n*

} *5 orders*

265. N = 15(14)(13)(12)(11)
N = 360,360 ways

15	14	13	12	11
1st prize	2nd prize	3rd prize	4th prize	5th prize

266. N = 5(4)(3)(2)
N = 120 numbers

5	4	3	2
1st digit	2nd digit	3rd digit	4th digit

267. ${}_nP_n = n!$

${}_6P_6 = 6! = 720$ ways

268. ${}_nP_n = n!$

${}_6P_6 = 6! = 720$ ways

269. Note: " BANANA"
Number of A's = 3
Number of N's = 2

$${}_nP_{p,q} = \frac{n!}{p!\,q!..} = \frac{6!}{3!\,2!} = 60 \text{ ways}$$

270. N = 10(8)(6)
N = 480 ways

10	8	6
M_1	M_2	M_3

271. N = 15(14)(13)(12)(11)
N = 360,360 ways

15	14	13	12	11
Pres.	V-Pres.	Sec.	Treas.	Aud.

272. With one flag:
$N_1 = 4$ signals
With two flags:
$N_2 = 4(3) = 12$ signals
With three flags:
$N_3 = 4(3)(2) = 24$ signals
With four flags:
$N_4 = 4(3)(2)(1) = 24$ signals

4			
4	3		
4	3	2	
4	3	2	1

N = 4 + 12 + 24 + 24 = 64 signals

273. Number of ways the 4 boys can be arranged = 4!
Number of ways the 4 girls can be arranged = 4!

N = (4! 4!)2 = 1152 ways

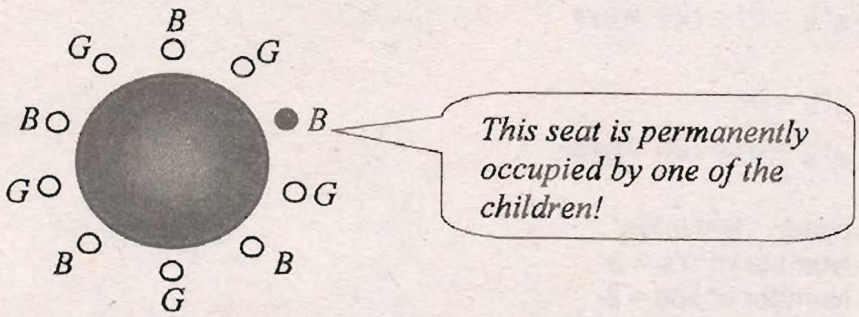
274. Note: Since the arrangement requires a definite order, then the said arrangement is under the principles of "permutation".

$${}_nP_r = \frac{n!}{(n-r)!}$$

$$_4P_2 = \frac{4!}{(4-2)!} = 12 \text{ arrangements}$$

275. Number of ways the boys can be arranged = (5 – 1)! = 4!
Number of ways the girls can be arranged = 5!

N = (4!)(5!) = 2880 ways



276. Note: Since the objects taken can be arrange in any way, then the said arrangement is under the principles of "combination".

$$_nC_r = \frac{n!}{(n-r)!r!}$$

$$_4C_2 = \frac{4!}{(4-2)!2!} = 6 \text{ combinations}$$

277. $$_{42}C_6 = \frac{42!}{(42-6)!6!} = 5{,}245{,}786 \text{ combinations}$$

278. $$_nC_{1,2,3\ldots n} = 2^n - 1$$

$$_3C_{1,2..} = 2^3 - 1 = 7 \text{ combinations}$$

279. $$_5C_{1,2..} = 2^5 - 1 = 31 \text{ combinations}$$

280. Number of ways of selecting a chemical eng'r: $_4C_2 = \frac{4!}{(4-2)!2!} = 6$

Number of ways of selecting a mechanical eng'r: $_3C_1 = \frac{3!}{(3-1)!1!} = 3$

N = 6(3) = 18 ways

281. $$_{10}C_7 = \frac{10!}{(10-7)!7!} = 120 \text{ ways}$$

282. $_{15}C_4 = \dfrac{15!}{(15-4)!4!} = 1365$ committees

283. Number of ways of hiring men:

$$_9C_7 = \frac{9!}{(9-7)!7!} = 36 \text{ ways}$$

Number of ways of hiring women:

$$_6C_4 = \frac{6!}{(6-4)!4!} = 15 \text{ ways}$$

N = 36(15) = 540 ways

284. Total number of games $= {}_{13}C_2 = \dfrac{13!}{(13-2)!2!} = 78$ games

Number of games that can be played per day: $\dfrac{13}{2} = 6.5$

= approximately 6 games per day

Number of days needed to complete the tournament:

$$= \frac{78}{6} = 13 \text{ days}$$

285. Number of ways to travel from A to B = 5 ways

Number of ways to travel from B to C = 4 ways

Number of ways to travel from C to B without using the same road to travel from B to C = 3 ways

Number of ways to travel from B to A without using the same road to travel from A to B = 4 ways

N = 5(4)(3)(4) = 240 ways

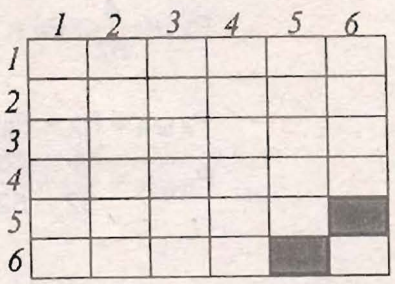
286. P = probability of the man to win a prize
P = number of tickets he bought x probability of winning the lottery

$$P = 5\left[\frac{2}{50}\right] = \frac{1}{5}$$

287. Number of trials with a sum of 11 = 2 trials

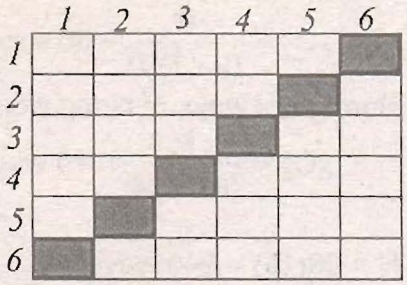
Total number of trials = $6^n = (6)^2 = 36$ trials

$$P = \frac{\text{number of successful trials}}{\text{total number trials}}$$



$$P = \frac{2}{36} = \frac{1}{18}$$

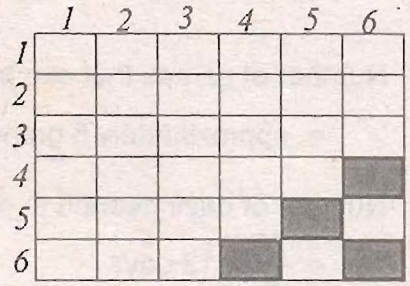
288. Number of trials with a sum of 7 = 6 trials
Total number of trials = 36 trials



$$P = \frac{\text{number of successful trials}}{\text{total number trials}}$$

$$P = \frac{6}{36} = \frac{1}{6}$$

289. Number of trials with a sum of 10 or 12 = 4 trials
Total number of trials = 36 trials



$$P = \frac{\text{number of successful trials}}{\text{total number trials}}$$

$$P = \frac{4}{36} = \frac{1}{9}$$

290. Let: P_K = probability of drawing a king
P_D = probability of drawing a diamond
$P_{K\,\&\,D}$ = probability of drawing a king at the time a diamond

$$P_K = \frac{4}{52};\ P_D = \frac{13}{52};\ P_{K\,\&\,D} = \frac{1}{52}$$

$$P_{K\,or\,D} = P_K + P_D - P_{K\,\&\,D}$$

$$P_{K\,or\,D} = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

291. Let: P_K = probability of drawing a king
P_R = probability of drawing a red card
$P_{K\,\&\,R}$ = probability of drawing a king at the same time a red card

$$P_K = \frac{4}{52};\ P_R = \frac{26}{52};\ P_{K\,\&\,R} = \frac{2}{52}$$

$$P_{K\,or\,R} = P_K + P_R - P_{K\,\&\,R}$$

$$P_{K\,or\,R} = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = 0.5385$$

292. $P={}_nC_r p^r q^{n-r}$

where: P = probability of getting 3 tails out from 3 trials
p = probability of getting a tail from a single trial
q = probability of not getting a tail from a single trial
n = number of trials
r = number of successful trials

$p=\frac{1}{2};\ q=\frac{1}{2};\ n=3;\ r=3$

Substitute: $P_{3T}={}_3C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0=\frac{3!}{(3-3)!3!}\left(\frac{1}{2}\right)^3=\frac{1}{8}$

293. $P={}_nC_r p^r q^{n-r}$

where: $p=\frac{1}{2};\ q=\frac{1}{2};\ n=4;\ r=2$

$$P_{2H}={}_4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2=\frac{4!}{(4-2)!2!}\left[\frac{1}{2}\right]^4=\frac{3}{8}$$

$$P_{3H}={}_4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1=\frac{4!}{(4-1)!3!}\left[\frac{1}{2}\right]^4=\frac{1}{4}$$

$$P_{4H}={}_4C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0=1\left[\frac{1}{2}\right]^4=\frac{1}{16}$$

$$P=\frac{3}{8}+\frac{1}{4}+\frac{1}{16}=\frac{11}{16}$$

294. $P={}_nC_r p^r q^{n-r}$

where: $p=\frac{1}{2};\ q=\frac{1}{2};\ n=3;\ r=3$

$$P_{3H}={}_3C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0=\frac{3!}{(3-3)!3!}\left[\frac{1}{2}\right]^3=\frac{1}{8}$$

$$P_{3T}={}_3C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0=\frac{3!}{(3-3)!3!}\left[\frac{1}{2}\right]^3=\frac{1}{8}$$

$$P_{3H\text{ or }3T}=P_{3H}+P_{3T}$$

$$P_{3H\text{ or }3T}=\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$$

295. Note: Probability of getting a credit = 1/3
Probability of not getting any credit = 2/3
Let: P_1 = probability that only one student gets a credit

$$P_1={}_3C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^2=\frac{3!}{(3-1)!1!}\left[\frac{4}{27}\right]=\frac{12}{27}$$

P_2 = probability that exactly two students get a credit

$$P_2 = {}_3C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1 = \frac{3!}{(3-2)!2!}\left[\frac{2}{27}\right] = \frac{6}{27}$$

P_3 = probability that all three students get a credit

$$P_3 = {}_3C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^0 = (1)\left[\frac{1}{27}\right] = \frac{1}{27}$$

P = probability that at least one student gets a credit
$P = P_1 + P_2 + P_3$

$$P = \frac{12}{27} + \frac{6}{27} + \frac{1}{27} = \frac{19}{27}$$

296. Note: The only way she can get zero is, if all her 3 answers were wrong.

Let: p = probability of getting a correct answer = 2/3
q = probability of getting a wrong answer = 1/3

$$P = {}_nC_r p^r q^{n-r}$$

$$P = {}_3C_3\left[\frac{1}{3}\right]^3\left[\frac{2}{3}\right]^0 = (1)\left[\frac{1}{3}\right]^3 = \frac{1}{27}$$

297. Note: Probability of getting a passing score in each subject is 0.8
Probability of failing in any of the three subjects is 0.2

Let: P_1 = probability of passing exactly two subjects

$$P_1 = {}_3C_2(0.8)^2(0.2)^1 = \frac{3!}{(3-2)!2!}(0.128) = 0.384$$

P_2 = probability of passing all the three subjects

$$P_2 = {}_3C_3(0.8)^3(0.2)^0 = (1)(0.512) = 0.512$$

P = probability of passing at least two subjects
$P = P_1 + P_2$
$P = 0.384 + 0.512 = 0.896$ or 89.6%

298. Note: Probability of getting a correct answer is 1/5
Probability of getting a wrong answer is 4/5

Let: P = probability of getting 6 correct answers out from 10 questions

$$P = {}_{10}C_6\left(\frac{1}{5}\right)^6\left(\frac{4}{5}\right)^4 = \frac{10!}{(10-6)!6!}\left(2.62 \times 10^{-5}\right) = 0.0055 \text{ or } 0.55\%$$

299. Let: P = probability of getting a red or a white ball from the box

$$P = \frac{\text{number of red or white balls}}{\text{total number of balls}} = \frac{14}{24} = \frac{7}{12}$$

300. Let: P_1 = probability of drawing a white ball in the first draw

$$P_1 = \frac{5}{9}$$

P_2 = probability of drawing a white ball in the second draw

Note: The 1st ball was returned in the bag before the 2nd ball was drawn

$$P_2 = \frac{5}{9}$$

P = probability that both balls drawn are all white

$$P = P_1 \times P_2$$

$$P = \frac{5}{9} \times \frac{5}{9} = \frac{25}{81}$$

301. Let: P_1 = probability of drawing a black ball in the first draw

$$P_1 = \frac{5}{8}$$

P_2 = probability of drawing a black ball in the second draw

Note: The 1st ball was not returned in the bag before the 2nd ball was drawn

$$P_2 = \frac{4}{7}$$

P = probability that both balls drawn are all black

$$P = P_1 \times P_2$$

$$P = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$$

302. Assume the first draw is black and the second draw is white:

$$P_1 = P_{black} \times P_{white}$$

$$P_1 = \frac{4}{10} \times \frac{6}{9} = \frac{24}{90}$$

Assume the first draw is white and the second draw is black:

$$P_2 = P_{white} \times P_{black}$$

$$P_2 = \frac{6}{10} \times \frac{4}{9} = \frac{24}{90}$$

Let: P = probability that one ball is black and the other is white

$$P = P_1 + P_2$$

$$P = \frac{24}{90} + \frac{24}{90} = 0.53$$

303. Assume the first draw is black and the second draw is white:

$$P_1 = P_{black} \times P_{white}$$

$$P_1 = \frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$$

Assume the first draw is white and the second draw is black:

$$P_2 = P_{white} \times P_{black}$$

$$P_2 = \frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$$

Let: P = probability that one ball is black and the other is white

$$P = P_1 + P_2$$

$$P = \frac{20}{81} + \frac{20}{81} = \frac{40}{81}$$

304. Probability that A is correct = 1/3, assuming he is to sit down first
Probability that B is correct = 1/2, assuming he is to sit down after A
Probability that C is correct = 1, assuming he is the last to sit down

Let: P = probability that A, B and C are correct

$$P = \frac{1}{3} \times \frac{1}{2} \times 1 = \frac{1}{6}$$

Alternate solution: ❐

Total number of possible arrangements = 3! = 6 arrangements

$$P = \frac{\text{correct arrangement}}{\text{total number of arrangements}} = \frac{1}{6}$$

305. Numbers from 1 to 20, which is divisible by 3 = 6 numbers (3,6,9,12,15,18)

Numbers from 1 to 20, which is divisible by 7 = 2 numbers (7,14)

Total numbers from 1 to 20, which is divisible by 3 or 7 = 8 numbers

Let: P = probability that the ticket number is divisible by 3 or 7

$$P = \frac{\text{numbers between 1 to 20 which is divisible by 3 or 7}}{\text{total numbers from 1 to 20}}$$

$$P = \frac{8}{20} = \frac{2}{5}$$

DAY 7

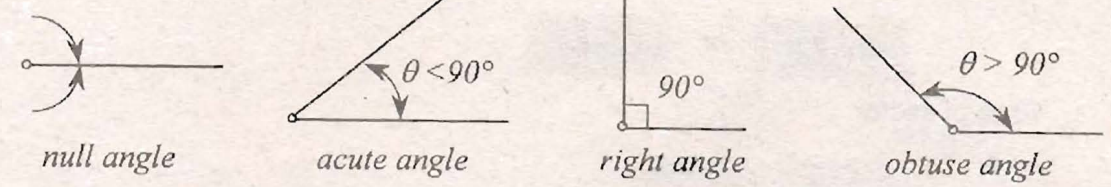
PLANE GEOMETRY

The term "**geometry**" was derived from the Greek words, "*ge*" meaning earth and "*metria*" meaning measurement. **Euclid** (c. 330 – c 275 B.C.) in his best known book in geometry "Elements" give more emphasis on **Plane geometry** which concerns with geometric figures constructed on a plane surface or geometrical shape of two dimensions (i.e angle, triangle, conic section, etc.) . **Archimedes** (287 – 212 B.C.) contributed so much to **Solid geometry** which concerns with three dimensional geometric figures such as cylinder, sphere, pyramid, angle between planes, etc. **Descriptive geometry**, developed by **Ptolemy** deals with the application of geometry to astronomy.

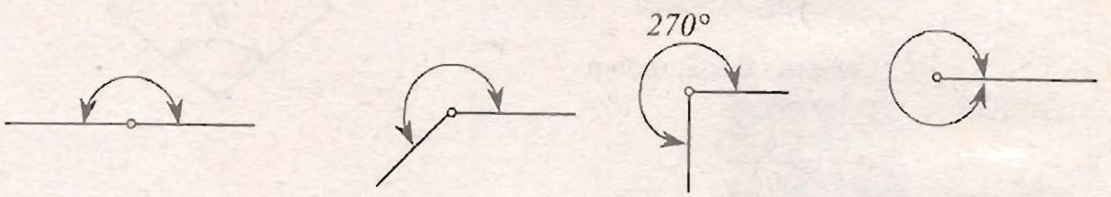
ANGLES

Angle is formed by two rays which extend from a common point called **vertex**.

Different types of angles:



null angle *acute angle* *right angle* *obtuse angle*



straight angle *reflex angle (> 180°)* *full angle or perigon*

Adjacent angles are two angles with a leg in common.
Complementary angles are two angles whose sum is a right angle (90°).
Supplementary angles are two angles whose sum is a straight angle (180°).
Vertical angles are angles formed by two intersecting lines. Vertical angles are equal.

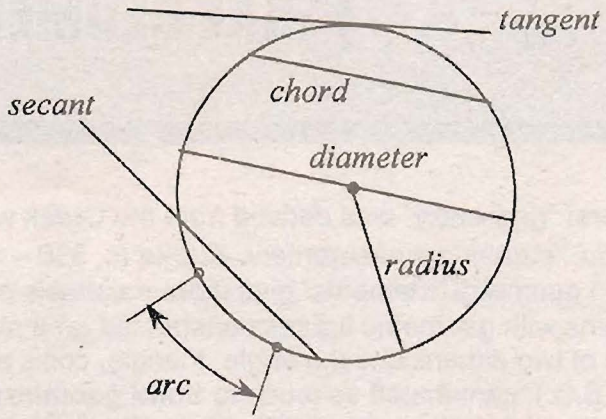
CIRCLES

Circle is a plane figure that is a locus of all points in the plane equidistant from a given point, the center of the circle.

Circumference is the length of the perimeter of a circle.

Sector is bounded by two radii and an included arc.

Segment is bounded by a chord and the arc subtending the chord.



Area of a circle:

$$A = \pi r^2 \quad \text{or} \quad A = \frac{\pi}{4} d^2$$

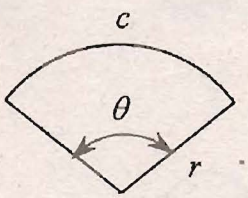
where: r = radius
d = diameter

Circumference of a circle:

$$C = 2\pi r \quad \text{or} \quad C = \pi d$$

Area of sector:

$$A = \frac{1}{2} rc = \frac{1}{2} r^2 \theta$$

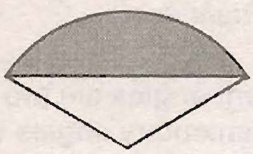


where : θ is in radian

$$A = \frac{\pi r^2 \theta}{360}$$

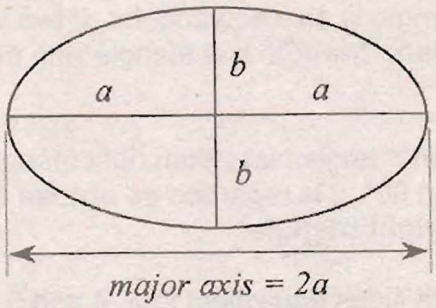
where: θ is in degrees

Area of segment:



A = Area of sector - Area of triangle

Ellipse is a locus of a point which moves so that the sum of its distances to the fixed points (foci) is constant and is equal to the length of the major axis.



Area of an ellipse:

$$A = \pi ab$$

where: a = semi-major axis
b = semi-minor axis

POLYGONS

Polygon is a closed plane figure with three or more angles. There are as many sides as angles in a polygon. The term "polygon" comes from Greek words *"poly"* meaning many and *"gonia"* meaning angle. Polygons are named according to the number of sides or vertices:

Number of sides	Name
3	Triangle
4	Quadrilateral or Tetragon
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon
1000	Chilliagon
n	n-gon

Regular polygon is a polygon having all sides equal and all interior angles equal.
Convex polygon is a polygon having each interior angle less than 180°.
Concave polygon is a polygon having one interior angle greater than 180°.
Diagonal is a line that connects two non-adjacent vertices.

Number of diagonals of a given polygon:

$$\text{Diagonals} = \frac{n}{2}(n-3)$$

Sum of interior angles:

$$S = (n-2)180°$$

where: n = number of sides of the polygon

TRIANGLES

Triangle is a polygon with three sides. If three sides of a triangle are equal, it is an **equilateral triangle**. An equilateral triangle is also equiangular. If two sides are equal, it is an **isosceles triangle**. **Scalene triangle** is a triangle with no two sides equal.

Acute triangle is a triangle with all interior angles less than right triangle (90°). If one of the interior angles is greater than 90°, it is regarded as **obtuse triangle**. If one interior angle is exactly 90°, it is a **right triangle**.

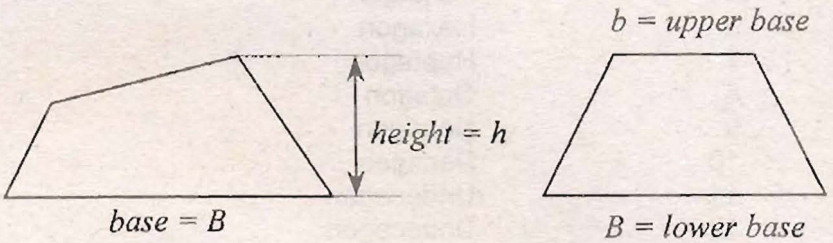
Egyptian triangle is a right triangle with sides equivalent to 3, 4 and 5 units.

Note: For the formulas for area of a triangle, refer to Trigonometry.

QUADRILATERALS

A **quadrilateral** (also known as **quadrangle** or **tetragon**) is a polygon with four sides.

In the U.S. standard, a quadrilateral with two sides parallel is called as **trapezoid** and **trapezium** if no two sides are parallel. It is opposite in the United Kingdom, where it is referred to as **trapezium** if two sides are parallel and **trapezoid** if no two sides are parallel.



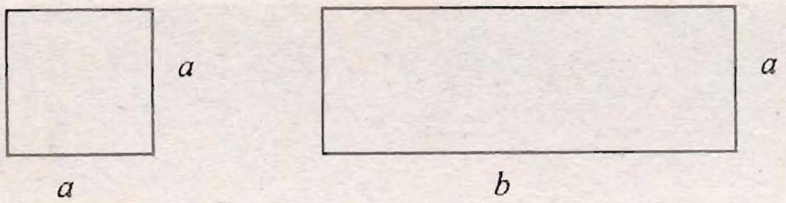
US :	*Trapezium*	*Trapezoid*
UK:	*Trapezoid*	*Trapezium*

Area of trapezoid:

$$A = \frac{1}{2}(B + b)h$$

PARALLELOGRAMS

A **parallelogram** is a quadrilateral with both pairs of opposite sides are parallel. A right-angled parallelogram is called a **rectangle**. A **square** is a rectangle with all four sides equal.



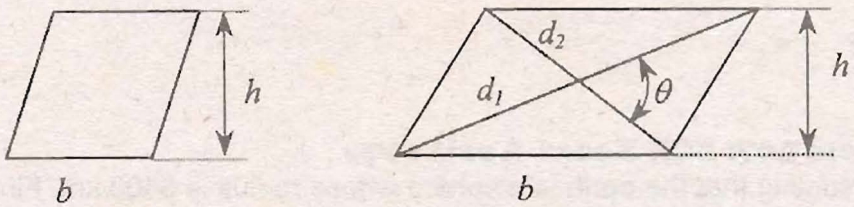
Area of square:

$$A = a^2$$

Area of rectangle:

$$A = ab$$

Rhomboid is a parallelogram whose adjacent sides are not equal while **rhombus** is a rhomboid with all sides equal.



Area of rhombus:

$$A = bh = \frac{1}{2} d_1 d_2$$

Area of rhomboid:

$$A = bh = \frac{1}{2} d_1 d_2 \sin\theta$$

where: d_1 and d_2 = diagonals
In the case of a rhombus, the value of $\theta = 90°$.



Tips: 1. **Complementary angles** are two angles whose sum is 90° or right angle.
2. **Supplementary angles** are two angle whose sum is 180° or straight angle.
3. **Explementary angles** are two angles whose sum is 360° or perigon.

Did you know that... the symbols + and – for plus and minus, respectively was introduced by German mathematician and astronomer, Johannes Regiomontanus in 1456!

Proceed to the next page for your 7th test. GOODLUCK ! ☞

Test 7

Time element: 3 hours & 45 minutes

Problem 306: ECE Board November 1998

Find the angle in mils subtended by a line 10 yards long at a distance of 5000 yards.

A. 1
B. 2
C. 2.5
D. 4

Problem 307: ECE Board April 1999

Assuming that the earth is a sphere whose radius is 6400 km. Find the distance along a 3 degree arc at the equator of the earth's surface.

A. 335.10 km
B. 533.10 km
C. 353.10 km
D. 353.01 km

Problem 308: EE Board April 1992

The angle subtended by an arc is 24°. If the radius of the circle is 45 cm, find the length of arc.

A. 16.85 cm
B. 17.85 cm
C. 18.85 cm
D. 19.85 cm

Problem 309: ME Board April 1990

A rat fell on a bucket of a water wheel with diameter of 600 cm which traveled an angle of 190° before it dropped from the bucket. Calculate for the linear cm that the rat was carried by the bucket before it fell.

A. 950
B. 965
C. 985
D. 995

Problem 310: ECE Board November 1992

Given a circle whose diameter AB equals 2 m. If two points C and D lie on the circle and angles ABC and BAD are 18° and 36°, respectively, find the length of the major arc CD.

A. 1.26 m
B. 1.36 m
C. 1.63 m
D. 1.45 m

Problem 311:

A certain angle has a supplement 5 times its complement. What is the angle?

A. 67.5°
B. 58.5°
C. 30°
D. 27°

Problem 312: ECE Board November 1998

Each angle of a regular dodecagon is equal to

A. 135°
B. 150°
C. 125°
D. 105°

Problem 313: CE Board May 1997

How many sides has a polygon if the sum of the interior angles is 1080°?

A. 5
B. 6
C. 7
D. 8

Problem 314: ECE Board March 1996

The sum of the interior angles of a polygon is 540°. Find the number of sides.

A. 3
B. 4
C. 5
D. 6

Problem 315: ECE Board April 1991

Find the sum of the interior angles of the vertices of a five pointed star inscribed in a circle.

A. 150°
B. 160°
C. 170°
D. 180°

Problem 316: ME Board April 1999

How many sides are in a polygon if each interior angle is 165 degrees?

A. 12
B. 24
C. 20
D. 48

Problem 317:

How many diagonals are there in a polygon of 20 sides?

A. 200
B. 170
C. 100
D. 158

Problem 318: ME Board April 1999

Find each interior angle of a hexagon.

A. 90°
B. 120°
C. 150°
D. 180°

Problem 319: EE Board April 1994

Given a triangle, C = 100°, a = 15 m, b = 20 m. Find c.

A. 26 m
B. 27 m
C. 28 m
D. 29 m

Problem 320: CE Board November 1994

In triangle ABC, angle A = 45° and C = 70°. The side opposite angle C is 40 m long. What is the length of the side opposite angle A?

A. 26.1 m
B. 27.1 m
C. 29.1 m
D. 30.1 m

Problem 321: CE Board May 1995

In triangle ABC, angle C = 70°, A = 45°, AB = 40 m. What is the length of the median drawn from vertex A to side BC?

A. 36.3 m
B. 36.6 m
C. 36.9 m
D. 37.2 m

Problem 322: EE Board April 1991

From a point outside of an equilateral triangle, the distances to the vertices are 10 m, 18 m and 10 m, respectively. What is the length of one side of a triangle?

A. 17.75 m
B. 18.50 m
C. 19.95 m
D. 20.50 m

Problem 323: EE Board April 1991

The sides of a triangle are 8 cm, 10 cm and 14 cm. Determine the radius of the inscribed circle.

A. 2.25 cm
B. 2.35 cm
C. 2.45 cm
D. 2.55 cm

Problem 324: CE Board May 1996

What is the radius of the circle circumscribing an isosceles right triangle having an area of 162 sq. cm.?

A. 12.73 m
B. 13.52 m
C. 14.18 m
D. 15.55 m

Problem 325: EE Board April 1991

The sides of a triangle are 8 cm, 10 cm and 14 cm. Determine the radius of the circumscribing circle.

A. 7.14 cm
B. 7.34 cm
C. 7.54 cm
D. 7.74 cm

Problem 326: CE Board May 1996

Two sides of a triangle are 50 m and 60 m long. The angle included between these sides is 30°. What is the interior angle opposite the longest side?

A. 93.74°
B. 92.74°
C. 90.74°
D. 86.38°

Problem 327: ECE Board March 1996

A circle with radius 6 cm has half its area removed by cutting off a border of uniform width. Find the width of the border.

A. 1.76 cm
B. 1.35 cm

C. 1.98 cm
D. 2.03 cm

Problem 328: ME Board April 1996

The area of a circle is 89.42 sq. inches. What is its circumference?

A. 32.25 in.
B. 33.52 in.
C. 35.33 in.
D. 35.55 in.

Problem 329: ECE Board April 1991

A square section ABCD has one of its sides equal to x. Point E is inside the square forming an equilateral triangle BEC having one side equal in length to the side of the square. Find the angle AED.

A. 130°
B. 140°
C. 150°
D. 160°

Problem 330: CE Board November 1995

The area of a circle circumscribing about an equilateral triangle is 254.47 sq. m. What is the area of the triangle in sq. m?

A. 100.25
B. 102.25
C. 104.25
D. 105.25

Problem 331: CE Board May 1995

What is the area in sq. cm of the circle circumscribed about an equilateral triangle with a side 10 cm long?

A. 104.7
B. 105.7
C. 106.7
D. 107.7

Problem 332: CE Board November 1992

The area of a triangle inscribed in a circle is 39.19 cm^2 and the radius of the circumscribed circle is 7.14 cm. If the two sides of the inscribed triangle are 8 cm and 10 cm, respectively, find the third side.

A. 11 cm
B. 12 cm
C. 13 cm
D. 14 cm

Problem 333: CE Board November 1994

The area of a triangle is 8346 sq. m and two of its interior angles are 37°25' and 56°17'. What is the length of the longest side?

A. 171.5 m
B. 181.5 m
C. 191.5 m
D. 200.5 m

Problem 334: ECE Board April 1998

The angle of a sector is 30° and the radius is 15 cm. What is the area of the sector in cm^2?

A. 59.8
B. 89.5
C. 58.9
D. 85.9

Problem 335: EE Board April 1992

Two perpendicular chords both 5 cm from the center of a circle divide the circle into four parts. If the radius of the circle is 13 cm, find the area of the smallest part.

A. 30 cm^2
B. 31 cm^2
C. 32 cm^2
D. 33 cm^2

Problem 336: ECE Board April 1998

The distance between the centers of the three circles which are mutually tangent to each other externally are 10, 12 and 14 units. The area of the largest circle is

A. 72 π
B. 23 π
C. 64 π
D. 16 π

Problem 337: ECE Board November 1993

The arc of a sector is 9 units and its radius is 3 units. What is the area of the sector in square units?

A. 12.5
B. 13.5
C. 14.5
D. 15.5

Problem 338: CE Board May 1998

A circle having an area of 452 sq. m is cut into two segments by a chord which is 6 m from the center of the circle. Compute the area of the bigger segment.

A. 354.89 sq. m
B. 363.68 sq. m
C. 378.42 sq. m
D. 383.64 sq. m

Problem 339: ECE Board April 1992

A swimming pool is constructed in the shape of two partially overlapping identical circles. Each of the circles has a radius of 9 m and each circle passes through the center of the other. Find the area of the swimming pool.

A. 380 m^2
B. 390 m^2
C. 400 m^2
D. 410 m^2

Problem 340: ME Board April 1991

Find the difference of the area of the square inscribed in a semi-circle having a radius of 15 m. The base of the square lies on the diameter of the semi-circle.

A. 171.5 cm^2
B. 172.5 cm^2
C. 173.5 cm^2
D. 174.5 cm^2

Problem 341: ECE Board November 1995

A rectangle ABCD which measures 18 cm. by 24 cm. is folded once, perpendicular to diagonal AC, so that the opposite vertices A and C coincide. Find the length of the fold.

A. 20.5 cm
B. 21.5 cm
C. 22.5 cm
D. 23.5 cm

Problem 342: ECE Board April 1998

A trapezoid has an area of 36 m^2 and an altitude of 2 m. Its two bases have ratio of 4:5. What are the lengths of the bases?

A. 12, 15
B. 7, 11
C. 8, 10
D. 16, 20

Problem 343: EE Board March 1998

A rhombus has diagonals of 32 and 20 inches. Determine its area.

A. 360 in^2
B. 280 in^2
C. 320 in^2
D. 400 in^2

Problem 344: ECE Board April 1998

If the sides of a parallelogram and an included angle are 6, 10 and 100°, respectively, find the length of the shorter diagonal.

A. 10.63
B. 10.37
C. 10.73
D. 10.23

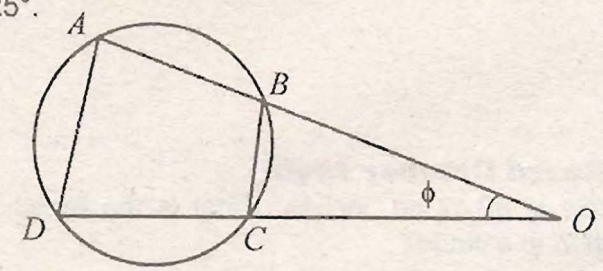
Problem 345: CE Board November 1996

Find the area of a quadrilateral having sides AB = 10 cm, BC = 5 cm, CD = 14.14 cm and DA = 15 cm, if the sum of the opposite angles is equal to 225°.

A. 96 sq. cm
B. 100 sq. cm
C. 94 sq. cm
D. 98 sq. cm

Problem 346: EE Board October 1992

Determine the area of the quadrilateral shown, OB = 80 cm, AO = 120 cm, OD = 150 cm and ϕ = 25°.



A. 2721.66 cm^2
B. 2271.66 cm^2
C. 2172.66 cm^2
D. 2217.66 cm^2

Problem 347: CE Board October 1997

Find the area of a quadrilateral have sides 12 m, 20 m, 8 m and 16.97 m. if the sum of the opposite angles is equal to 225°, find the area of the quadrilateral.

A. 100 m^2
B. 124 m^2
C. 168 m^2
D. 158 m^2

Problem 348: ME Board October 1996, ME Board April 1997

The area of a regular hexagon inscribed in a circle of radius 1 is

A. 1.316
B. 2.945
C. 2.598
D. 3.816

Problem 349: EE Board April 1990

Find the area (in cm^2) of a regular octagon inscribed in a circle of radius 10 cm?

A. 283
B. 289
C. 298
D. 238

Problem 350: GE Board February 1992

A regular hexagon is inscribed in a circle whose diameter is 20 m. Find the area of the 6 segments of the circle formed by the sides of the hexagon.

A. 36.45 sq. m
B. 63.54 sq. m
C. 45.63 sq. m
D. 54.36 sq. m

Problem 351: EE Board April 1993

Find the area of a regular pentagon whose side is 25 m and apothem is 17.2 m.

A. 1075 m^2
B. 1085 m^2
C. 1080 m^2
D. 1095 m^2

Problem 352: ME Board October 1996

The area of a circle is 89.42 sq. inches. What is the length of the side of a regular hexagon inscribed in a circle?

A. 5.533 in.
B. 5.335 in.
C. 6.335 in.
D. 7.335 in.

Problem 353: EE Board April 1990

In a circle of diameter of 10 m, a regular five-pointed star touching its circumference is inscribed. What is the area of that part not covered by the star?

A. 40.5 sq. m
B. 45.5 sq. m
C. 50.5 sq. m
D. 55.5 sq. m

Problem 354: EE Board March 1998

A regular pentagon has sides of 20 cm. An inner pentagon with sides of 10 cm is inside and concentric to the larger pentagon. Determine the area inside and concentric to the larger pentagon but outside of the smaller pentagon.

A. 430.70 cm^3
B. 573.26 cm^3
C. 473.77 cm^3
D. 516.14 cm^3

Problem 355: EE Board March 1999

Determine the area of a regular 6-star polygon if the inner regular hexagon has 10 cm sides.

A. 441.66 cm^2
B. 467.64 cm^2
C. 519.60 cm^2
D. 493.62 cm^2

ANSWER KEY

306. B	319. B	332. D	345. B
307. A	320. D	333. B	346. A
308. C	321. A	334. C	347. C
309. D	322. C	335. B	348. C
310. A	323. C	336. C	349. A
311. A	324. A	337. B	350. D
312. B	325. A	338. B	351. A
313. D	326. D	339. D	352. B
314. C	327. A	340. C	353. C
315. D	328. B	341. C	354. D
316. B	329. C	342. D	355. C
317. B	330. D	343. C	
318. B	331. A	344. C	

RATING

- ☐ 43-50 Topnotcher
- ☐ 33-42 Passer
- ☐ 25-32 Conditional
- ☐ 0-24 Failed

If FAILED, repeat the test.

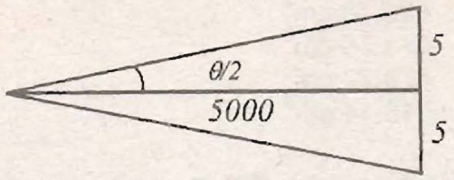
SOLUTIONS TO TEST 7

306. $\tan\frac{\theta}{2} = \frac{5}{5000}$

$\frac{\theta}{2} = \tan^{-1}\frac{1}{1000}$

$\theta = 2(0.0573°) = 0.1146°$

$\theta = 0.1146° \times \frac{6400 \text{ mils}}{360°} = 2 \text{ mils}$

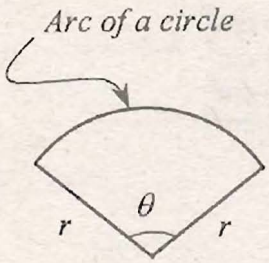


307. $C = r\theta$

where: C = length of the arc
r = radius of the circle
θ = included angle in radians

$C = 6400\left[3° \times \frac{2\pi \text{ radians}}{360°}\right]$

C = 335.10 km.



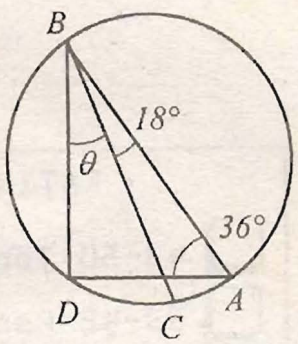
308. $C = r\theta$

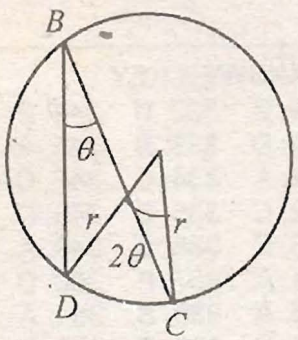
$C = 45 \times \left[24^\circ \times \frac{2\pi \text{ rad}}{360^\circ}\right] = 18.85 \text{ cm}$

309. $C = r\theta$

$C = 300 \times \left[190^\circ \times \frac{2\pi \text{ rad}}{360^\circ}\right] = 995 \text{ cm.}$

310.





Note: 1. For the same intercepted arc (arc CD), the value of the central angle is twice that of the inscribed angle.
2. If one side of the triangle inscribed in a circle is equal to the diameter of the circle, then the said triangle is a right triangle.

Solving for θ:

$18 + \theta + 36 = 90$

$\theta = 36°$
$2\theta = 72°$

Let: C = length of arc CD
$C = r(2\theta)$

$$C = 1 \text{ x} \left[72° \text{ x} \frac{2\pi \text{ radians}}{360°} \right] = 1.26 \text{ m.}$$

311. Let: θ = the angle
$90° - \theta$ = the complement of angle θ
$180° - \theta$ = the supplement of angle θ

$180 - \theta = 5(90 - \theta)$
$180 - \theta = 450 - 5\theta$
$4\theta = 270$
$\theta = 67.5°$

312. $\theta = \dfrac{(n-2)(180^\circ)}{n}$

Note: A dodecagon has 12 sides, thus n = 12.

$$\theta = \frac{(12-2)(180^\circ)}{12} = 150°$$

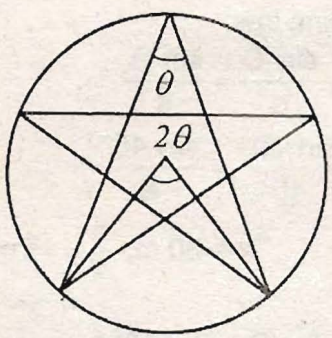
313. Sum of interior angles = (n - 2)(180°)
$1080° = (n - 2)(180°)$
$n - 2 = 6$
$n = 8$ sides

314. Sum of interior angles = (n - 2)(180°)
$540° = (n - 2)(180°)$
$n - 2 = 3$
$n = 5$ sides

315. $2\theta = \dfrac{360°}{5} = 72°$

$\theta = 36°$

Sum of the interior angles
of the 5 vertices = 5θ
$= 5(36°)$
$= 180°$



316. $\theta = \dfrac{(n-2)(180^\circ)}{n}$

$165° = \dfrac{(n-2)(180°)}{n}$

$165n = 180n - 360$

$15n = 360$

$n = 24$ sides

317. Let: N = number of diagonals

$$N = \frac{n}{2}(n-3)$$

$$N = \frac{20}{2}(20-3) = 170 \text{ diagonals}$$

318. $\theta = \dfrac{(n-2)(180^\circ)}{n}$

Note: A hexagon has 6 sides, thus n = 6.

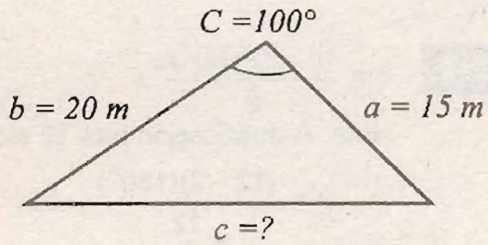
$$\theta = \frac{(6-2)(180^\circ)}{6} = 120^\circ$$

319. By cosine law:

$$c = \sqrt{a^2 + b^2 - 2ab\cos C}$$

$$= \sqrt{(15)^2 + (20)^2 - 2(15)(20)\cos 100^\circ}$$

$c = 27\text{m}$

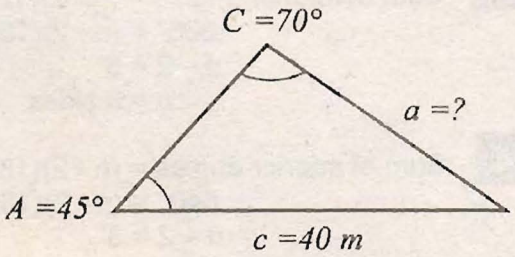


320. By sine law:

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$a = c\left[\frac{\sin A}{\sin C}\right]$$

$$a = 40\left[\frac{\sin 45^\circ}{\sin 70^\circ}\right] = 30.1\text{m}$$



321. By sine law:

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 70°}{40} = \frac{\sin 45°}{a}$$

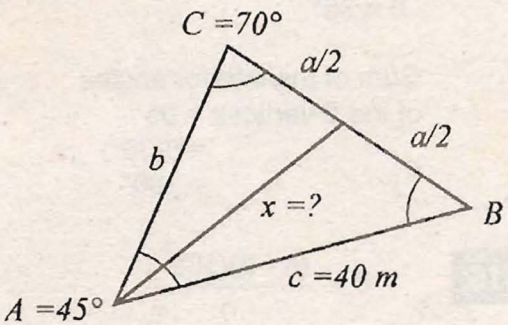
$$a = 30 \text{ m}; \quad \frac{a}{2} = 15\,\text{m}$$

$$A + B + C = 180°$$

$$45° + B + 70° = 180°$$

$$B = 65°$$

By cosine law:



$$x = \sqrt{c^2 + \left(\frac{a}{2}\right)^2 - 2(c)\left(\frac{a}{2}\right)\cos B}$$

$$= \sqrt{(40)^2 + (15)^2 - 2(40)(15)\cos 65^\circ}$$

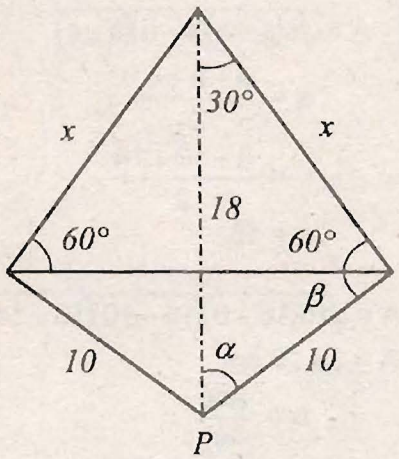
$$x = 36.3 \text{ m}$$

322. By sine law:

$$\frac{\sin 30}{10} = \frac{\sin(60+\beta)}{18}$$

$$60 + \beta = \sin^{-1}\frac{18 \sin 30^\circ}{10}$$

$$\beta = 4.158^\circ$$



$$\alpha + \beta + 30^\circ + 60^\circ = 180^\circ$$

$$\alpha + 4.158^\circ + 30^\circ + 60^\circ = 180^\circ$$

$$\alpha = 85.842^\circ$$

By sine law:

$$\frac{\sin 85.842^\circ}{x} = \frac{\sin 30^\circ}{10}$$

$$x = 19.95 \text{ cm}$$

323. $A = \sqrt{s(s-a)(s-b)(s-c)}$

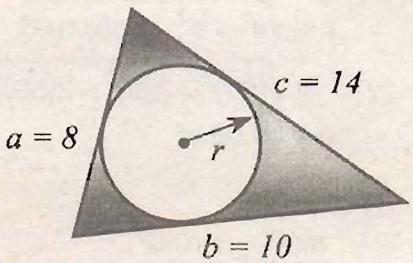
$$s = \frac{a+b+c}{2}$$

$$= \frac{8+10+14}{2}$$

$$s = 16$$

$$A = \sqrt{16(16-8)(16-10)(16-14)}$$

$$A = 39.19 \text{ cm}^2$$



$$A = r\,s$$

$$39.19 = r\,(16)$$

$$r = 2.45 \text{ cm}$$

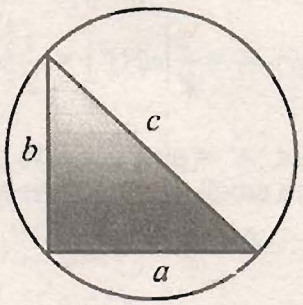
324. Note: Since an isosceles right triangle, "a" must be equal to "b" and the diameter of the circumscribing circle must be equal to the hypotenuse "c" of the inscribed right triangle.

$$A = \frac{1}{2}ab = \frac{1}{2}(a)(a) = \frac{1}{2}a^2$$

$$162 = \frac{1}{2}a^2$$

$$a = 18$$

Using Phytagorean theorem:



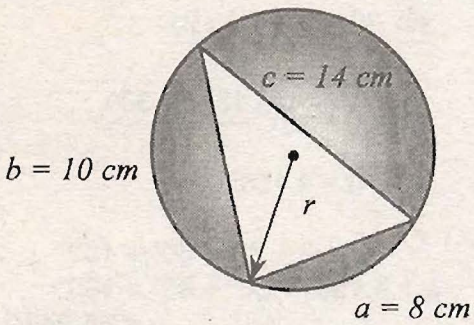
$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{(18)^2 + (18)^2} = 25.445 \text{ cm.}$$

$$c = 2r$$
$$2r = 25.445$$
$$r = 12.73 \text{ cm}$$

325. $A = \sqrt{s(s-a)(s-b)(s-c)}$

$$s = \frac{a+b+c}{2}$$
$$= \frac{8+10+14}{2}$$
$$s = 16$$

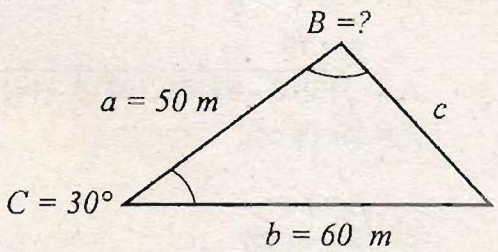


$$A = \sqrt{16(16-8)(16-10)(16-14)}$$
$$A = 39.19 \text{ cm}^2$$

$$A = \frac{abc}{4r}$$
$$39.19 = \frac{8(10)(14)}{4r}$$
$$r = 7.14 \text{ cm}$$

326. By cosine law:

$$c = \sqrt{a^2 + b^2 - 2ab\cos C}$$
$$= \sqrt{(50)^2 + (60)^2 - 2(50)(60)\cos 30^\circ}$$
$$c = 30.06 \text{ m}$$



By sine law:

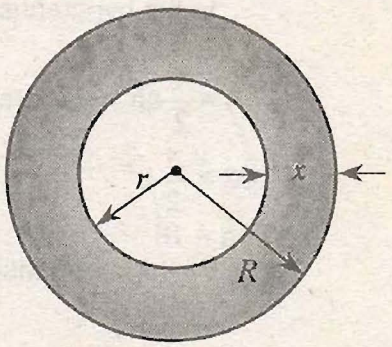
$$\frac{\sin B}{60} = \frac{\sin 30^\circ}{30.06}$$
$$B = \sin^{-1}\frac{60\sin 30^\circ}{30.06}$$
$$B = 86.38^\circ$$

327. Note: Since half of the area was removed, then the area (A) left is also one-half of the total area.

$$A = \frac{1}{2}\left[\pi(R)^2\right] = \frac{1}{2}\left[\pi(6)^2\right] = 18\pi$$

But "A" is also equal to the area of the small circle. (Refer to figure)

$$A = \pi r^2$$



$18\pi = \pi r^2$

$r = 4.24$ cm.

$x = R - r$

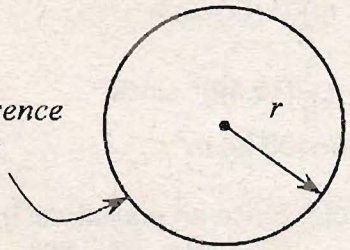
$x = 6 - 4.24 = 1.76$ cm.

328.

$A = \pi r^2$

$89.42 = \pi r^2$

$r = 5.335$ in



Let: C = circumference of the circle

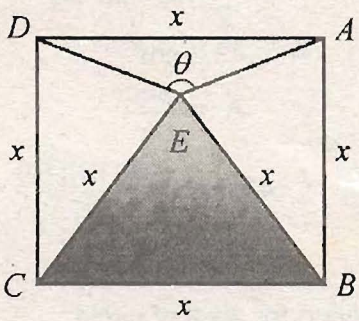
$C = 2\pi r$

$= (2\pi)(5.335)$

$C = 33.52$ in.

329. Note: Since equilateral triangle,

$\angle BEC = \angle BCE = \angle CBE = 60°$



$\angle BCE + \angle ECD = 90°$

$60° + \angle ECD = 90°$

$\angle ECD = 30°$

$\angle CED + \angle EDC + \angle ECD = 180°$

Since isosceles triangle, $\angle CED = \angle CDE$

$2\angle CED + 30° = 180°$

$$\angle CED = \frac{180° - 30°}{2} = 75°$$

By symmetry, $\angle CED = \angle BEA$ $b = 10$ *cm*

$\angle BEC + \angle CED + \angle BEA + \theta = 360°$

$60° + 75° + 75° + \theta = 360°$

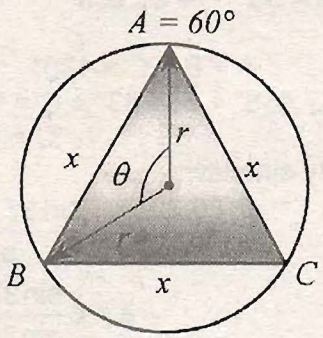
$\theta = 150°$

330.

$A = \pi r^2$

$254.47 = \pi r^2$

$r = 9$ m.



Note: Since an equilateral triangle,

$\angle A = \angle B = \angle C = 60°$

$$\theta = \frac{360°}{3} = 120°$$

By cosine law:

$x = \sqrt{r^2 + r^2 - 2rr\cos\theta}$

$= \sqrt{(9)^2 + (9)^2 - 2(9)(9)\cos 120°}$

$x = 15.59$ m

$$A = \frac{1}{2}(x)(x)\sin A$$

$$= \frac{1}{2}(15.59)^2 \sin 60°$$

$A = 105.24$ m^2

331. Note: Since an equilateral triangle,
$\angle A = \angle B = \angle C = 60°$

$$A = \frac{1}{2}bc\sin A$$

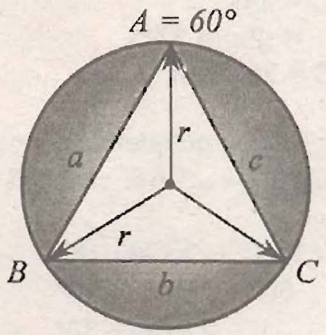
$$= \frac{1}{2}(10)^2 \sin 60°$$

$A = 43.3$ cm^2

$$A = \frac{abc}{4r}$$

$$43.3 = \frac{(10)(10)(10)}{4r}$$

$r = 5.774$ cm



Solving for area of circle:

$$A = \pi r^2$$

$$= \pi(5.774)^2$$

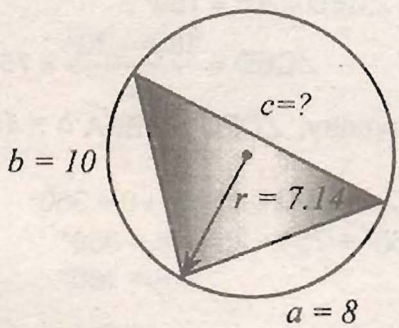
$A = 104.7$ cm^2

332.

$$A = \frac{abc}{4r}$$

$$39.19 = \frac{8(10)c}{4(7.14)}$$

$c = 14$ cm.



333. $A = 37°25' = 37.416°$
$B = 56°17' = 56.283°$

$$A + B + C = 180°$$

$$37.416° + 56.283° + C = 180°$$

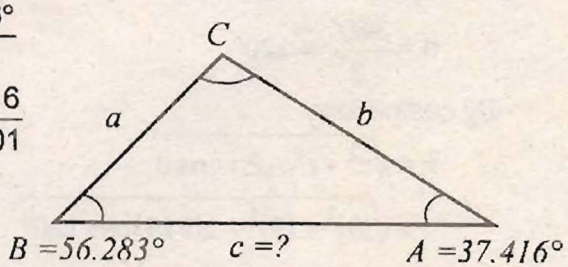
$$C = 86.301°$$

By sine law:

$$\frac{\sin 86.301°}{c} = \frac{\sin 37.416°}{a}$$

$$a = c\frac{\sin 37.416}{\sin 80.301}$$

$$a = 0.609\,c$$



$$A = \frac{1}{2}ac\sin B$$

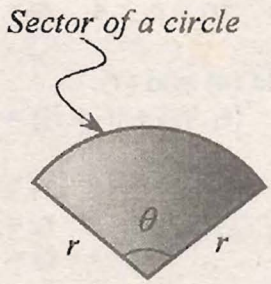
$$8346 = \frac{1}{2}(0.609c)(c)(\sin 56.283)$$

$$16692 = 0.5065\,c^2$$

$$c = 181.5 \text{ m.}$$

334. $A = \frac{1}{2}r^2\theta$

where: A = area of the sector
r = radius of the circle
θ = included angle in radians

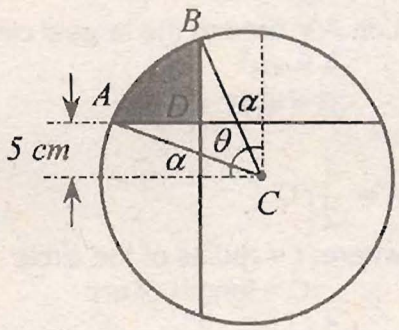


$$A = \frac{1}{2}(15)^2\left[30° \times \frac{2\pi \text{ radians}}{360°}\right] = 58.9 \text{ cm}^2$$

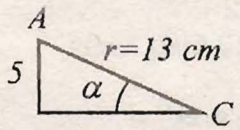
335. $\sin\alpha = \frac{5}{13}$

$$\alpha = 22.62°$$

$$2\alpha + \theta = 90°$$
$$2(22.62) + \theta = 90°$$
$$\theta = 44.76°$$



Let: A = area of the shaded part
A_S = area of sector ABC
A_T = area of triangle CDB

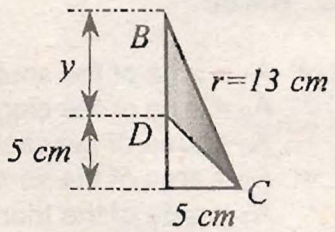


$$A_S = \frac{1}{2}r^2\theta$$

$$A_S = \frac{1}{2}(13)^2\left[44.76° \times \frac{2\pi \text{ radians}}{360°}\right] = 66 \text{ cm}^2$$

By Phythagorean theorem:

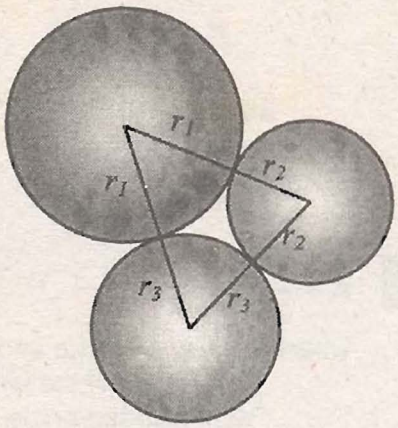
$$(13)^2 = (5)^2 + (y + 5)^2$$
$$144 = (y + 5)^2$$
$$12 = y + 5$$
$$y = 7$$



$$A = A_S - 2A_T$$

$$A = 66 - 2\left[\frac{1}{2}(7)(5)\right] = 31 \text{ cm}^2$$

336.



$r_1 + r_2 = 12$ ☞ ❶

$r_2 + r_3 = 10$ ☞ ❷

$r_1 + r_3 = 14$ ☞ ❸

Subtract (3) by (2):

$(r_1 + r_3) - (r_2 + r_3) = 14 - 10$

$r_1 - r_2 = 4$ ☞ ❹

Add (4) and (1):

$(r_1 - r_2) + (r_1 + r_2) = 4 + 12$

$2r_1 = 16$

$r_1 = 8$

$r_2 = 12 - 8 = 4$

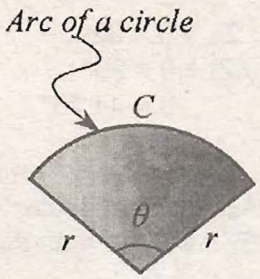
$r_3 = 10 - 4 = 6$

Let: A = area of the largest circle

$A = \pi r_1^2$

$A = \pi(8)^2 = 64\pi$

337.



$A = \frac{1}{2} r C$

where: r = radius of the circle

C = length of arc

$A = \frac{1}{2}(3)(9)$

A = 13.5 sq. units

338.

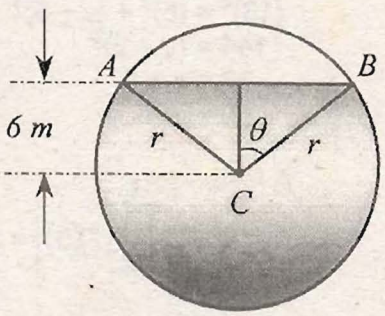
$A_C = \pi r^2$

$452 = \pi r^2$

r = 12 m.

$\cos\theta = \frac{6}{12}$

$\theta = 60°$



Let: A_1 = area of the smaller segment

A_2 = area of the bigger segment

A_C = area of the circle

A_S = area of the sector

A_T = area of the triangle ABC

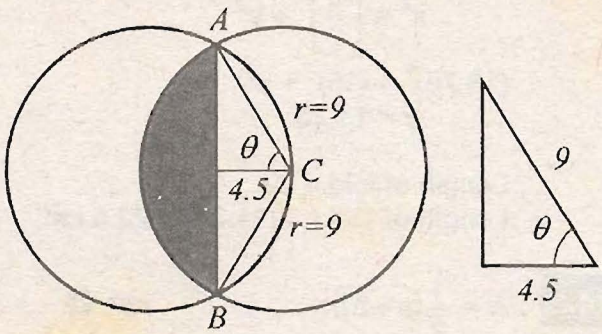
$A_1 = A_S - A_T$

$= \frac{1}{2} r^2(2\theta) - \frac{1}{2} r^2 \sin 2\theta$

$= \frac{1}{2}(12)^2\left[120° \times \frac{2\pi}{360°}\right] - \frac{1}{2}(12)^2 \sin 120°$

$A_1 = 88.44$

$A_2 = A_C - A_1$
$= 452 - 88.44$
$A_2 = 363.56\ m^2$



339. $\cos\theta = \dfrac{4.5}{9}$

$\theta = 60°$

Let: A = area of the pool
A_T = area of triangle ABC
A_S = area of the sector
A_G = area of the segment (shaded portion)

$A_G = A_S - A_T$

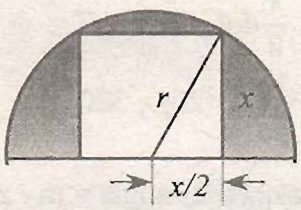
$= \dfrac{1}{2}r^2(2\theta) - \dfrac{1}{2}r^2\sin(2\theta)$

$A_G = \dfrac{1}{2}(9)^2\left[120°x\dfrac{2\pi\ \text{radians}}{360°}\right] - \dfrac{1}{2}(9)^2\sin 120° = 49.75\,m^2$

$A = 2A_C - 2A_G$
$A = 2(\pi)(9)^2 - 2(49.75) = 409.44\ m^2$

340. $r^2 = x^2 + \left(\dfrac{x}{2}\right)^2$

$r^2 = 1.25\,x^2$ ☞ ❶



Substitute r = 15 to (1):
$1.25\,x^2 = 15^2$
$x^2 = 180$

Let: A_1 = area of the semi-circle
A_2 = area of the square
A = difference of A_1 and A_2

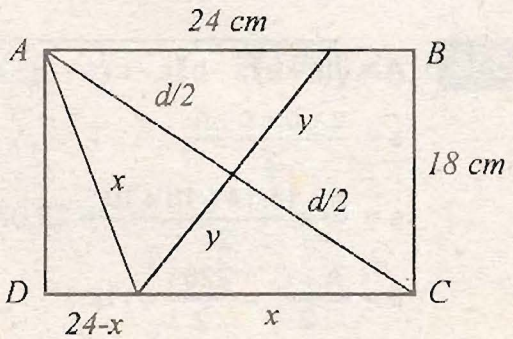
$A = A_1 - A_2$

$= \dfrac{1}{2}\pi r^2 - x^2$

$A = \dfrac{1}{2}\pi(15)^2 - 180 = 173.43\ cm^2$

341. $d = \sqrt{18^2 + 24^2} = 30$ cm.

$x^2 = 18^2 + (24 - x)^2$
$x^2 = 324 + 576 - 48x + x^2$
$x = 18.75$ cm



$$x^2 = \left(\frac{d}{2}\right)^2 + y^2$$

$$(18.75)^2 = (15)^2 + y^2$$

$$y = 11.25$$

Length of fold = 2y

Length of fold = 2(11.25) = 22.5 cm.

342. $A = \frac{1}{2}(a+b)h$ ☞ ❶

$\frac{a}{b} = \frac{4}{5}$; $a = \frac{4}{5}b$ ☞ ❷

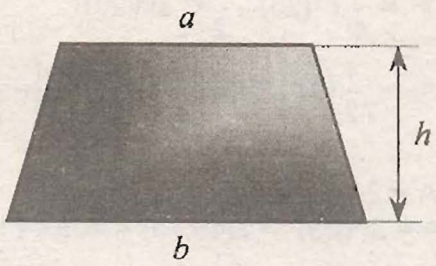
Substitute (2) in (1):

$$36 = \frac{1}{2}\left(\frac{4}{5}b + b\right)(2)$$

$$36 = \frac{9}{5}b$$

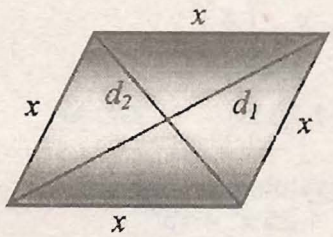
$$b = 20$$

$$a = \frac{4}{5}b = \frac{4(20)}{5} = 16$$



343. $A = \frac{1}{2}d_1d_2$

$A = \frac{1}{2}(32)(20) = 320\ \text{in}^2$



344. Total interior angle = (n - 2)(180°)

= (4 – 2)(180°) = 360°

$2\theta + 2(100°) = 360°$

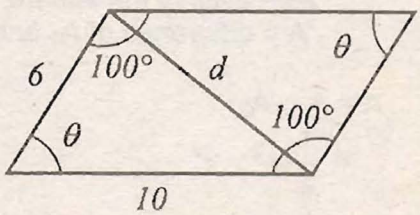
$\theta = 80°$

By cosine law:

$d^2 = 6^2 + 10^2 - 2(6)(10)\cos 80°$

$d^2 = 115.162$

$d = 10.73$

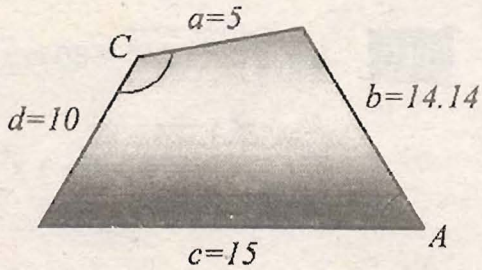


345. $A = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\theta}$

$$s = \frac{a+b+c+d}{2}$$

$$s = \frac{5+14.14+15+10}{2} = 22.07$$

$$\theta = \frac{A+C}{2} = \frac{225°}{2} = 112.5°$$



Substitute:

$$A = \sqrt{(22.07-5)(22.07-14.14)(22.07-15)(22.07-10)-5(14.14)(15)(10)\cos^2 112.5^\circ}$$

$$= \sqrt{(17.07)(7.93)(7.07)(12.07)-(1553.066)}$$

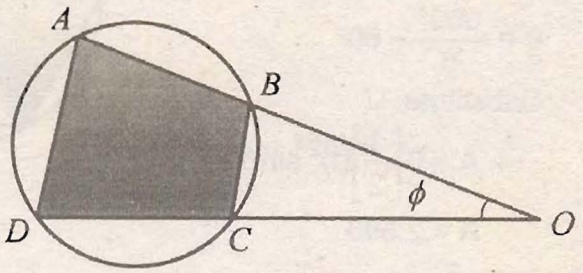
$$A = 100 \text{ cm}^2$$

346. By secant law:

$AO \cdot BO = CO \cdot DO$

$120 \cdot 80 = CO \cdot 150$

$OC = 64$ cm.



Let: A = area of the quadrilateral

A_1 = area of triangle AOD

A_2 = area of triangle BOC

$$A = A_1 - A_2$$

$$= \frac{1}{2}(AO)(DO)\sin\phi - \frac{1}{2}(BO)(CO)\sin\phi$$

$$= \frac{1}{2}(120)(150)\sin 25^\circ - \frac{1}{2}(80)(64)\sin 25^\circ$$

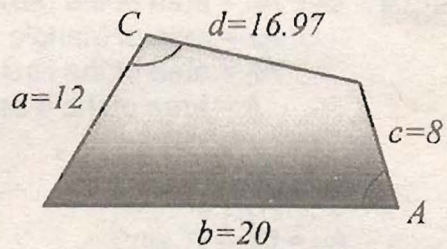
$$A = 2721.66 \text{ cm}^2$$

347. $$A = \sqrt{(s-a)(s-b)(s-c)(s-d)-abcd\cos^2\theta}$$

$$s = \frac{a+b+c+d}{2}$$

$$s = \frac{12+20+8+16.97}{2} = 28.485$$

$$\theta = \frac{A+C}{2} = \frac{225^\circ}{2} = 112.5^\circ$$



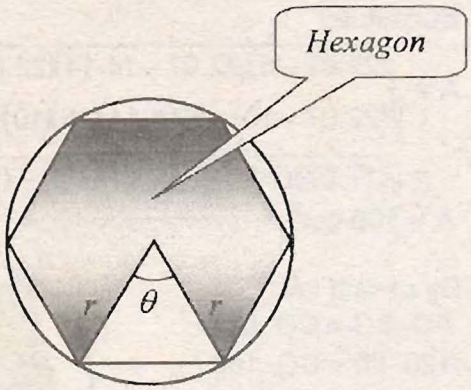
Substitute:

$$A = \sqrt{(28.485-12)(28.485-20)(28.485-8)(28.485-16.97)-12(20)(8)(16.97)\cos^2 112.5^\circ}$$

$$= \sqrt{(16.485)(8.485)(20.485)(11.515)-(4771.582)}$$

$$A = 168 \text{ m}^2$$

348. Let: A = area of the hexagon
A_T = area of triangle



$$A = 6A_T$$

$$A = 6\left(\frac{1}{2}\right)r^2 \sin\theta$$

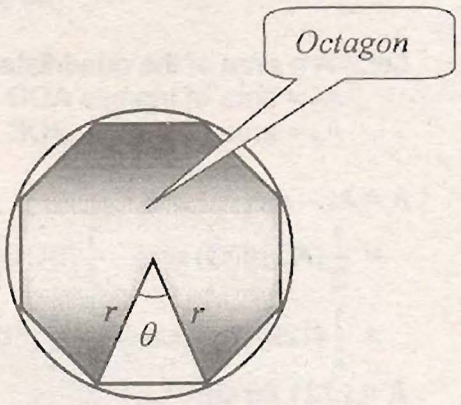
$$\theta = \frac{360°}{6} = 60°$$

Substitute:

$$A = 6\left[\frac{1}{2}\right](1)^2 \sin 60°$$

$$A = 2.598$$

349.



$$A = 8A_T$$

$$A = 8\left(\frac{1}{2}\right)r^2 \sin\theta$$

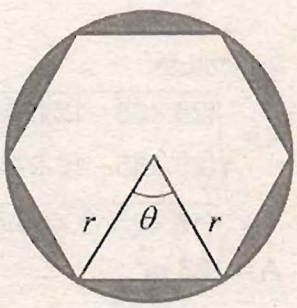
$$\theta = \frac{360°}{8} = 45°$$

Substitute:

$$A = 8\left[\frac{1}{2}\right](10)^2 \sin 45°$$

$$A = 283 \text{ cm}^2$$

350. Let: A_H = area of the hexagon
A_T = area of triangle
A_C = area of the circle
A = area of the 6 segments of the circle



$$A_H = 6A_T$$

$$A_H = 6\left(\frac{1}{2}\right)r^2 \sin\theta$$

$$\theta = \frac{360°}{6} = 60°$$

Substitute:

$$A_H = 6\left[\frac{1}{2}\right](10)^2 \sin 60°$$

$$A_H = 259.8$$

$$A = A_C - A_H$$

$$A = \pi(10)^2 - 259.8 = 54.36 \text{ m}^2$$

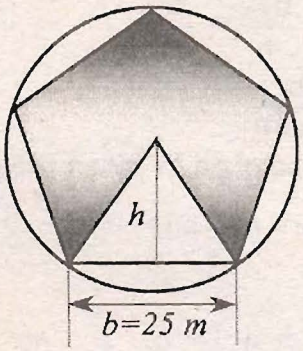
351.

$A = 5A_T$

$= 5\left(\frac{1}{2}\right)(bh)$

$= \frac{5}{2}(25)(17.2)$

$A = 1075 m^2$



352.

$\theta = \frac{360°}{6} = 60°$

$A_C = \pi r^2$

$89.42 = \pi r^2$

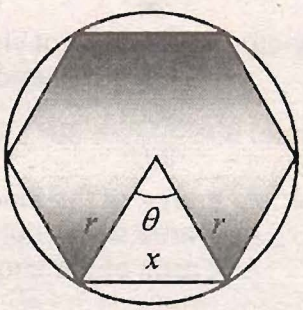
$r = 5.335$ in.

By cosine law:

$x = \sqrt{r^2 + r^2 - 2(r)(r)\cos\theta}$

$= \sqrt{2(5.335)^2 - 2(5.335)^2 \cos 60^0}$

$x = 5.335$ in.



353. Let: A = area not covered by the star

A_S = area of the star

A_C = area of the circle

$2\theta = \frac{360°}{5}$

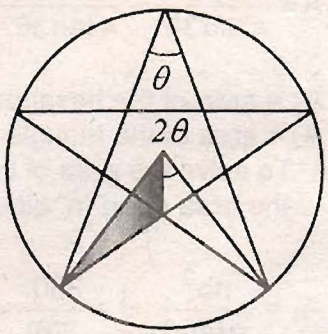
$\theta = 36°$

$\theta/2 = 18°$

$\theta + \frac{\theta}{2} + \alpha = 180°$

$36° + 18° + \alpha = 180°$

$\alpha = 126°$

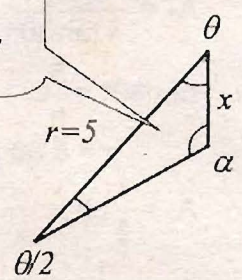


By sine law:

$\frac{\sin 18°}{x} = \frac{\sin 126°}{5}$

$x = 1.91$ m

Area of the shaded portion!



$A = A_C - A_S$

$= \pi r^2 - 10\left(\frac{1}{2}\right)(5)(x)\sin\theta$

$= \pi(5)^2 - 10\left(\frac{1}{2}\right)(5)(1.91)\sin 36°$

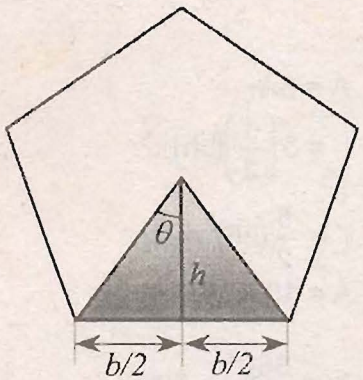
$A = 50.5 m^2$

354. $\tan\theta = \dfrac{b/2}{h} = \dfrac{b}{2h}$

$h = \dfrac{b}{2\tan\theta}$

A_P = area of an "n" sided polygon

$A_P = n\left[\dfrac{1}{2}bh\right]$

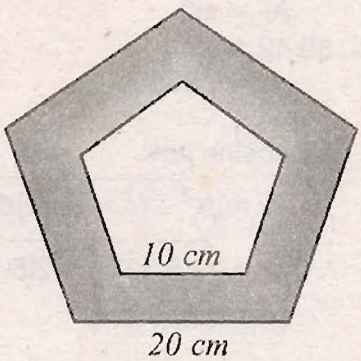


$A_P = \dfrac{n}{2}b\left[\dfrac{b}{2\tan\theta}\right] = \dfrac{nb^2}{4\tan\theta}$ ☞ *General formula*

$\theta = \dfrac{360°}{2n}$

where: n = number of sides
b = length of each side

Note: $\theta = \dfrac{360°}{2(5)} = 36°$, for a pentagon



Let: A_1 = area of the inner pentagon
A_2 = area of the outer pentagon
A = area of the shaded portion

$A = A_2 - A_1$

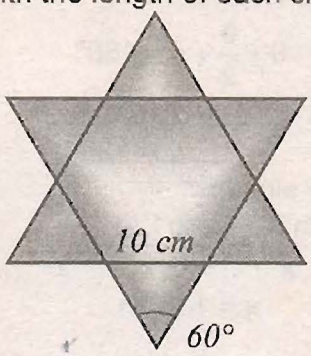
$A = \dfrac{5(20)^2}{4\tan 36°} - \dfrac{5(10)^2}{4\tan 36°} = 516.14\ \text{cm}^2$

355. Let: A_1 = area of the hexagon
A_2 = area of the triangle

Note: To solve the area of the hexagon, use the general formula to solve the area of an "n" sided polygon with the length of each side given.

$A_1 = \dfrac{nb^2}{4\tan\theta}$; $\theta = \dfrac{360°}{2(6)} = 30°$

$A_1 = \dfrac{6(10)^2}{4\tan 30°} = 259.8\ \text{cm}^2$

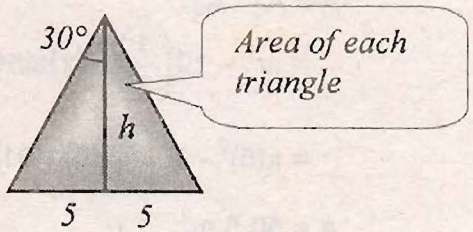


$\tan 30° = \dfrac{5}{h}$

$h = 8.66$ cm

$A_2 = \dfrac{1}{2}bh$

$A_2 = \dfrac{1}{2}(10)(8.66) = 43.3\ \text{cm}^2$



Total area = $A_1 + 6A_2$
= 259.8 + 6(43.3)
Total area = 519.6 cm^2

DAY 8

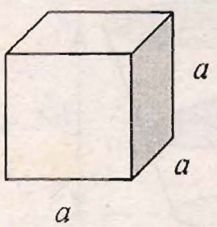
SOLID GEOMETRY

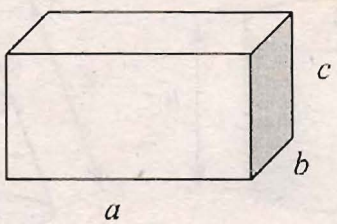
POLYHEDRONS

Polyhedron is a solid whose faces are plane polygons. A regular polyhedron is a solid with all its faces identical regular polygons. There are only five regular polyhedrons, namely tetrahedron, hexahedron (cube), octahedron, dodecahedron and icosahedron. These solids are also known as **Platonic solids** in honor of Plato (427 – 348 B.C.).

Polyhedron	No. of Faces	Faces	No.of Edges	No. of Vertices	Volume
Tetrahedron	4	Triangle	6	4	$\frac{1}{12}e^3\sqrt{2}$
Hexahedron	6	Square	12	8	e^3
Octahedron	8	Triangle	12	6	$\frac{1}{3}e^3\sqrt{2}$
Dodecahedron	12	Pentagon	30	20	$\frac{e^3}{4}\left(15+7\sqrt{5}\right)$
Icosahedron	20	Triangle	30	12	$\frac{5}{12}e^3\left(3+\sqrt{5}\right)$

Cube is a polyhedron with all six faces a square.
Rectangular parallelepiped is a polyhedron with all six faces a rectangle.





Volume of cube:

$V = a^3$

Volume of rectangular parallelepiped:

$V = abc$

Surface area of cube:

$$A = 6a^2$$

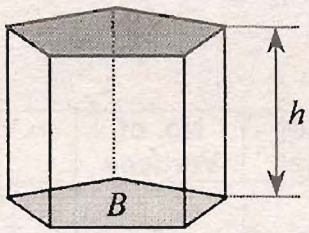
Surface area of rectangular parallelepiped:

$$A = 2(ab + bc + ca)$$

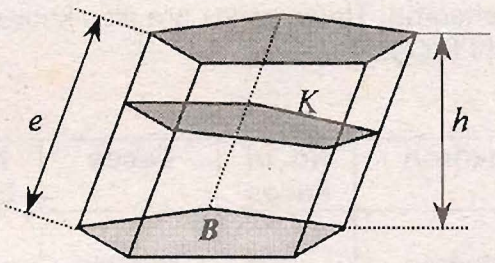
PRISMS

Prism is a polyhedron with two faces (bases) parallel and congruent and whose remaining faces (lateral faces) are parallelograms.

Right prism is one which has its lateral faces perpendicular to the base.
Oblique prism is one which has it lateral faces not perpendicular to the base.
Truncated prism is a portion of a prism contained between the base and a plane that is not parallel to the base.



right prism



oblique prism

Volume of prism:

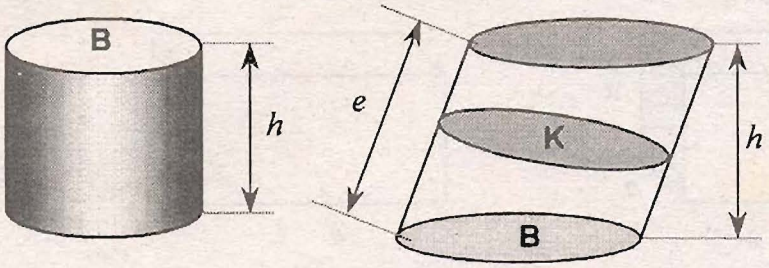
$$V = Bh = Ke$$

Lateral area of prism:

$$A = e\,p_k$$

where:
B = area of the base
h = altitude of prism
K = area at right section
e = lateral edge
p_k = perimeter of right section

Cylinder is a solid bounded by a closed cylindrical surface and two parallel planes.



Volume of cylinder:

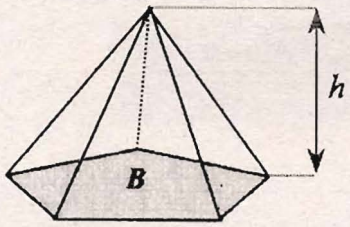
$$V = Bh = Ke$$

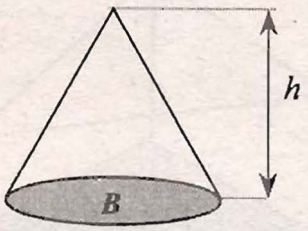
where:
B = area of the base
h = altitude of prism
K = area at right section
e = lateral edge
p_k = perimeter of right section

Lateral area of cylinder:

$$A = e\,p_k$$

Pyramid is a polyhedron of which one face, called the base, is a polygon of any number of sides and the other faces are triangles which have a common vertex. **Cone** is a solid bounded by a conical surface (lateral surface) whose directrix is a closed curve, and a plane (base) which cuts all the elements.



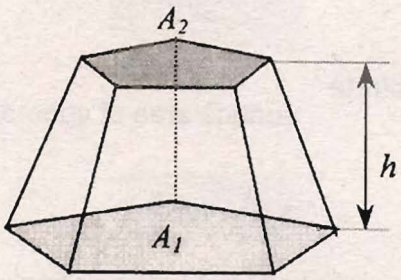


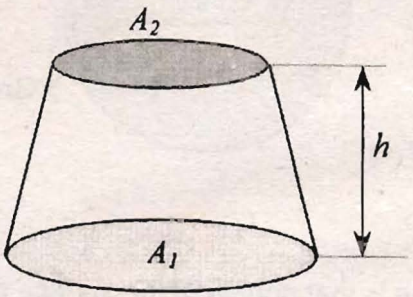
Volume of pyramid / cone:

$$V = \frac{1}{3}Bh$$

where: B = area of the base
h = altitude

Frustum (of a pyramid/cone) is a portion of the pyramid / cone included between the base and a section parallel to the base.





Volume of frustum of pyramid / cone:

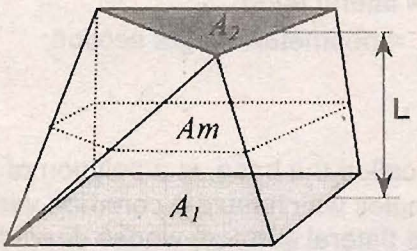
$$V = \frac{h}{3}\left(A_1 + A_2 + \sqrt{A_1 A_2}\right)$$

where: R = bigger radius
r = smaller radius

For cone:

$$V = \frac{\pi h}{3}\left(R^2 + r^2 + Rr\right)$$

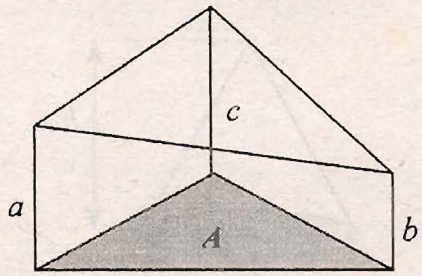
Prismatoid is a polyhedron having for bases two polygons in parallel planes and for lateral faces triangles or trapezoids with one side lying in one base, and the opposite vertex or side lying in the other base of the polyhedron.



Volume of prismatoid:

$$V = \frac{L}{6}(A_1 + 4Am + A_2)$$

This formula is known as **Prismoidal formula**

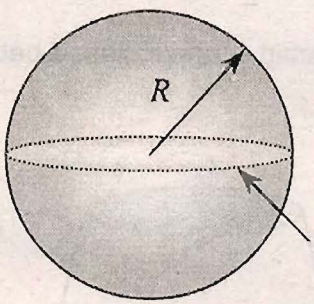


Truncated prism

Volume of truncated prism:

$$V = \frac{A(a+b+c)}{3}$$

Sphere is a solid bounded by a closed surface every point of which is equidistant from a fixed point called center.



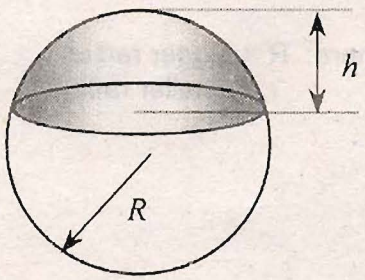
Volume of sphere:

$$V = \frac{4}{3}\pi R^3$$

Surface area of sphere:

$$A = 4\pi R^2$$

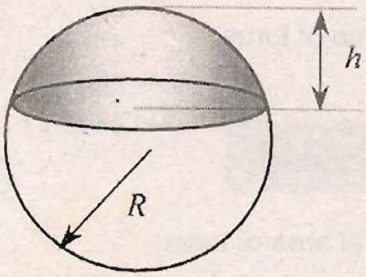
Zone is that portion of the surface of a sphere included between two parallel planes.



Area of zone:

$$A = 2\pi Rh$$

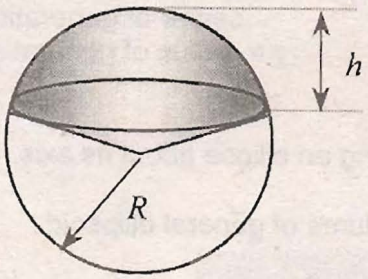
Spherical segment is a solid bounded by a zone and the planes of the zone's base.



Volume of spherical segment:

$$V = \frac{\pi h^2}{3}(3R - h)$$

Spherical sector is a solid generated by rotating a sector of a circle about an axis which passes through the center of the circle but which contains no point inside the sector.

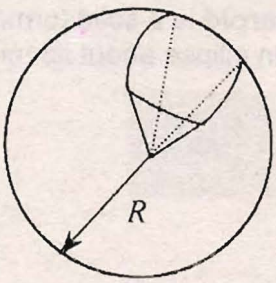


Volume of spherical sector:

$$V = \frac{1}{3}AR$$

where : A = area of zone

Spherical pyramid is a pyramid formed by a portion of a surface of a sphere as base and whose elements are the edges from the vertices of the base to the center of the sphere.

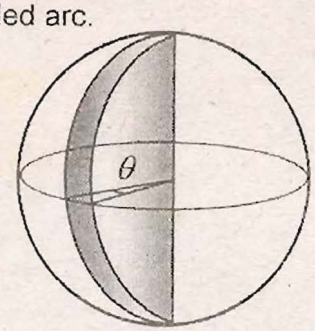


Volume of spherical pyramid:

$$V = \frac{\pi R^3 E}{540}$$

where : E = spherical excess of polygon ABCD in degrees

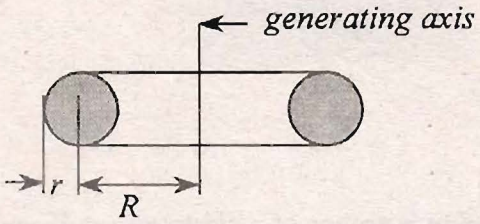
Spherical wedge is a portion of a sphere bounded by two half great circles and an included arc.



Volume of spherical wedge:

$$V = \frac{\pi R^3 \theta}{270}$$

Torus is a solid formed by revolving a circle about a line not intersecting it.



Volume of torus:

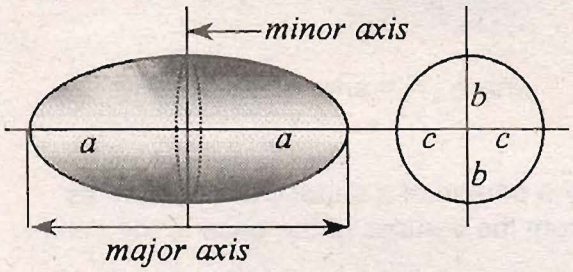
$$V = 2\pi^2 Rr^2$$

Lateral area of torus:

$$A = 4\pi^2 Rr$$

where : R = distance from axis to
center of generating circle
r = radius of generating circle

Ellipsoid (Spheroid) is a solid formed by revolving an ellipse about its axis.



Volume of general ellipsoid:

$$V = \frac{4}{3}\pi abc$$

Prolate spheroid is a solid formed by revolving an ellipse about its major axis.

$$V = \frac{4}{3}\pi ab^2$$

Oblate spheroid is a solid formed by Revolving an ellipse about its minor axis.

$$V = \frac{4}{3}\pi a^2 b$$



Tips: 1. **Lateral area** is the total area of the faces of the polyhedron excluding the bases.

2. **Surface area** refers to the total area of the faces of the polyhedron including the bases.

3. **Anchor ring or Doughnut** is another term for a torus.

4. **Rhomboid** is another term of a parallelogram.

5. **Geoid** is the actual shape of the earth although earth most of the time is regarded as a spheroid or ellipsoid.

6. **Cavalieri's Theorem** states that solids of equal height have equal volume if sections parallel to and equidistant from their bases have equal area. This is named after **Bonaventura Cavalieri** (1598 – 1647).

Did you know that... the Pascal's triangle which is used to determine the coefficient of a binomial expansion was named after the French mathematician, philosopher and physicist Blaise Pascal but did not claim recognition for the discovery because such triangle was first introduced by a Chinese mathematician, Chu Shih-chieh in 1303!

Proceed to the next page for your fourth test. GOODLUCK ! ☞

Test 8

Time element: 2 hours & 30 minutes

Problem 356: ME Board October 1991

A circular piece of cardboard with a diameter of 1 m will be made into a conical hat 40 cm high by cutting a sector off and joining the edges to form a cone. Determine the angle subtended by the sector removed.

A. 144°
B. 148°
C. 152°
D. 154°

Problem 357: CE Board November 1994

What is the area in sq. m of the zone of a spherical segment having a volume of 1470.265 cu. m if the diameter of the sphere is 30 m?

A. 465.5 m^2
B. 565.5 m^2
C. 665.5 m^2
D. 656.5 m^2

Problem 358: CE Board May 1995

A sphere having a diameter of 30 cm is cut into 2 segments. The altitude of the first segment is 6 cm. What is the ratio of the area of the second segment to that of the first?

A. 4:1
B. 3:1
C. 2:1
D. 3:2

Problem 359: CE Board November 1996

If the edge of a cube is increased by 30%, by how much is the surface area increased?

A. 30 %
B. 33 %
C. 60 %
D. 69 %

Problem 360: ECE Board April 1995

Each side of a cube is increased by 1%. By what percent is the volume of the cube increased?

A. 1.21 %
B. 2.8 %
C. 3.03 %
D. 3.5 %

Problem 361: ECE Board November 1992

Given a sphere of diameter, d. What is the percentage increase in its diameter when the surface area increases by 21 %?

A. 5 %
B. 10 %
C. 21 %
D. 33 %

Problem 362: ECE Board November 1992

Given a sphere of diameter, d. What is the percentage increase in its volume when the surface area increases by 21 %?

A. 5 %
B. 10 %
C. 21 %
D. 33 %

Problem 363: EE Board October 1991

How many times does the volume of a sphere increases if the radius is doubled?

A. 4 times
B. 2 times
C. 6 times
D. 8 times

Problem 364: CE Board May 1997

A circular cone having an altitude of 9 m is divided into 2 segments having the same vertex. If the smaller altitude is 6 m, find the ratio of the volume of the small cone to the big cone.

A. 0.186
B. 0.296
C. 0.386
D. 0.486

Problem 365: CE Board November 1997

Find the volume of a cone to be constructed from a sector having a diameter of 72 cm and a central angle of 210°.

A. 12367.2 cm^3
B. 13232.6 cm^3
C. 13503.4 cm^3
D. 14682.5 cm^3

Problem 366: CE Board May 1998

Find the volume of a cone to be constructed from a sector having a diameter of 72 cm and a central angle of 150°.

A. 5533.32 cm^3
B. 6622.44 cm^3
C. 7710.82 cm^3
D. 8866.44 cm^3

Problem 367: CE Board November 1996

A conical vessel has a height of 24 cm and a base diameter of 12 cm. It holds water to a depth of 18 cm above its vertex. Find the volume (in cm^3) of its content.

A. 188.40
B. 298.40
C. 381.70
D. 412.60

Problem 368: CE Board May 1995

What is the height of a right circular cone having a slant height of $\sqrt{10x}$ and a base diameter of 2x?

A. 2x
B. 3x
C. 3.317x
D. 3.162x

Problem 369: CE Board November 1995

The ratio of the volume to the lateral area of a right circular cone is 2:1. If the altitude is 15 cm, what is the ratio of the slant height to the radius?

A. 5:6
B. 5:4
C. 5:3
D. 5:2

Problem 370: CE Board November 1994

A regular triangular pyramid has an altitude of 9 m and a volume of 187.06 cu. m. What is the base edge in meters?

A. 12
B. 13
C. 14
D. 15

Problem 371: CE Board November 1995

The volume of the frustum of a regular triangular pyramid is 135 cu. m. The lower base is an equilateral triangle with an edge of 9 m. The upper base is 8 m above the lower base. What is the upper base edge in meters?

A. 2
B. 3
C. 4
D. 5

Problem 372: EE Board April 1992

What is the volume of a frustum of a cone whose upper base is 15 cm in diameter and lower base 10 cm. in diameter with an altitude of 25 cm?

A. 3018.87 cm^3
B. 3180.87 cm^3
C. 3108.87 cm^3
D. 3081.87 cm^3

Problem 373: EE Board April 1993

In a portion of an electrical railway cutting, the areas of cross section taken every 50 m are 2556, 2619, 2700, 2610 and 2484 sq. m. Find its volume.

A. 522,600 m^3
B. 520,500 m^3
C. 540,600 m^3
D. 534,200 m^3

Problem 374: ME Board April 1996

Determine the volume of a right truncated triangular prism with the following definitions: Let the corners of the triangular base be defined by A, B and C. The length of AB = 10 ft., BC = 9 ft. and CA =12 ft. The sides A, B and C are perpendicular to the triangular base and have the height of 8.6 ft., 7.1 ft. and 5.5 ft. respectively.

A. 413 ft^3
B. 311 ft^3
C. 313 ft^3
D. 391 ft^3

Problem 375: CE Board November 1995

A circular cylinder with a volume of 6.54 cu. m is circumscribed about a right prism whose base is an equilateral triangle of side 1.25 m. What is the altitude of the cylinder in meters?

A. 3.50
B. 3.75
C. 4.00
D. 4.25

Problem 376: CE Board May 1996

A circular cylinder is circumscribed about a right prism having a square base one meter on an edge. The volume of the cylinder is 6.283 cu.m. Find its altitude in meters.

A. 4.00
B. 3.75
C. 3.50
D. 3.25

Problem 377: CE Board November 1997

The bases of a right prism is a hexagon with one of each side equal to 6 cm. The bases are 12 cm apart. What is the volume of the right prism?

A. 1211.6 cm^3
B. 2211.7 cm^3
C. 1212.5 cm^3
D. 1122.4 cm^3

Problem 378: EE Board April 1996

Two vertical conical tanks are joined at the vertices by a pipe. Initially the bigger tank is full of water. The pipe valve is open to allow the water to flow to the smaller tank until it is full. At this moment, how deep is the water in the bigger tank? The bigger tank has a diameter of 6 ft and a height of 10 ft, the smaller tank has a diameter of 6 ft and a height of 8 feet. Neglect the volume of water in the pipeline.

A. $\sqrt[3]{200}$
B. $\sqrt[3]{50}$
C. $\sqrt[3]{25}$
D. $\sqrt[4]{50}$

Problem 379:

The central angle of a spherical wedge is 1 radian. Find its volume if its radius is 1 unit.

A. 2/3
B. 1/2

C. 3/4
D. 2/5

Problem 380:

A regular octahedron has an edge 2m. Find its volume (in m^3).

A. 3.77
B. 1.88
C. 3.22
D. 2.44

Problem 381: CE Board May 1996

A mixture compound of equal parts of two liquids, one white and the other black, was placed in a hemispherical bowl. The total depth of the two liquids is 6 inches. After standing for a short time, the mixture separated, the white liquid settling below the black. If the thickness of the segment of the black liquid is 2 inches, find the radius of the bowl in inches.

A. 7.33
B. 7.53
C. 7.73
D. 7.93

Problem 382: CE Board November 1996

The volume of water in a spherical tank having a diameter of 4 m is 5.236 m^3. Determine the depth of the water in the tank.

A. 1.0
B. 1.2
C. 1.4
D. 1.8

Problem 383:

An ice cream cone is filled with ice cream and a surmounted ice cream in the form of a hemisphere on top of the cone. If the hemispherical surface is equal to the lateral area of the cone, find the total volume (in cubic inches) of ice cream if the radius of the hemisphere is 1 inch and assuming the diameter of hemisphere is equal to the diameter of the cone.

A. 3.45
B. 3.91
C. 4.12
D. 4.25

Problem 384: ME Board April 1997

A cubical container that measures 2 inches on a side is tightly packed with 8 marbles and is filled with water. All 8 marbles are in contact with the walls of the container and the adjacent marbles. All of the marbles are of the same size. What is the volume of water in the container?

A. $0.38\ in^3$
B. $2.5\ in^3$
C. $3.8\ in^3$
D. $4.2\ in^3$

Problem 385: CE Board May 1997

The corners of a cubical block touched the closed spherical shell that encloses it. The volume of the box is 2744 cubic cm. What volume in cubic centimeter inside the shell is not occupied by the block?

A. 2714.56
B. 3714.65
C. 4713.56
D. 4613.74

ANSWER KEY

356. A	366. C	376. A
357. B	367. C	377. D
358. A	368. B	378. A
359. D	369. D	379. A
360. C	370. A	380. A
361. B	371. B	381. A
362. D	372. C	382. A
363. D	373. A	383. B
364. B	374. B	384. C
365. C	375. C	385. C

RATING

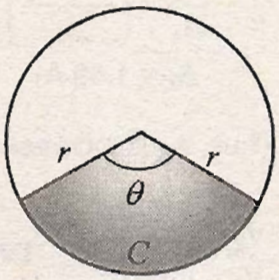
- ☐ 26-30 Topnotcher
- ☐ 20-25 Passer
- ☐ 15-19 Conditional
- ☐ 0-14 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 8

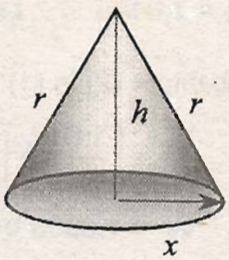
356. $x = \sqrt{r^2 - h^2}$

$x = \sqrt{(50)^2 - (40)^2} = 30$ cm.



Let:
C_1 = circumference of the circle
C_2 = circumference of the base of the cone
C = length of arc

$C = C_1 - C_2$
$= 2\pi r - 2\pi x$
$C = 2\pi(50) - 2\pi(30) = 40\pi$

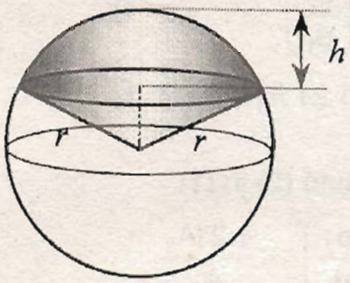


$C = r\theta$
$40\pi = (50)\theta$
$\theta = \frac{40\pi}{50} \times \frac{360°}{2\pi} = 144°$

357. $V = \frac{\pi h^2}{3}(3r - h)$

$1470.265 = \frac{\pi h^2}{3}[3(15) - h]$

$1410.795 = \pi h^2(45 - h)$
$1404 = 45h^2 - h^3$
$h = 6$



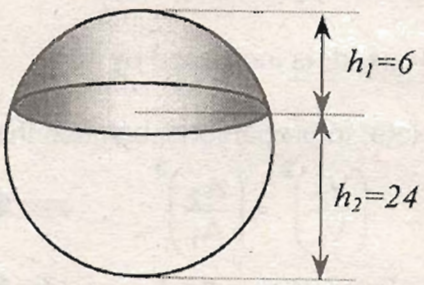
$A = 2\pi rh$
$A = 2\pi(15)(6) = 565.5\ m^2$

358. $h_2 = 30 - 6 = 24$ cm.

$\frac{A_2}{A_1} = \frac{2\pi rh_2}{2\pi rh_1}$

$= \frac{h_2}{h_1}$

$= \frac{24}{6} = 4$



Thus, $A_2 : A_1 = 4 : 1$

359. $\frac{A_2}{A_1} = \left(\frac{x_2}{x_1}\right)^2$ ☞ ❶

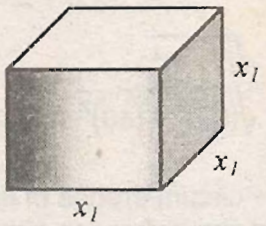
$x_2 = 1.3\ x_1$ ☞ ❷

Substitute (2) in (1):

$$\frac{A_2}{A_1} = \left(\frac{1.31x_1}{x_1}\right)^2$$

$$A_2 = 1.69\, A_1$$

Thus, A_2 is increased by 69%.



360.

$$\frac{V_2}{V_1} = \left(\frac{x_2}{x_1}\right)^3 \quad \text{☞ ❶}$$

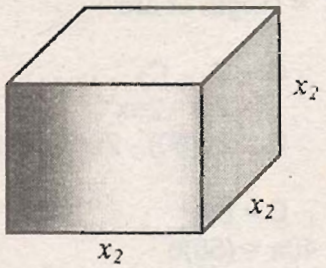
$$x_2 = 1.01\, x_1 \quad \text{☞ ❷}$$

Substitute (2) in (1):

$$\frac{V_2}{V_1} = \left(\frac{1.01x_1}{x_1}\right)^3$$

$$V_2 = 1.0303\, V_1$$

Thus, V_2 is increased by 3.03%



361.

$$\frac{A_2}{A_1} = \left(\frac{d_2}{d_1}\right)^2 \quad \text{☞ ❶}$$

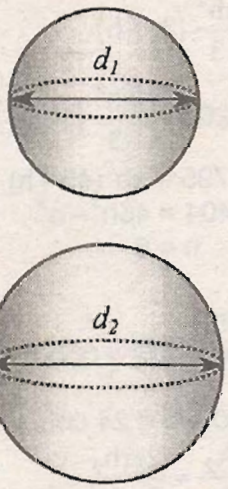
$$A_2 = 1.21\, A_1 \quad \text{☞ ❷}$$

Substitute (2) in (1):

$$\left(\frac{d_2}{d_1}\right)^2 = \frac{1.21A_1}{A_1}$$

$$d_2 = 1.1d_1$$

Thus, d_2 is increased by 10%



362. Note: In proportions, balance the resulting units on both sides.

$$\left(\frac{V_2}{V_1}\right)^2 = \left(\frac{A_2}{A_1}\right)^3 \quad \text{☞ ❶}$$

$$A_2 = 1.21\, A_1 \quad \text{☞ ❷}$$

Substitute (2) in (1):

$$\left(\frac{V_2}{V_1}\right)^2 = \left(\frac{1.21A_1}{A_1}\right)^3$$

$$\frac{V_2}{V_1} = (1.21)^{\frac{3}{2}}$$

$$V_2 = 1.33\, V_1$$

Thus, V_2 is increased by 33.1%

363. $\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^3$ ☞ ❶

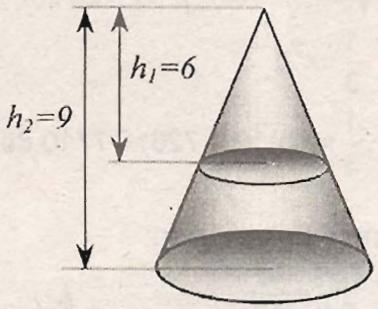
$r_2 = 2\,r_1$ ☞ ❷

Substitute (2) in (1):

$$\frac{V_2}{V_1} = \left(\frac{2r_1}{r_1}\right)^3$$

$$V_2 = 8\,V_1$$

364.

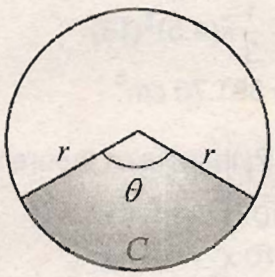


$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

$$= \left(\frac{6}{9}\right)^3$$

$$\frac{V_1}{V_2} = 0.296$$

365. Let: C_1 = circumference of the circle
C_2 = circumference of the base of the cone
C = length of arc



$$C_2 = C_1 - C$$

$$2\pi x = 2\pi r - r\theta$$

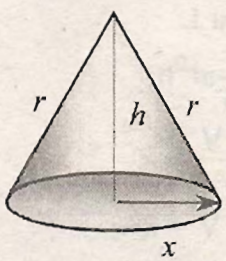
$$x = r - \frac{r\theta}{2\pi}$$

$$= 36 - \frac{36}{2\pi}\left[150°x\frac{2\pi}{360°}\right]$$

$$x = 21 \text{ cm.}$$

$$h = \sqrt{r^2 - x^2}$$

$$h = \sqrt{(36)^2 - (21)^2} = 29.24 \text{ cm.}$$



$$V = \frac{1}{3}\pi x^2 h$$

$$V = \frac{1}{3}\pi(21)^2(29.24) = 13503.4 \text{ cm}^3$$

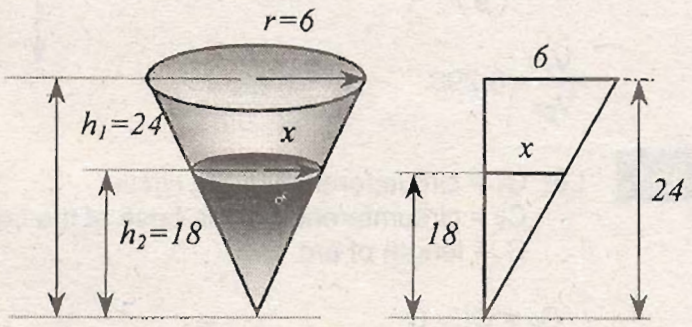
366. Let: C_1 = circumference of the circle
C_2 = circumference of the base of the cone
C = length of arc

$$C_2 = C_1 - C$$
$$2\pi x = 2\pi r - r\theta$$
$$x = r - \frac{r\theta}{2\pi}$$
$$= 36 - \frac{36}{2\pi}\left[210° x \frac{2\pi}{360°}\right]$$
$$x = 15 \text{ cm.}$$

$$h = \sqrt{r^2 - x^2}$$
$$h = \sqrt{(36)^2 - (15)^2} = 32.726 \text{ cm.}$$
$$V = \frac{1}{3}\pi x^2 h$$
$$V = \frac{1}{3}\pi(15)^2(32.726) = 7710.88 \text{ cm}$$

367. By ratio & proportion:

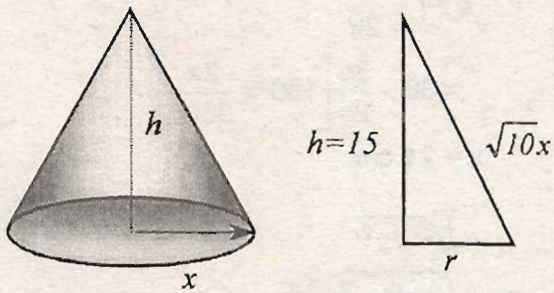
$$\frac{6}{24} = \frac{x}{18}$$
$$x = 4.5$$



$$V = \frac{1}{3}\pi x^2 h_2$$
$$= \frac{1}{3}\pi(4.5)^2(18)$$
$$V = 381.70 \text{ cm}^3$$

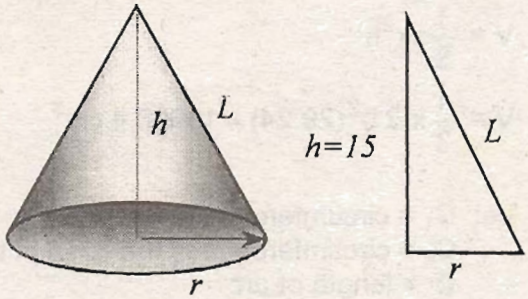
368. By Pythagorean theorem:

$$\left(\sqrt{10}x\right)^2 = h^2 + x^2$$
$$10x^2 = h^2 + x^2$$
$$h^2 = 9x^2$$
$$h = 3x$$



369.

$$A_L = \pi r L$$
$$V = \frac{1}{3}\pi r^2 h$$
$$\frac{V}{A_L} = 2$$
$$V = 2A_L$$
$$\frac{1}{3}\pi r^2 h = 2(\pi r L)$$
$$r^2 h = 6rL$$
$$\frac{L}{r} = \frac{h}{6}$$

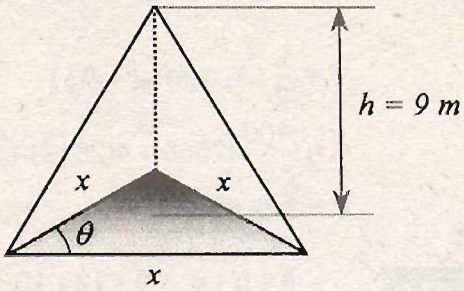


$$\frac{L}{r} = \frac{15}{6} = \frac{5}{2}$$

370. $V = \frac{1}{3}(\text{Base Area})h$

$$V = \frac{1}{3}\left[\frac{1}{2}x^2 \sin\theta\right]h$$

Note: $\theta = 60°$, since equilateral triangle.

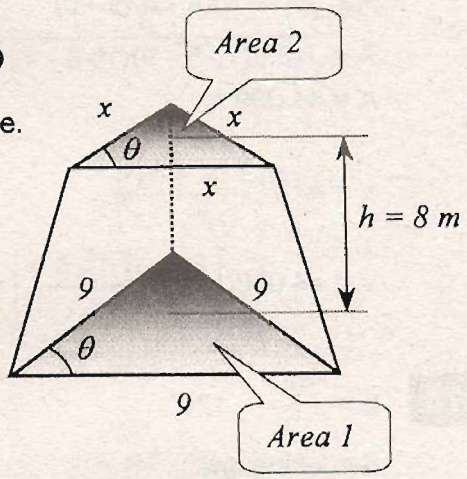


$$187.06 = \frac{1}{3}\left(\frac{1}{2}\right)x^2 \sin 60°\,(9)$$

$x = 12$ m.

371. $V = \frac{h}{3}\left[A_1 + A_2 + \sqrt{A_1 A_2}\right]$ ☞ ❶

Note: $\theta = 60°$, since equilateral triangle.



$$A_1 = \frac{1}{2}(9)^2 \sin 60°$$

$A_1 = 35.074$ ☞ ❷

$$A_2 = \frac{1}{2}(x)^2 \sin 60°$$

$A_2 = 0.433\,x^2$ ☞ ❸

Substitute (2) & (3) in (1):

$$135 = \frac{8}{3}\left[35.074 + 0.433x^2 + \sqrt{(35.074)(0.433x^2)}\right]$$

$50.625 = 35.074 + 0.433\,x^2 + 3.897\,x$

$x^2 + 9x - 36 = 0$

$(x - 3)(x + 12) = 0$

$x = 3$m.

372. $V = \frac{h}{3}\left[A_1 + A_2 + \sqrt{A_1 A_2}\right]$ ☞ ❶

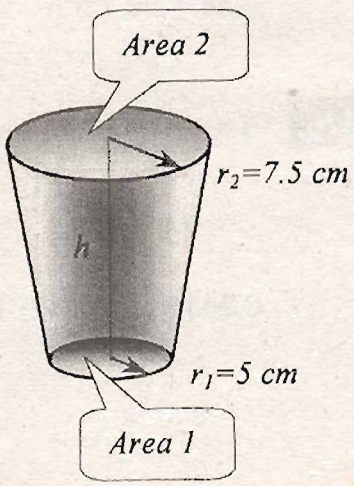
$A_1 = \pi r_1^2 = \pi(5)^2 = 25\pi$

$A_2 = \pi r_2^2 = \pi(7.5)^2 = 56.25\pi$

Substitute A_1 and A_2 in (1):

$$V = \frac{25}{3}\left[25\pi + 56.25\pi + \sqrt{(25\pi)(56.25\pi)}\right]$$

$V = 3108.87\ \text{cm}^3$



373. Note: Since the areas being cut is at the same distance, then the given solid is a *prismatoid*. And since there are 5 different areas being cut then, this is equivalent to 2 prismatoids.

$$V = \frac{L}{2}(A_1 + 4A_m + A_2)$$

$$V = \frac{100}{6}[2556 + 4(2619) + 2700] + \frac{100}{6}[2700 + 4(2610) + 2484]$$

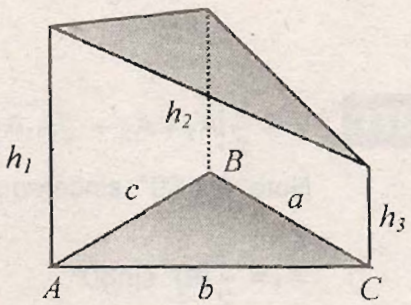
$$V = 522{,}600 \text{ m}^3$$

374. $$s = \frac{a+b+c}{2} = \frac{9+12+10}{2} = 15.5$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{15.5(15.5-9)(15.5-12)(15.5-10)}$$

$$A = 44.039 \text{ ft.}$$



$$V = A\left[\frac{h_1 + h_2 + h_3}{3}\right]$$

$$V = 44.039\left[\frac{8.6 + 7.1 + 5.5}{3}\right] = 311 \text{ ft}^3$$

375. $$\theta = \frac{360°}{3} = 120°$$

By cosine law:

$$x^2 = r^2 + r^2 - 2r^2 \cos\theta$$

$$(1.25)^2 = 2r^2 - 2r^2 \cos 120°$$

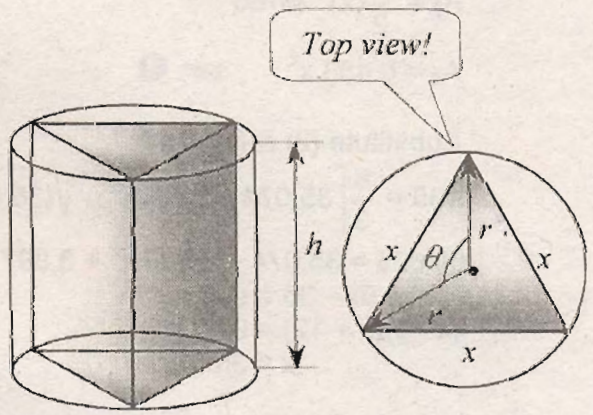
$$1.5625 = 3r^2$$

$$r = 0.72 \text{ m}$$

$$V = \pi r^2 h$$

$$6.54 = \pi(0.72)^2 h$$

$$h = 4 \text{ m.}$$

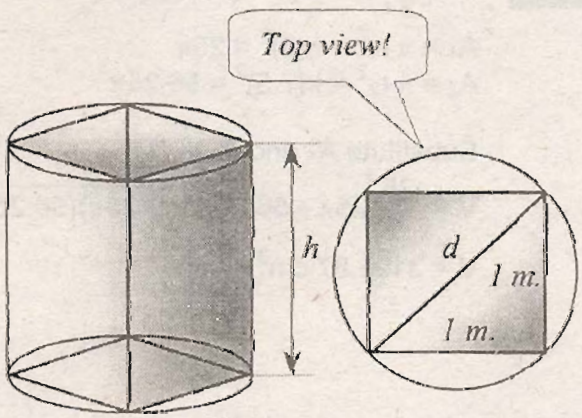


376. $$d = \sqrt{(1)^2 + (1)^2} = 1.4142 \text{ m}$$

$$V = \frac{\pi}{4} d^2 h$$

$$6.283 = \frac{\pi}{4}(1.4142)^2 h$$

$$h = 4 \text{ m.}$$



377. Note: A = area of one base
b = length of each side of the base

$$A = \frac{nb^2}{4\tan\theta}$$

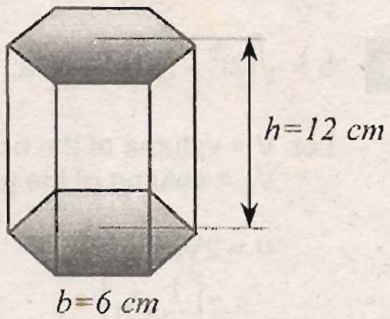
$$\theta = \frac{360°}{2n} = \frac{360°}{2(6)} = 30°$$

Substitute:

$$A = \frac{(6)(6)^2}{4\tan 30°} = 93.53 \text{ cm}^2$$

$$V = Ah$$

$$V = 93.53(12) = 1122.4 \text{ cm}^3$$



378. Let: V_1 = total volume of the bigger tank
V_2 = total volume of the smaller tank
V = volume left in the bigger tank

$$V_1 = \frac{1}{3}\left[\frac{\pi d_1^2}{4}\right]h_1 = \frac{\pi}{12}(6)^2(10) = 94.247 \text{ ft}^3$$

$$V_2 = \frac{1}{3}\left[\frac{\pi d_2^2}{4}\right]h_2 = \frac{\pi}{12}(6)^2(8) = 75.398 \text{ ft}^3$$

$$V = V_1 - V_2$$

$$V = 94.247 - 75.398 = 18.849 \text{ ft}^3$$

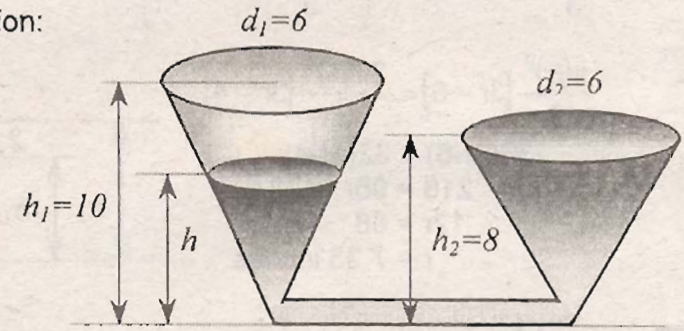
By ratio and proportion:

$$\frac{V_1}{V} = \left(\frac{h_1}{h}\right)^3$$

$$\frac{94.247}{18.849} = \left(\frac{10}{h}\right)^3$$

$$5 = \frac{1000}{h^3}$$

$$h = \sqrt[3]{200} \text{ ft.}$$

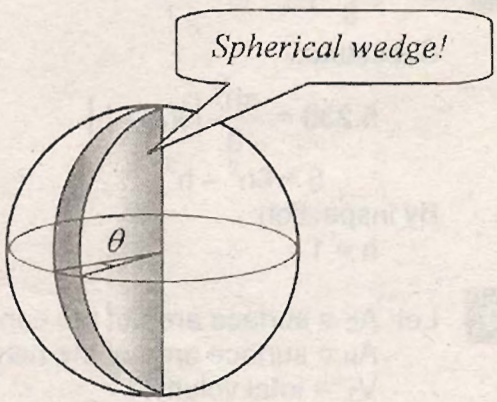


379. Let: V = volume of the wedge
V_C = volume of the sphere

By ratio and proportion:

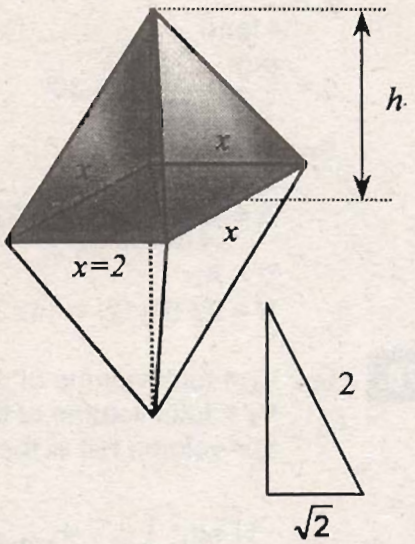
$$\frac{V_C}{2\pi} = \frac{V}{\theta}$$

$$V = \frac{V_C}{2\pi} = \frac{(4/3)\pi r^3\theta}{2\pi}$$



$$V = \frac{4(1)^3(1)}{6} = \frac{2}{3} \text{ cubic units}$$

380. $h = \sqrt{(2)^2 - (\sqrt{2})^2} = \sqrt{2}$



Let: V = volume of the octahedron
V_P = volume of the pyramid

$$V = 2V_P$$
$$= 2\left[\frac{1}{3}x^2h\right]$$
$$V = 2\left[\frac{1}{3}(2)^2\sqrt{2}\right] = 3.77 \text{ m}^3$$

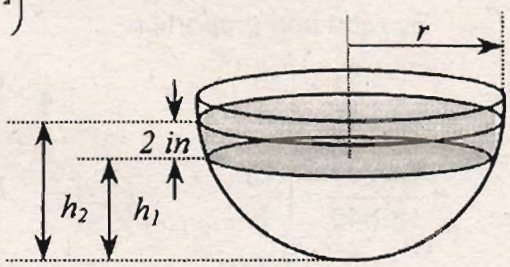
381. Let: V_B = volume of the black mixture
V_W = volume of the white mixture

$V_B = V_W$ ☞ ❶

$V_T = V_B + V_W$ ☞ ❷

Substitute (2) in (1):

$$V_T = 2\,V_W$$
$$\frac{\pi(h_1)^2}{3}[3r - h_1] = 2\left(\frac{\pi(h_2)^2}{3}[3r - h_2]\right)$$
$$\frac{\pi(6)^2}{3}[3r - 6] = \frac{2\pi(4)^2}{3}[3r - 4]$$
$$36(2r\text{-}6) = 32(3r\text{-}4)$$
$$108r - 216 = 96r - 128$$
$$12r = 88$$
$$r = 7.33 \text{ inches}$$

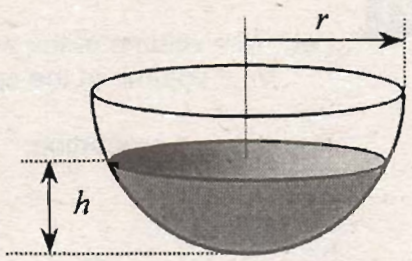


382. $V = \frac{\pi h^2}{3}(3r - h)$

Substitute:

$$5.236 = \frac{\pi h^2}{3}[3(2) - h]$$
$$5 = 6h^2 - h^3$$

By inspection:
h = 1



383. Let: A_C = surface area of the cone
A_H = surface area of the hemisphere
V_T = total volume
V_C = volume of the cone

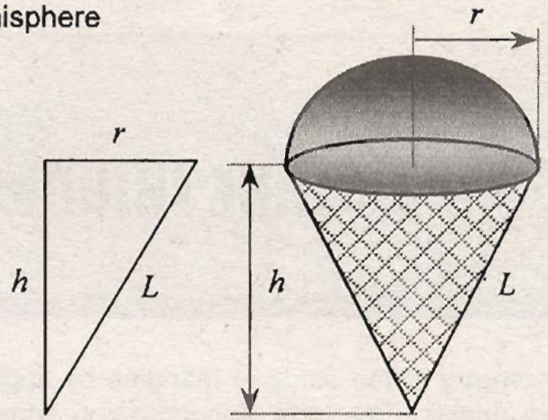
V_H = volume of the hemisphere

$A_C = A_H$

$$\pi rL = \frac{1}{2}\left[4\pi r^2\right]$$

$L = 2r$

$L = 2(1) = 2$ in.



$h = \sqrt{L^2 - r^2}$

$h = \sqrt{(2)^2 - (1)^2} = 1.732$ in.

$V_T = V_C + V_H$

$$= \frac{1}{3}\pi r^2 h + \frac{1}{2}\left[\frac{4}{3}\pi r^3\right]$$

$$= \left(\frac{\pi}{3}\right)(1)^2(1.732) + \frac{4\pi}{6}(1)^3$$

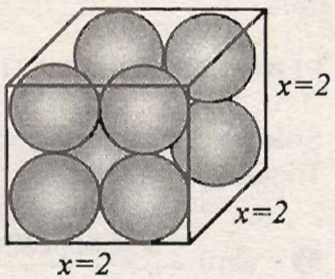
$V_T = 3.91$ in^3

384. Let: r = radius of each marble

V_W = volume of water inside the cube

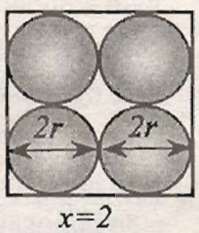
V_C = volume of the cube

V_M = volume of each marble



$4r = x = 2$

$r = 0.5$ in.



$V_W = V_C - 8V_M$

$$= (x)^3 - 8\left[\frac{4}{3}\pi(r)^3\right]$$

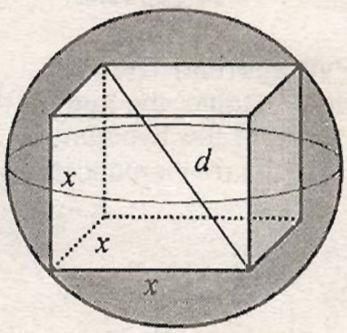
$$V_W = (2)^3 - 8\left[\frac{4}{3}\pi(0.5)^3\right] = 3.8 \text{ in}^3$$

385. Let: V = volume inside the sphere but outside the box

V_S = volume of the sphere

V_B = volume of the box

$V_B = x^3$



$2744 = x^3$

$x = 14$ cm.

$d = \sqrt{x^2 + x^2 + x^2}$

$d = \sqrt{3(14)^2} = 24.24$ cm.

$r = 12.12$ cm.

$V = V_S - V_B$

$$= \frac{4}{3}\pi r^3 - 2744 = \frac{4}{3}\pi(12.12)^3 - 2744$$

$V = 4713.555$ cm^3

DAY 9

PLANE TRIGONOMETRY

Trigonometry is the study of triangles by applying the relations between the sides and the angles. The term "trigonometry" comes from the Greek words "trigonon" which means "triangle" and "metria" meaning "measurements.

Plane Trigonometry deals with triangles in the two dimensions of the plane while **Spherical Trigonometry** concerns with triangles extracted from the surface of a sphere.

A. PLANE TRIGONOMETRY

1. Solutions to right triangles:
Trigonometric functions:

❶ $\sin\theta = \dfrac{\text{side opposite}}{\text{hypotenuse}}$

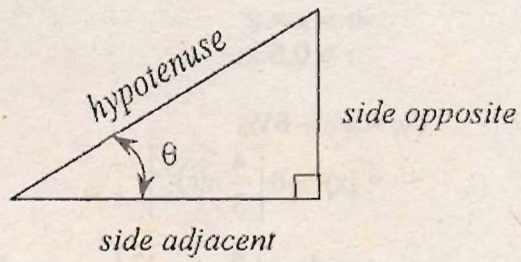
❷ $\cos\theta = \dfrac{\text{side adjacent}}{\text{hypotenuse}}$

❸ $\tan\theta = \dfrac{\text{side opposite}}{\text{side adjacent}}$

❹ $\cot\theta = \dfrac{\text{side adjacent}}{\text{side opposite}}$

❺ $\sec\theta = \dfrac{\text{hypotenuse}}{\text{side adjacent}}$

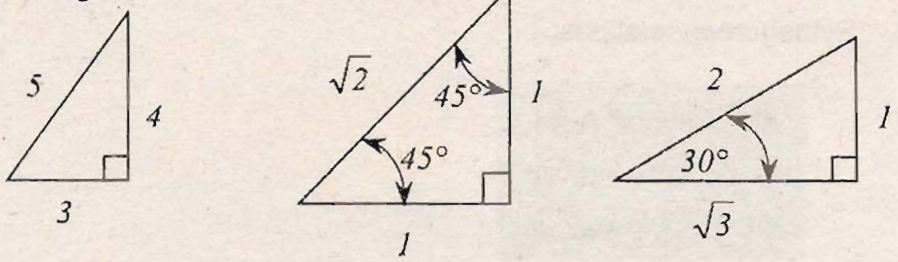
❻ $\csc\theta = \dfrac{\text{hypotenuse}}{\text{side opposite}}$



The Pythagorean Theorem:
In a right triangle, the sum of the squares of the length of the sides is equal to the square of the hypotenuse. This was formulated by **Pythagoras** (c.580 – c 500 B.C.). Its mathematical expression is

$$a^2 + b^2 = c^2$$

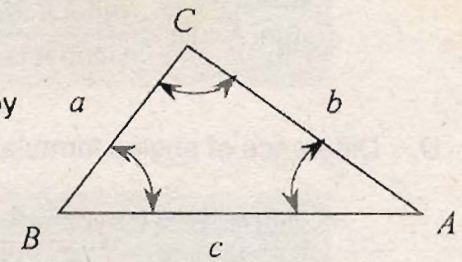
Special Triangles:



2. Solutions to oblique triangles:

Law of sines: This was first demonstrated by **Ptolemy** of Alexandria in about 150 A.D.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Law of cosines: This was first described by a French mathematician, **Francois Viete** (1540 – 1603).

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Law of tangents: This was first described by a Danish mathematician and Physician **Thomas Fincke** (1561-1656) in 1583.

$$\frac{a-b}{a+b} = \frac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)}$$

3. Fundamental trigonometric identities:

A. Reciprocal relations:

$$\sin A = \frac{1}{\csc A} \qquad \cot A = \frac{1}{\tan A}$$
$$\cos A = \frac{1}{\sec A} \qquad \sec A = \frac{1}{\cos A}$$
$$\tan A = \frac{1}{\cot A} \qquad \csc A = \frac{1}{\sin A}$$

B. Pythagorean relations:

$$\sin^2 A + \cos^2 A = 1$$
$$1 + \cot^2 A = \csc^2 A$$
$$1 + \tan^2 A = \sec^2 A$$

C. Sum of angles formulas:

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

D. Difference of angles formulas:

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

E. Double angle formulas:

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

F. Powers of functions:

$$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$$
$$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$$
$$\tan^2 A = \frac{1 - \cos 2A}{1 + \cos 2A}$$

G. Functions of half angles:

$$\sin\frac{A}{2}=\pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\frac{A}{2}=\pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\frac{A}{2}=\frac{1-\cos A}{\sin A}=\frac{\sin A}{1+\cos A}$$

H. Sum of two functions:

$$\sin A+\sin B=2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\cos A+\cos B=2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\tan A+\tan B=\frac{\sin(A+B)}{\cos A\cos B}$$

I. Difference of two functions:

$$\sin A-\sin B=2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

$$\cos A-\cos B=-2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

$$\tan A-\tan B=\frac{\sin(A-B)}{\cos A\cos B}$$

J. Product of two functions:

$$2\sin A\sin B=\cos(A-B)-\cos(A+B)$$

$$2\sin A\cos B=\sin(A+B)+\sin(A-B)$$

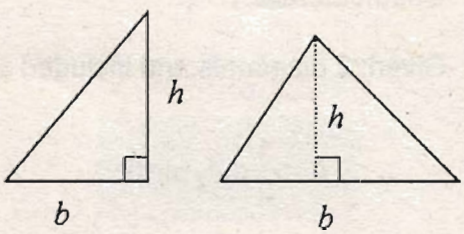
$$2\cos A\cos B=\cos(A+B)+\cos(A-B)$$

4. Plane area:

A. Triangles:

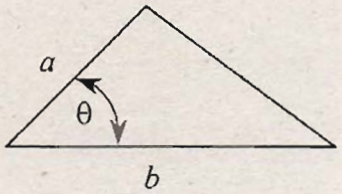
1. Given: base and altitude

$$A=\frac{1}{2}bh$$



2. Given: 2 sides and included angle

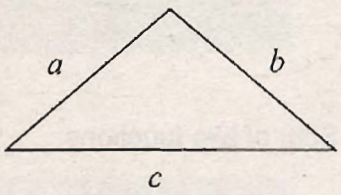
$$A = \frac{1}{2}ab\sin\theta$$



3. Given: 3 sides
   Using Heron's Formula: Named after Heron of Alexandria (1st Century AD)

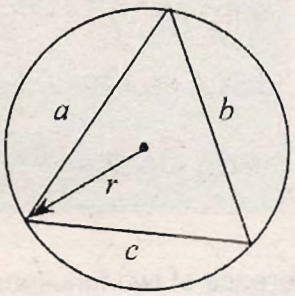
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where: $s = \frac{a+b+c}{2}$



4. Triangle inscribed in a circle:

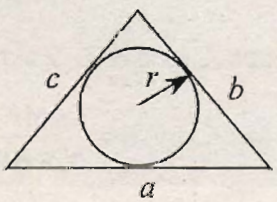
$$A = \frac{abc}{4r}$$



5. Triangle circumscribing a circle:

$$A = rs$$

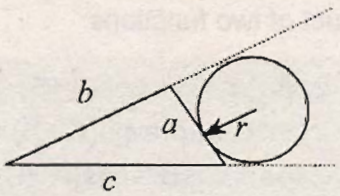
where: $s = \frac{a+b+c}{2}$



6. Triangle with escribed circle:

$$A = r(s-a)$$

where: $s = \frac{a+b+c}{2}$

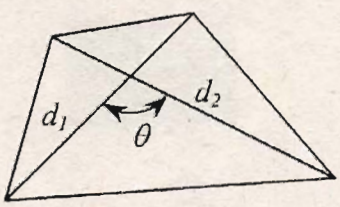


and a is the side physically tangent to the circle

B. Quadrilaterals:

1. Given: 2 diagonals and included angle

$$A = \frac{1}{2}d_1 d_2 \sin\theta$$



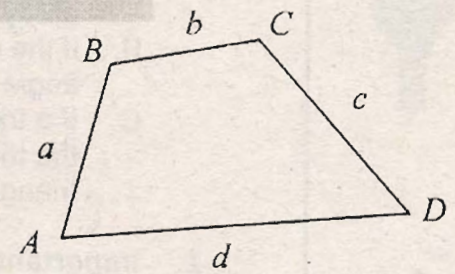
2. Given: 4 sides and 2 opposite angles:

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\theta}$$

where:

$$s = \frac{a+b+c+d}{2}$$

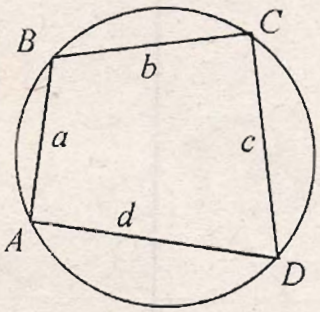
$$\theta = \frac{A+C}{2} = \frac{B+D}{2}$$



3. Cyclic quadrilateral: (All vertices lie on a circle)

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

Note: A + C = 180°
B + D = 180°



Ptolemy's Theorem states that " *The sum of the two pairs of opposite sides of a convex quadrilateral inscribed in a circle is equal to the product of the lengths of the diagonals*".

$$ac + bd = d_1 d_2$$

where : d_1 and d_2 are diagonals of a quadrilateral

This theorem was named after the geographer, mathematician and astronomer, **Ptolemy** or **Claudius Ptolemaeus** (c.100 – c.168 A.D.) of Alexandria.



Tips: 1. Important Theorems:

A. For the same intercepted arc, the central angle is always twice the inscribed angle.
B. If the arc intercepted is the same the inscribed angle is also the same.
C. If a triangle is inscribed in a circle and one side of the triangle is the diameter of the circle, then the triangle is a right triangle.

2. Important properties of a triangle:

A. The sum of two sides of a triangle is greater than the third side and their difference is less than the third side.
B. The perpendicular bisectors of the sides and the bisectors of the angles of a triangle, meet in points which are the center of the circumscribed circle and the inscribed circle, respectively.
C. The altitudes of a triangle meet in a point.
D. The medians of a triangle are concurrent at a point which is 2/3 of the distance from any vertex to the midpoint of the opposite sides.

Did you know that... the symbol "x" for multiplication was introduced by English mathematician, William Oughtred (1575 – 1660) in 1631!

Proceed to the next page for your 9th test. GOODLUCK ! ☞

Test 9

Time element: 4.0 hours

Problem 386: ECE Board April 1999

Sin (B – A) is equal to _____, when B = 270 degrees and A is an acute angle.

A. – cos A
B. cos A
C. – sin A
D. sin A

Problem 387: ECE Board April 1999

If $\sec^2 A$ is 5/2, the quantity $1 - \sin^2 A$ is equivalent to

A. 2.5
B. 1.5
C. 0.4
D. 0.6

Problem 388: ECE Board April 1999

$(\cos A)^4 - (\sin A)^4$ is equal to _____.

A. cos 4A
B. cos 2A
C. sin 2A
D. sin 4A

Problem 389: ECE Board April 1999

Of what quadrant is A, if sec A is positive and csc A is negative?

A. IV
B. II
C. III
D. I

Problem 390: ME Board October 1996

Angles are measured from the positive horizontal axis, and the positive direction is counterclockwise. What are the values of sin B and cos B in the 4th quadrant?

A. sin B > 0 and cos B < 0
B. sin B < 0 and cos B < 0
C. sin B > 0 and cos B > 0
D. sin B < 0 and cos B > 0

Problem 391: ECE Board November 1998

Csc 520° is equal to

A. cos 20°
B. csc 20°
C. tan 45°
D. sin 20°

Problem 392: ECE Board April 1993

Solve for θ in the following equation: Sin 2θ = cos θ

A. 30°
B. 45°
C. 60°
D. 15°

Problem 393: CE Board November 1993

If sin 3A = cos 6B, then

A. A + B = 90°
B. A + 2B = 30°
C. A + B = 180°
D. None of these

Problem 394: EE Board October 1996

Solve for x, if tan 3x = 5 tan x.

A. 20.705°
B. 30.705°
C. 35.705°
D. 15.705°

Problem 395: EE Board October 1997

If sin x cos x + sin 2x = 1, what are the values of x?

A. 32.2°, 69.3°
B. –20.67°, 69.3°
C. 20.90°, 69.1°
D. –32.2°, 69.3°

Problem 396: EE Board April 1997

Solve for G is csc (11G – 16 degrees) = sec (5G + 26 degrees).

A. 7 degrees
B. 5 degrees
C. 6 degrees
D. 4 degrees

Problem 397: EE Board April 1992

Find the value of A between 270° and 360° if $2 \sin^2 A - \sin A = 1$.

A. 300°
B. 320°
C. 310°
D. 330°

Problem 398: CE Board November 1993

If $\cos 65° + \cos 55° = \cos \theta$, find θ in radians.

A. 0.765
B. 0.087
C. 1.213
D. 1.421

Problem 399: CE Board November 1992

Find the value of $\sin (\text{arc} \cos \frac{15}{17})$.

A. 8/11
B. 8/19
C. 8/15
D. 8/17

Problem 400: EE Board October 1991

The sine of a certain angle is 0.6, calculate the cotangent of the angle.

A. 4/3
B. 5/4
C. 4/5
D. 3/4

Problem 401: EE Board March 1998

If $\sec 2A = \frac{1}{\sin 13A}$, determine the angle A in degrees.

A. 5°
B. 6°
C. 3°
D. 7°

Problem 402: CE Board November 1992

If $\tan x = \frac{1}{2}$, $\tan y = \frac{1}{3}$, what is the value of $\tan (x + y)$?

A. 1/2
B. 1/6
C. 2

D. 1

Problem 403: CE Board November 1993

Find the value of y in the given: $y = (1 + \cos 2\theta) \tan \theta$.

A. $\sin \theta$
B. $\cos \theta$
C. $\sin 2\theta$
D. $\cos 2\theta$

Problem 404: CE Board May 1992

Find the value of $\dfrac{\sin\theta + \cos\theta \tan\theta}{\cos\theta}$

A. $2 \sin \theta$
B. $2 \cos \theta$
C. $2 \tan \theta$
D. $2 \cot \theta$

Problem 405: ME Board April 1996

Simplify the equation $\sin^2 \theta (1 + \cot^2 \theta)$

A. 1
B. $\sin^2 \theta$
C. $\sin^2 \theta \sec^2 \theta$
D. $\sec^2 \theta$

Problem 406: ME Board October 1995

Simplify the expression $\sec \theta - (\sec \theta) \sin^2 \theta$

A. $\cos^2 \theta$
B. $\cos \theta$
C. $\sin^2 \theta$
D. $\sin \theta$

Problem 407: ME Board April 1998

Arc tan [2 cos (arc sin $[(3^{1/2}) / 2]$) is equal to

A. $\pi/3$
B. $\pi/4$
C. $\pi/16$
D. $\pi/2$

Problem 408: EE Board October 1992

Evaluate arc cot [2cos (arc sin 0.5)]

A. 30°
B. 45°
C. 60°
D. 90°

Problem 409: ECE Board March 1996

Solve for x in the given equation: Arc tan (2x) + arc tan (x) = $\frac{\pi}{4}$

A. 0.149
B. 0.281
C. 0.421
D. 0.316

Problem 410: EE Board March 1998

Solve for x in the equation: arc tan (x + 1) + arc tan (x – 1) = arc tan (12).

A. 1.5
B. 1.34
C. 1.20
D. 1.25

Problem 411: ECE Board November 1998

Solve for A for the given equation $\cos^2 A = 1 - \cos^2 A$.

A. 45, 125, 225, 335 degrees
B. 45, 125, 225, 315 degrees
C. 45, 135, 225, 315 degrees
D. 45, 150, 220, 315 degrees

Problem 412: ECE Board April 1991

Evaluate the following: $\frac{\sin 0° + \sin 1° + \sin 2° + \cdots + \sin 89° + \sin 90°}{\cos 0° + \cos 1° + \cos 2° + \cdots + \cos 89° + \cos 90°}$

A. 1
B. 0
C. 45.5
D. 10

Problem 413: ECE Board April 1991

Simplify the following: $\frac{\cos A + \cos B}{\sin A - \sin B} + \frac{\sin A + \sin B}{\cos A - \cos B}$

A. 0
B. sin A

C. 1
D. cos A

Problem 414: ECE Board April 1991

Evaluate: $\dfrac{2\sin\theta\cos\theta - \cos\theta}{1 - \sin\theta + \sin^2\theta - \cos^2\theta}$

A. sin θ
B. cos θ
C. tan θ
D. cot θ

Problem 415: ECE Board April 1994

Solve for the value of "A" when sin A = 3.5 x and cos A = 5.5x.

A. 32.47°
B. 33.68°
C. 34.12°
D. 35.21°

Problem 416: ECE Board November 1996

If sin A = 2.511x, cos A = 3.06x and sin 2A = 3.939x, find the value of x?

A. 0.265
B. 0.256
C. 0.562
D. 0.625

Problem 417: CE Board May 1994

If coversed sin θ = 0.134, find the value of θ.

A. 30°
B. 45°
C. 60°
D. 90°

Problem 418: ME Board April 1991

A man standing on a 48.5 meter building high, has an eyesight height of 1.5 m from the top of the building, took a depression reading from the top of another nearby building and nearest wall, which are 50° and 80° respectively. Find the height of the nearby building in meters. The man is standing at the edge of the building and both buildings lie on the same horizontal plane.

A. 39.49
B. 35.50
C. 30.74
D. 42.55

Problem 419: ECE Board April 1998

Points A and B 1000 m apart are plotted on a straight highway running East and West. From A, the bearing of a tower C is 32° W of N and from B the bearing of C is 26° N of E. Approximate the shortest distance of tower C to the highway.

A. 364 m
B. 374 m
C. 384 m
D. 394 m

Problem 420: ECE Board November 1998

Two triangles have equal bases. The altitude of one triangle is 3 units more than its base and the altitude of the other triangle is 3 units less than its base. Find the altitudes, if the areas of the triangles differ by 21 square units.

A. 6 and 12
B. 3 and 9
C. 5 and 11
D. 4 and 10

Problem 421: GE Board August 1994

A ship started sailing S 42°35' W at the rate of 5 kph. After 2 hours, ship B started at the same port going N 46°20' W at the rate of 7 kph. After how many hours will the second ship be exactly north of ship A?

A. 3.68
B. 4.03
C. 5.12
D. 4.83

Problem 422: ME Board April 1993

An aerolift airplane can fly at an airspeed of 300 mph. If there is a wind blowing towards the cast at at 50 mph, what should be the plane's compass heading in order for its course to be 30°? What will be the plane's ground speed if it flies in this course?

A. 19.7°, 307.4 mph
B. 20.1°, 309.4 mph
C. 21.7°, 321.8 mph
D. 22.3°, 319.2 mph

Problem 423: ECE Board April 1998

A man finds the angle of elevation of the top of a tower to be 30°. He walks 85 m nearer the tower and finds its angle of elevation to be 60°. What is the height of the tower?

A. 76.31 m
B. 73.31 m

C. 73.16 m
D. 73.61 m

Problem 424: ECE Board April 1994

A pole cast a shadow 15 m long when the angle of elevation of the sun is 61°. If the pole is leaned 15° from the vertical directly towards the sun, determine the length of the pole.

A. 54.23 m
B. 48.23 m
C. 42.44 m
D. 46.21 m

Problem 425: ME Board November 1994

A wire supporting a pole is fastened to it 20 feet from the ground and to the ground 15 feet from the pole. Determine the length of the wire and the angle it makes with the pole.

A. 24 ft, 53.13°
B. 24 ft, 36.87°
C. 25 ft, 53.13°
D. 25 ft, 36.87°

Problem 426: CE Board November 1997

The angle of elevation of the top of tower B from the top of tower A is 28° and the angle of elevation of the top of tower A from the base of tower B is 46°. The two towers lie in the same horizontal plane. If the height of tower B is 120 m, find the height of tower A.

A. 66.3 m
B. 79.3 m
C. 87.2 m
D. 90.7 m

Problem 427: CE Board November 1997

Points A and B are 100 m apart and are of the same elevation as the foot of a building. The angles of elevation of the top of the building from points A and B are 21° and 32° respectively. How far is A from the building in meters?

A. 259.28
B. 265.42
C. 271.64
D. 277.29

Problem 428: ECE Board November 1991

The captain of a ship views the top of a lighthouse at an angle of 60° with the horizontal at an elevation of 6 meters above sea level. Five minutes later, the same captain of the ship views the top of the same lighthouse at an angle of 30° with the horizontal. Determine the speed of the ship if the lighthouse is known to be 50 meters above sea level.

A. 0.265 m/sec
B. 0.155 m/sec
C. 0.169 m/sec
D. 0.210 m/sec

Problem 429: ME Board April 1997

An observer wishes to determine the height of a tower. He takes sights at the top of the tower from A and B, which are 50 feet apart, at the same elevation on a direct line with the tower. The vertical angle at point A is 30° and at point B is 40°. What is the height of the tower?

A. 85.60 feet
B. 92.54 feet
C. 110.29 feet
D. 143.97 feet

Problem 430: ME Board April 1993

A PLDT tower and a monument stand on a level plane. The angles of depression of the top and bottom of the monument viewed from the top of the PLDT tower at 13° and 35° respectively. The height of the tower is 50 m. Find the height of the monument.

A. 29.13 m
B. 30.11 m
C. 32.12 m
D. 33.51 m

Problem 431: ECE Board November 1998

If an equilateral triangle is circumscribed about a circle of radius 10 cm, determine the side of the triangle.

A. 34.64 cm
B. 64.12 cm
C. 36.44 cm
D. 32.10 cm

Problem 432: EE Board October 1997

The two legs of a triangle are 300 and 150 m each, respectively. The angle opposite the 150 m side is 26°. What is the third side?

A. 197.49 m

B. 218.61 m
C. 341.78 m
D. 282.15 m

Problem 433: EE Board October 1997

The sides of a triangular lot are 130 m., 180 m and 190 m. The lot is to be divided by a line bisecting the longest side and drawn from the opposite vertex. Find the length of the line.

A. 120 m
B. 130 m
C. 125 m
D. 128 m

Problem434: EE Board October 1997

The sides of a triangle are 195, 157 and 210, respectively. What is the area of the triangle?

A. 73,250 sq. units
B. 10,250 sq. units
C. 14,586 sq. units
D. 11,260 sq. units

Problem 435: ECE Board April 1998

The sides of a triangle are 8, 15 and 17 units. If each side is doubled, how many square units will the area of the new triangle be?

A. 240
B. 420
C. 320
D. 200

ANSWER KEY

386. A	399. D	412. A	425. D
387. C	400. A	413. A	426. B
388. B	401. B	414. D	427. A
389. A	402. D	415. A	428. C
390. D	403. C	416. B	429. B
391. B	404. C	417. C	430. D
392. A	405. A	418. A	431. A
393. B	406. B	419. B	432. C
394. A	407. B	420. D	433. C
395. C	408. A	421. B	434. C
396. B	409. B	422. C	435. A
397. D	410. B	423. D	
398. B	411. C	424. A	

RATING

☐ 43-50 Topnotcher
☐ 33-42 Passer
☐ 25-32 Conditional
☐ 0-24 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 9

386. $\sin(270° - A) = \sin 270° \cos A - \sin A \cos 270°$
$= (-1)\cos A - \sin A\,(0)$
$\sin(270° - A) = -\cos A$

387. $\sin^2 A + \cos^2 A = 1$
$1 - \sin^2 A = \cos^2 A$ ☞ ❶
$\sec^2 A = \frac{5}{2}$ ☞ ❷

Note: $\cos A = \frac{1}{\sec A}$, thus $\cos^2 A = \frac{1}{\sec^2 A}$

Substitute (2) in (1):

$1 - \sin^2 A = \frac{1}{\sec^2 A} = \frac{1}{5/2} = 0.4$

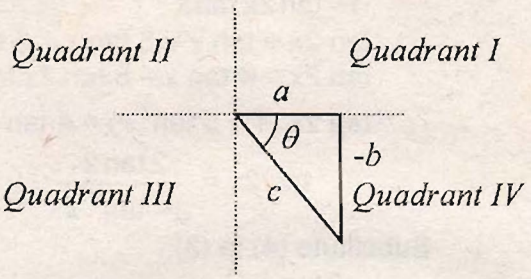
388. $\cos^4 A - \sin^4 A = \cos^2 A \cos^2 A - \sin^2 A \sin^2 A$
$= \cos^2 A\,(1 - \sin^2 A) - \sin^2 A\,(1 - \cos^2 A)$
$= \cos^2 A - \cos^2 A \sin^2 A - \sin^2 A + \sin^2 A \cos^2 A$
$= \cos^2 A - \sin^2 A$
$= \cos 2A$

Note: $\cos 2A = \cos^2 A - \sin^2 A$

389. In the fourth quadrant:

$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{c}{a}$

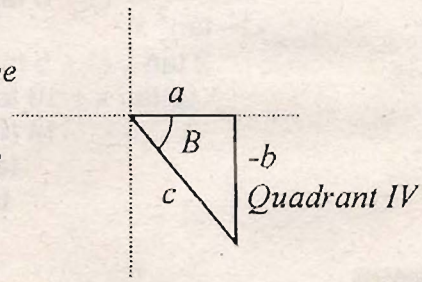
$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite side}} = -\frac{c}{b}$



390. In the fourth quadrant:

$\sin B = \frac{\text{opposite side}}{\text{hypotenuse}} = -\frac{b}{c}$ ☞ *Negative*

$\cos B = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{a}{c}$ ☞ *Positive*



Thus, $\sin B < 0$ and $\cos B > 0$

391. $\csc 520° = \csc(520° - 360°)$
$\csc 520° = \csc 160°$

$\csc 160° = \csc(180° - 160°)$
$\csc 160° = \csc 20°$

Thus $\csc 520° = \csc 20°$

392. $\sin 2\theta = \cos \theta$ ☞ ❶

Note: $\sin 2\theta = 2 \sin \theta \cos \theta$ ☞ ❷

Substitute (2) in (1):

$$2 \sin \theta \cos \theta = \cos \theta$$
$$2 \sin \theta = 1$$
$$\sin \theta = 0.5$$
$$\theta = 30°$$

393. $\sin 3A = \cos 6B$ ☞ ❶

Note: $\cos 6B = \sin (90° - 6B)$ ☞ ❷

Substitute (2) in (1):

$$\sin 3A = \sin (90° - 6B)$$
$$3A = 90° - 6B$$
$$A = 30° - 2B$$
$$A + 2B = 30°$$

394. $\tan 3x = 5 \tan x$ ☞ ❶

$$\tan 3x = \tan (2x + x) = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$ ☞ ❷

Substitute (2) in (1):

$$\frac{\tan 2x + \tan x}{1 - \tan 2x \tan x} = 5 \tan x$$
$$\tan 2x + \tan x = 5 \tan x - 5 \tan 2x \tan^2 x$$
$$\tan 2x = 4 \tan x - 5 \tan 2x \tan^2 x$$
$$\tan 2x (1 + 5 \tan^2 x) = 4 \tan x$$ ☞ ❸

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$ ☞ ❹

Substitute (4) in (3):

$$\frac{2 \tan x}{1 - \tan^2 x} (1 + 5 \tan^2 x) = 4 \tan x$$
$$2 \tan x (1 + 5 \tan^2 x) = 4 \tan x (1 - \tan^2 x)$$
$$2 \tan x + 10 \tan^3 x = 4 \tan x - 4 \tan^3 x$$
$$14 \tan^3 x = 2 \tan x$$
$$\tan^2 x = 0.142857$$
$$\tan x = 0.3779642$$
$$x = 20.705°$$

395. $\sin x \cos x + \sin 2x = 1$ ☞ ❶

Note: $2 \sin x \cos x = \sin 2x$

$$\sin x \cos x = 0.5 \sin 2x$$ ☞ ❷

Substitute (2) in (1):

$$0.5 \sin 2x + \sin 2x = 1$$
$$1.5 \sin 2x = 1$$
$$\sin 2x = 0.6667$$
$$2x = 41.8$$

$x = 20.9°$

Note: Complementary angles have the same values of their sine functions.
Thus, the other angle is equal to: $90° - 20.9° = 69.1°$

396. $\csc (11G - 16°) = \sec (5G + 26°)$

$$\frac{1}{\sin (11G - 16°)} = \frac{1}{\cos (5G + 26°)}$$

$\cos (5G + 26°) = \sin (11G - 16°)$ ☞ ❶

Note: $\sin \theta = \cos (90 - \theta)$

Let: $\theta = 11G - 16°$

$\sin (11G - 16°) = \cos [90° - (11G - 16°)]$

$\sin (11G - 16°) = \cos (106° - 11G)$ ☞ ❷

Substitute (2) in (1):

$\cos (5G + 26°) = \cos (106° - 11G)$

$5G + 26° = 106° - 11G$

$G = 5°$

397. $2 \sin^2 A - \sin A = 1$

$\sin^2 A - 0.5 \sin A = 0.5$

By completing square:

$(\sin A - 0.25)^2 = 0.5 + (0.25)^2$

$(\sin A - 0.25)^2 = 0.5625$

$\sin A - 0.25 = \pm 0.75$

Take minus sign:

$\sin A = 0.25 - 0.75 = - 0.5$

$A = - 30°$ or

$A = - 30° + 360° = 330°$ ☞ *Answer*

398. $\cos 65° + \cos 55° = \cos \theta$

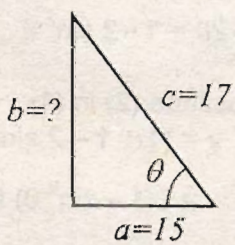
$\cos \theta = 0.99619$

$$\theta = 5° \times \frac{2\pi \text{ radians}}{360°} = 0.087 \text{ radian}$$

399. $x = \sin (\cos^{-1} \frac{15}{17})$

Let: $\theta = \cos^{-1} \frac{15}{17}$

$\cos \theta = \frac{15}{17}$

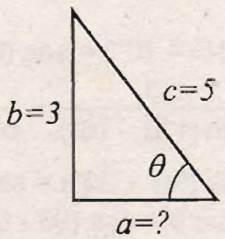


$b = \sqrt{c^2 - a^2} = \sqrt{(17)^2 - (15)^2} = 8$

$$x = \sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{b}{c}$$

$$x = \frac{8}{17}$$

400. Let: θ = angle



$$\sin\theta = 0.6 = \frac{3}{5}$$

$$a = \sqrt{c^2 - b^2} = \sqrt{(5)^2 - (3)^2} = 4$$

$$\cot\theta = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{a}{b}$$

$$\cot\theta = \frac{4}{3}$$

401.

$$\sec 2A = \frac{1}{\sin 13A}$$

$$\frac{1}{\cos 2A} = \frac{1}{\sin 13A}$$

$$\cos 2A = \sin 13A \quad \text{☞ ❶}$$

Note: $\sin\theta = \cos(90° - \theta)$
Let: $\theta = 13A$

$$\sin 13A = \cos(90° - 13A) \quad \text{☞ ❷}$$

Substitute (2) in (1):

$$\cos 2A = \cos(90° - 13A)$$
$$2A = 90° - 13A$$
$$A = 6°$$

402.

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\left(\frac{1}{3}\right)} = 1$$

403.

$$y = (1 + \cos 2\theta)\tan\theta \quad \text{☞ ❶}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\cos 2\theta = (1 - \sin^2\theta) - \sin^2\theta$$
$$\cos 2\theta = 1 - 2\sin^2\theta \quad \text{☞ ❷}$$

Substitute (2) in (1):

$$y = (1 + 1 - 2\sin^2\theta)\tan\theta = (2 - 2\sin^2\theta)\tan\theta$$

$$= 2(1 - \sin^2\theta)\tan\theta = 2(\cos^2\theta)\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$= 2\cos\theta\sin\theta$$
$$y = \sin 2\theta$$

404. $x = \dfrac{\sin\theta + \cos\theta \tan\theta}{\cos\theta} = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta \tan\theta}{\cos\theta}$

$= \tan\theta + \tan\theta$

$x = 2 \tan\theta$

405. $x = \sin^2\theta\,(1 + \cot^2\theta) = \sin^2\theta \left[1 + \left(\dfrac{\cos\theta}{\sin\theta}\right)^2\right]$

$x = \sin^2\theta \left[\dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}\right] = \dfrac{\sin^2\theta}{\sin^2\theta}$

$x = 1$

406. $x = \sec\theta - (\sec\theta)\sin^2\theta = \sec\theta\,[1 - \sin^2\theta]$

$= \sec\theta\,(\cos^2\theta) = \dfrac{1}{\cos\theta}(\cos^2\theta)$

$x = \cos\theta$

407. $x = \tan^{-1}\left[2\cos\left(\sin^{-1}\dfrac{\sqrt{3}}{2}\right)\right] = \tan^{-1} 2\cos 60°$

Thus,

$x = \tan^{-1}(2 \cos 60°) = \tan^{-1}(1)$

$= 45° \left[\dfrac{2\pi \text{ radians}}{360°}\right]$

$x = \dfrac{\pi}{4}$ radian

408. $x = \cot^{-1}[2 \cos(\sin^{-1} 0.5)] = \cot^{-1}[2 \cos 30°]$

$x = \cot^{-1}(1.732)$

$\cot x = 1.732$

$\dfrac{1}{\tan x} = 1.732$

$\tan x = \dfrac{1}{1.732} = 0.57736$

$x = 30°$

409. $\tan^{-1}(2x) + \tan^{-1} x = \dfrac{\pi}{4}$ ☞ ❶

Let: $\tan A = 2x$

$A = \tan^{-1} 2x$ ☞ ❷

$\tan B = x$

$B = \tan^{-1} x$ ☞ ❸

Substitute (2) and (3) in (1);

$$A + B = \frac{\pi}{4} = 45°$$

$$\tan (A + B) = \tan 45°$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\frac{2x + x}{1 - 2x(x)} = 1$$

$$3x = 1 - 2x^2$$

$$2x^2 + 3x - 1 = 0$$

Using the quadratic formula:

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-1)}}{2(2)} = \frac{-3 \pm 4.123}{4}$$

$$x = \frac{-3 + 4.123}{4} = 0.28$$

410. $\arctan (x + 1) + \arctan (x - 1) = \arctan (12)$ ☞ ❶

Let: $\tan A = x + 1$

$$A = \tan^{-1} (x + 1) \quad ☞ ❷$$

$$\tan B = x - 1$$

$$B = \tan^{-1} (x - 1) \quad ☞ ❸$$

Substitute (2) and (3) in (1):

$$A + B = \tan^{-1} 12$$

$$\tan (A + B) = \tan (\tan^{-1} 12)$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 12$$

$$\frac{(x + 1) + (x - 1)}{1 - (x + 1)(x - 1)} = 12$$

$$2x = 12 - 12(x^2 - x + x - 1)$$

$$2x = 12 - 12x^2 + 12$$

$$12x^2 + 2x - 24 = 0$$

Using the quadratic formula:

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(12)(-24)}}{2(12)} = \frac{-2 \pm 34}{24}$$

$$x = \frac{-2 + 34}{24} = 1.33$$

411. $\cos^2 A = 1 - \cos^2 A.$

$$2 \cos^2 A = 1$$

$$\cos^2 A = 0.5$$

$$\cos A = \pm 0.707$$

If $\cos A = +0.707$
$A = 45°$ or $315°$
If $\cos A = -0.707$
$A = 135°$ or $225°$

412. $$\frac{\sin^2\theta + \sin^2 1 + \sin^2 3 \ldots \sin^2 89 + \sin^2 90}{\cos^2\theta + \cos^2 1 + \cos^2 3 \ldots \cos^2 89 + \cos^2 90}$$

Note: $\sin^2 A + \cos^2 B = 1$ and $\cos^2 A + \cos^2 B = 1$, provided A and B are complementary angles, $(A + B = 90)$.

Thus, the equation can be written as

$$= \frac{(\sin^2 0 + \sin^2 90) + (\sin^2 1 + \sin^2 89) \ldots\ldots (\sin^2 44 + \sin^2 46)(\sin^2 45)}{(\cos^2 0 + \cos^2 90) + (\cos^2 1 + \cos^2 89) \ldots\ldots (\cos^2 44 + \cos^4 46)(\cos^2 45)}$$

$= 1$ ☞ *Answer*

413. $$\frac{\cos\theta + \cos B}{\cos A - \sin B} + \frac{\sin A + \sin B}{\cos A - \cos B}$$

$$= \frac{(\cos A + \cos B)(\cos A + \cos B) + (\sin A - \sin B)(\sin A + \sin B)}{(\sin A - \sin B)(\cos A - \cos B)}$$

$$= \frac{\cos^2 A - \cos^2 B + \sin^2 A - \sin^2 B}{(\sin A - \sin B)(\cos A - \cos B)}$$

$= 0$

414. $$x = \frac{2\sin\theta\cos\theta - \cos\theta}{1 - \sin\theta + \sin^2\theta - \cos^2\theta} = \frac{\cos\theta(2\sin\theta - 1)}{(1 - \cos^2\theta) + \sin^2\theta - \sin\theta}$$

$$= \frac{\cos\theta(2\sin\theta - 1)}{\sin^2\theta + \sin^2\theta - \sin\theta} = \frac{\cos\theta(2\sin\theta - 1)}{2\sin^2\theta - \sin\theta}$$

$$= \frac{\cos\theta(2\sin\theta - 1)}{\sin\theta(2\sin\theta - 1)} = \frac{\cos\theta}{\sin\theta}$$

$x = \cot\theta$

415. $\sin A = 3.5x$ ☞ ❶

$\cos A = 5.5x$ ☞ ❷

Divide (1) by (2):

$$\frac{\sin A}{\cos A} = \frac{3.5x}{5.5x}$$

$\tan A = 0.63636$
$A = 32.47°$

416. $\sin A = 2.511x$; $\cos A = 3.06x$; $\sin 2A = 3.939x$

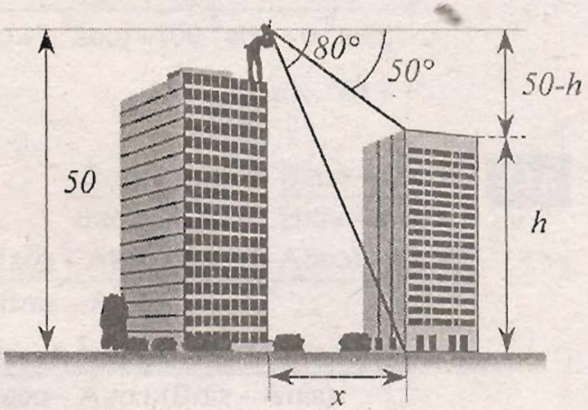
Note: $\sin 2A = 2 \sin A \cos A$
Substitute:
$3.939x = 2(2.511x)(3.06x)$
$3.939x = 15.367x^2$
$x = 0.256$

417. coversed $\sin\theta = 0.134$ ☞ ❶

Note: coversed $\sin\theta = 1 - \sin\theta$ ☞ ❷

Substitute (2) in (1):
$0.134 = 1 - \sin\theta$
$\sin\theta = 1 - 0.134$
$\sin\theta = 0.866$
$\theta = 60°$

418.



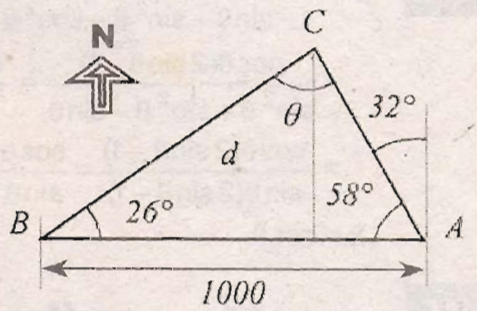
$$\tan 80° = \frac{50}{x}$$
$x = 8.816$ m.

$$\tan 50° = \frac{50 - h}{8.816}$$
$10.506 = 50 - h$
$h = 39.49$ m.

419. $\theta = 180° - (26° + 58°) = 96°$
By sine law:

$$\frac{\sin 96°}{1000} = \frac{\sin 58°}{BC}$$
$BC = 852.719$ m.

$$\sin 26° = \frac{d}{BC}$$
$d = BC \sin 26°$
$= 852.719 \sin 26°$
$d = 374$ m.



420. $h_1 = b + 3$ ☞ ❶

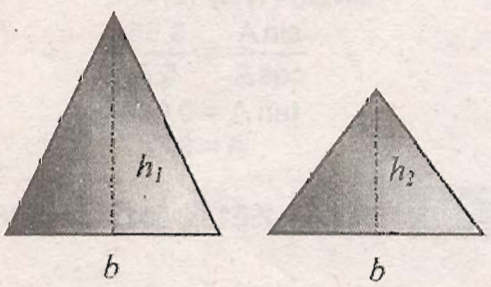
$h_2 = b - 3$ ☞ ❷

$A_1 = A_2 + 21$

$$\frac{1}{2}bh_1 = \frac{1}{2}bh_2 + 21$$ ☞ ❸

Substitute (1) and (2) in (3):

$$\frac{1}{2}b(b+3) = \frac{1}{2}b(b-3) + 21$$



$$\left[\frac{b^2+3b}{2}=\frac{b^2-3b}{2}+21\right]2$$

$$b^2+3b=b^2-3b+42$$
$$6b=42$$
$$b=7$$

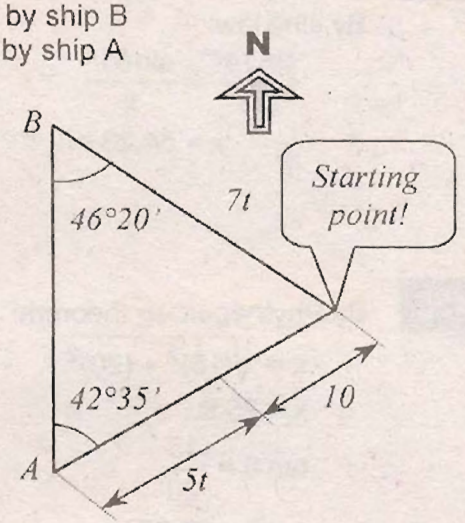
Thus,

$h_1 = 7+3 = 10$ units

$h_2 = 7-3 = 4$ units

421. Note: $7t$ = total distance traveled by ship B

$10+5t$ = total distance traveled by ship A



By sine law:

$$\frac{\sin 42°35'}{7t}=\frac{\sin 46°20'}{10+5t}$$

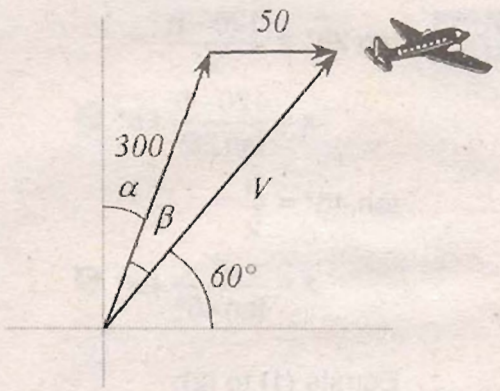
$$(10+5t)\left(\frac{\sin 42°35'}{\sin 46°20'}\right)=7t$$

$$9.354+4.677t=7t$$
$$2.323t=9.354$$
$$t=4.03 \text{ hrs.}$$

422. By sine law:

$$\frac{50}{\sin\beta}=\frac{300}{\sin 60°}$$

$$\beta = 8.3°$$



$$\alpha = 30° - \theta$$
$$\alpha = 30° - 8.3° = 21.7°$$

$$\delta + 60° + \beta = 180°$$
$$\delta + 60° + 8.3° = 180°$$
$$\delta = 111.7°$$

By sine law:

$$\frac{\sin 111.7°}{V}=\frac{\sin 8.3°}{50}$$

$$V = 321.8 \text{ mph}$$

423.

$$\tan 30° = \frac{h}{85+x}$$

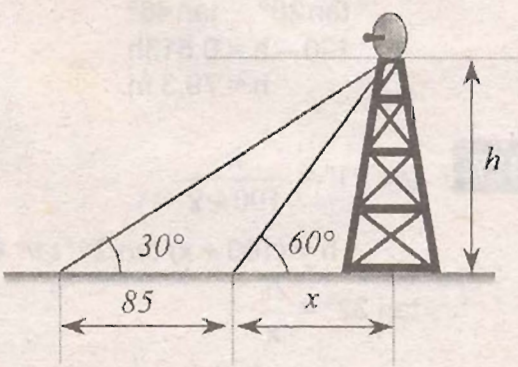
$$h = (85+x)\tan 30° \quad ☞ ❶$$

$$\tan 60° = \frac{h}{x}$$

$$h = x \tan 60° \quad ☞ ❷$$

Equate (1) to (2):

$$(85+x)\tan 30° = x \tan 60°$$



$$85 + x = 3x$$
$$x = 42.5 \text{ m.}$$

Substitute x = 42.5 in (2):

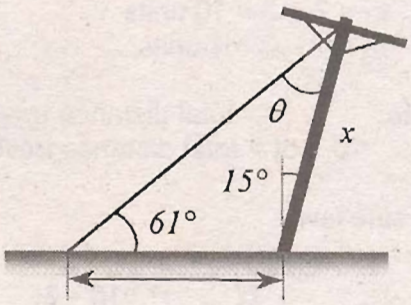
$$h = 42.5 \tan 60°$$
$$h = 73.61 \text{ m}$$

424. $\theta + 61° + 90° + 15° = 180°$

$$\theta = 14°$$

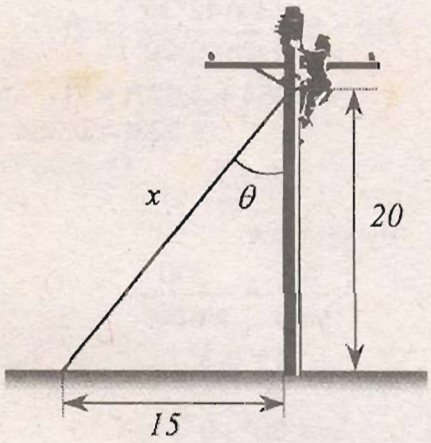
By sine law:

$$\frac{\sin 14°}{15} = \frac{\sin 61°}{x}$$
$$x = 54.23 \text{ m}$$



425. By Phythagorean theorem:

$$x = \sqrt{(15)^2 + (20)^2}$$
$$x = 25 \text{ ft.}$$
$$\tan\theta = \frac{15}{20}$$
$$\theta = 36.87°$$



426. $\tan 28° = \dfrac{120 - h}{x}$

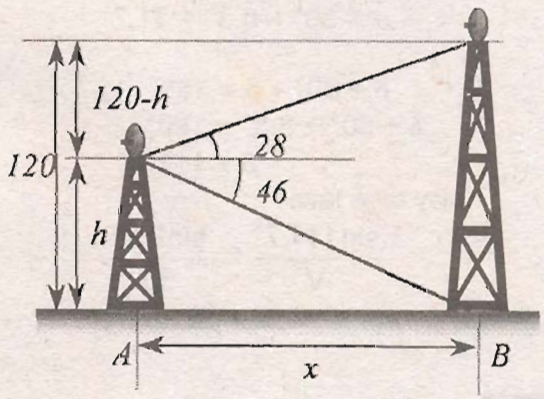
$$x = \frac{120 - h}{\tan 28°}$$ ☞ ❶

$$\tan 46° = \frac{h}{x}$$

$$x = \frac{h}{\tan 46°}$$ ☞ ❷

Equate (1) to (2):

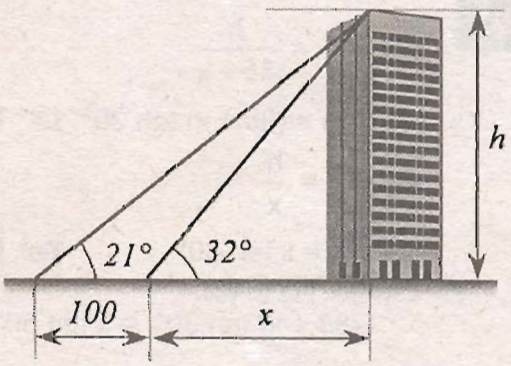
$$\frac{120 - h}{\tan 28°} = \frac{h}{\tan 46°}$$
$$120 - h = 0.513h$$
$$h = 79.3 \text{ m.}$$



427. $\tan 21° = \dfrac{h}{100 + x}$

$$h = (100 + x)\tan 21°$$ ☞ ❶

$$\tan 32° = \frac{h}{x}$$



$h = x \tan 32°$ ☞ ❷

Equate (1) to (2):

$(100 + x) \tan 21° = x \tan 32°$

$100 + x = 1.6278\, x$

$x = 159.286$ m.

Thus, the distance of point A from the building is = 100 + 159.286
= 259.286 m.

428. $\tan 60° = \dfrac{44}{x}$

$x = 25.4$ m.

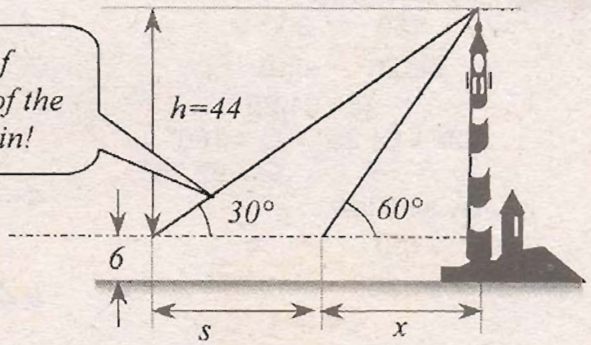
$\tan 30° = \dfrac{44}{s + x}$

$s + x = 76.21$

$s + 25.4 = 76.21$

$s = 50.81$ m.

$V = \dfrac{s}{t} = \dfrac{50.81}{5(60)} = 0.169$ m/sec



429. $\tan 40° = \dfrac{h}{x}$

$x = \dfrac{h}{\tan 40°}$ ☞ ❶

$\tan 30° = \dfrac{h}{50 + x}$

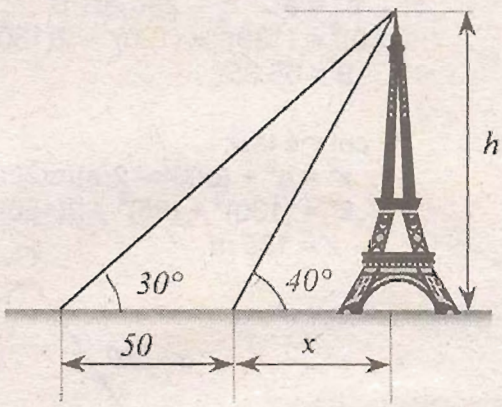
$x = \dfrac{h}{\tan 30°} - 50$ ☞ ❷

Equate (1) to (2):

$\dfrac{h}{\tan 40°} = \dfrac{h}{\tan 30°} - 50$

$1.19175\, h = 1.73205\, h - 50$

$h = 92.54$ ft.



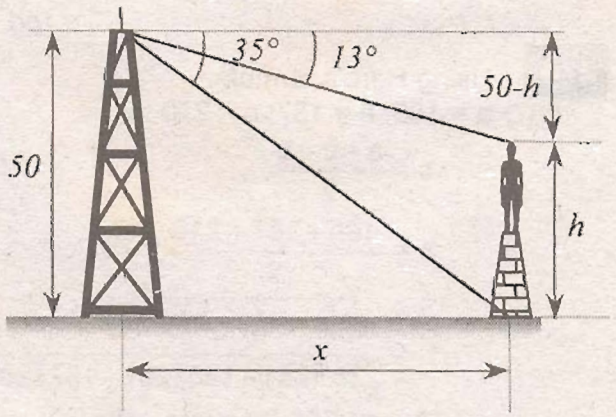
430. $\tan 35° = \dfrac{50}{x}$

$x = 71.407$ m.

$\tan 13° = \dfrac{50 - h}{x}$

$\tan 13° = \dfrac{50 - h}{71.407}$

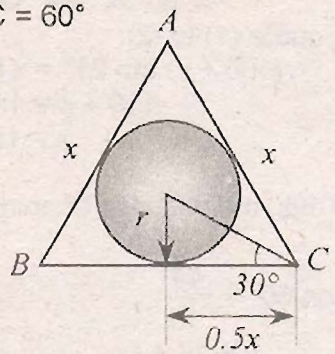
$h = 33.51$ m.



431. Note: Since equilateral triangle, A = B = C = 60°

$$\tan 30° = \frac{r}{0.5x} = \frac{10}{0.5x}$$

$$x = 34.64 \text{ cm.}$$



432. By sine law:

$$\frac{150}{\sin 26°} = \frac{300}{\sin B}$$

$$B = 61.25°$$

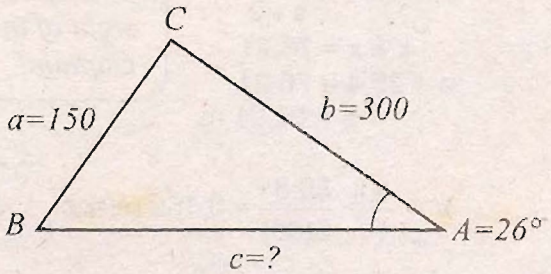
$$26° + 61.25° + C = 180°$$

$$C = 92.75°$$

By sine law:

$$\frac{150}{\sin 26°} = \frac{c}{\sin 92.75°}$$

$$c = 341.78 \text{ m.}$$



433. By cosine law:

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$(180)^2 = (130)^2 + (190)^2 - 2(130)(190) \cos B$$

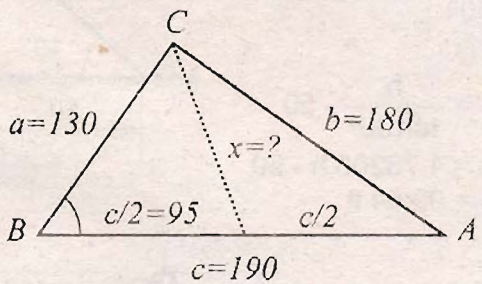
$$B = 65.35°$$

By cosine law:

$$x^2 = a^2 + (c/2)^2 - 2(a)(c/2) \cos B$$

$$x^2 = (130)^2 + (95)^2 - 2(130)(95) \cos 65.35°$$

$$x = 125 \text{ m.}$$



434. Using Hero's formula:

a = 195; b = 157; c = 210

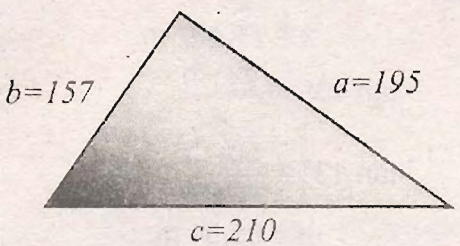
$$s = \frac{a+b+c}{2}$$

$$s = \frac{195+157+210}{2} = 281$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{281(281-195)(281-157)(281-210)}$$

A = 14,586.2 square units



435. Using Heron's formula:

$a = 16;\ b = 30;\ c = 34$

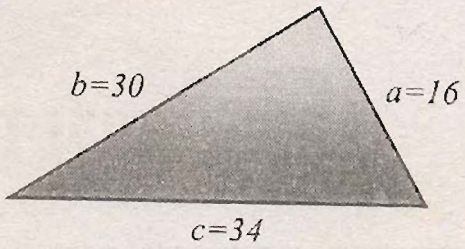
$$s = \frac{a+b+c}{2}$$

$$s = \frac{16+30+34}{2} = 40$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{40(40-16)(40-30)(40-34)}$$

$A = 240$ square units

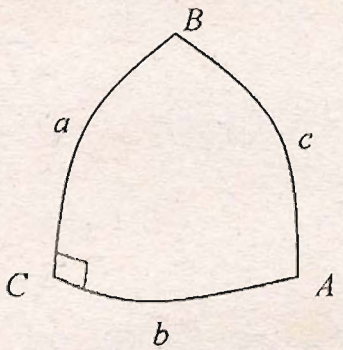


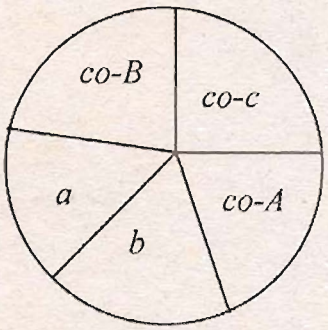
DAY 10

SPHERICAL TRIGONOMETRY

Spherical Trigonometry concerns with triangles extracted from the surface of a sphere.

1. Solution to right triangles:





To solve a right triangle, draw a circle with 5 parts. The 5 parts corresponds to the 3 sides and 2 angles of the triangle (excluding the 90°angle). Then apply Napier's rules:

Rule 1: (Tan-Ad rule)
"The sine of any middle part is equal to the product of the tangent of the adjacent parts."

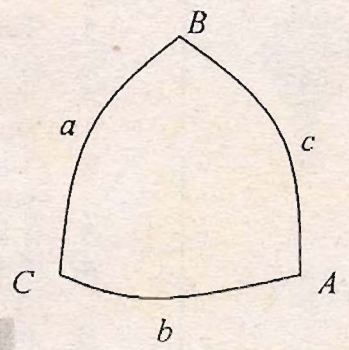
Rule 2: (Co-Op rule)
"The sine of any middle part is equal to the product of the cosine of the opposite parts."

Any of the 5 parts can be the middle part. For example, if "a" is the middle part, then "co-B" and "b" are the adjacent parts and "co-c" and "co-A" are the opposite parts, thus

$$\sin a = \tan(\text{co}\ B)\tan b$$
but $\tan(\text{co-}B) = \cot B$, $\therefore$ $\boxed{\sin a = \cot B \tan b}$

$$\sin a = \cos(\text{co}\ c)\cos(\text{co}\ A)$$
but $\cos(\text{co-}c) = \sin c$
$\cos(\text{co-}A) = \sin A$ $\therefore$ $\boxed{\sin a = \sin c \sin A}$

2. Solution to oblique triangles:



A. Law of sines

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

B. Law of cosines for the sides

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

C. Law of cosines for the angles

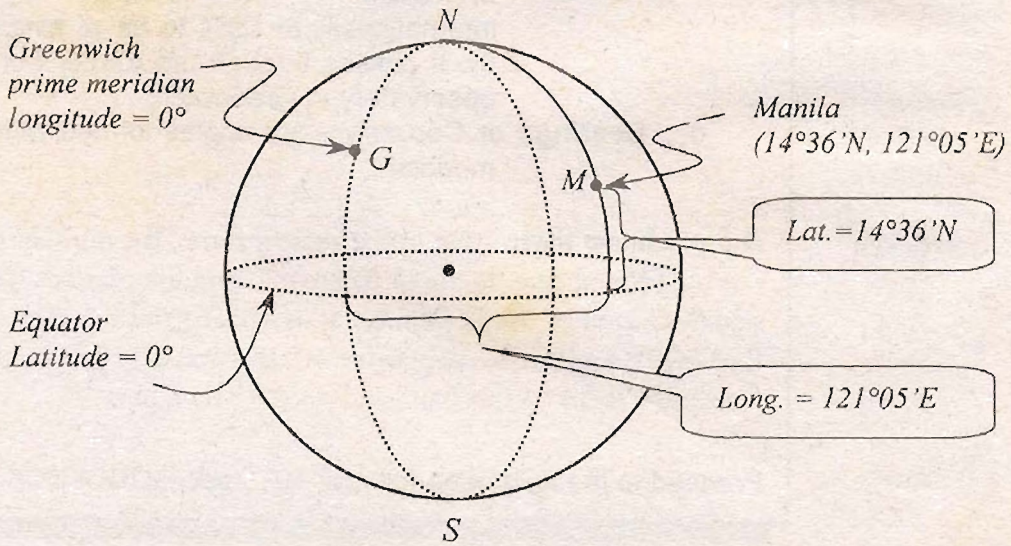
$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$
$$\cos B = -\cos A \cos C + \sin A \sin C \cos b$$
$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

3. Area of the spherical triangle:

$$A = \frac{\pi R^2 E}{180}$$

where: E = spherical excess in degrees
$E = (A + B + C) - 180°$

TERRESTRIAL SPHERE





Tips:

1. **Terrestrial Sphere:**

A. Important constants:

1. Radius of the earth = 3959 miles
2. 1 minute of the great circle area on the surface of the earth = 1 NM
3. 1 NM (nautical mile) = 6080 feet
4. 1 statute mile = 5280 feet

B. Important terms:

1. **North & South poles** – are the poles of the earth which are on its axis.
2. **Equator** – an imaginary line passing through the center of the earth and perpendicular to the axis of the earth.
3. **Parallels or Latitudes** – are curves parallel to the equator. Latitude of a point is the smallest angle formed between the radius from the point to the Earth's center an the plane of the equator. It is measured from 0° to 90° North or South.
4. **Meridians or Longitudes**– are curves perpendicular to the equator. Longitude of a point is the angle between the plane of its meridian and the plane of the Greenwich meridian. It is measured from 0° to 180° East or West.
   Meridians are all great circles while parallels are not unless it is at the equator.
5. **Prime meridian** (also known as **Greenwich meridian**) – the meridian agreed internationally in 1864 to be of longitude 0°. It passes through the former London observatory at Greenwich.
6. **Bearings or Courses** – are angles formed with the meridian.

Did you know that... that in the ancient times, the number 40 was used to indicate "many or too many" and the number 1001 signifies a kind of "finite infinity" as in AliBaba and the 40 thieves, Moses leave his people for 40 days and 40 nights and the famous Arabian "A thousand and one (1001) nights".

Proceed to the next page for your 10th test. GOODLUCK ! ☞

Test 10

Time element: 1.0 hour & 30 minutes

Problem 436:
If Greenwich mean time (GMT) is 6 A.M., what is the time at a place located 30° East longitude?

A. 7 A.M.
B. 8 A.M.
C. 9 A.M.
D. 4 A.M.

Problem 437:
If the longitude of Tokyo is 139°E and that of Manila is 121°E, what is the time difference between Tokyo and Manila?

A. 1 hour and 12 minutes
B. 1 hour and 5 minutes
C. 1 hour and 8 minutes
D. 1 hour and 10 minutes

Problem 438:
One degree on the equator of the earth is equivalent to

A. 1 minute
B. 4 minutes
C. 30 minutes
D. 1 hour

Problem 439: CE Board May 1997
A spherical triangle ABC has an angle C = 90° and sides a = 50° and c = 80°. Find the value of "b" in degrees.

A. 73.22
B. 74.33
C. 75.44
D. 76.55

Problem 440:
Solve the remaining side of the spherical triangle whose given parts are A = B = 80° and a = b = 89°.

A. 158°12'
B. 162°21'
C. 168°31'
D. 172°12'

Problem 441:
Solve for side b of a right spherical triangle ABC whose parts are a = 46°, c = 75° and C = 90°.

A. 74°
B. 68°
C. 48°
D. 74°

Problem 442:
Given a right spherical triangle whose given parts are a = 82°, b = 62° and C = 90°. What is the value of the side opposite the right angle?

A. 83°30'
B. 84°45'
C. 86°15'
D. 85°15'

Problem 443:
Determine the value of the angle B of an isosceles spherical triangle ABC whose given parts are b = c = 54°28' and a = 92°30'.

A. 89°45'
B. 55°45'
C. 84°25'
D. 41°45'

Problem 444:
Solve for angle A in the spherical triangle ABC, given a = 106°25', c = 42°16' and B = 114°53'.

A. 45°54'
B. 80°42'
C. 97°09'
D. 72°43'

Problem 445:
Solve for angle C of the oblique triangle ABC given, a = 80°, c = 115° and A = 72°.

A. 61°
B. 85°
C. 95°
D. 119°

Problem 446:
Determine the spherical excess of the spherical triangle ABC given a = 56°, b = 65° and c = 78°.

A. 33°33'
B. 68°37'
C. 91°57'
D. 98°45'

Problem 447:
What is the spherical excess of a spherical triangle whose angles are all right angles?

A. 45°
B. 90°
C. 60°
D. 30°

Problem 448: ECE Board April 1997
The area of spherical triangle ABC whose parts are A = 93°40', B = 64°12', C = 116°51' and the radius of the sphere is 100 m is

A. 15613 sq. m.
B. 16531 sq. m
C. 18645 sq. m
D. 25612 sq. m

Problem 449:
A spherical triangle has an area of 327.25 sq. km. What is the radius of the sphere if its spherical excess is 30°?

A. 20 km
B. 22 km
C. 25 km
D. 28 km

Problem 450: EE Board April 1997

A ship on a certain day is at latitude 20° N and longitude 140° E. After sailing for 150 hours at a uniform speed along a great circle route, it reaches a point at latitude 10°S and longitude 170° E. If the radius of the earth is 3959 miles, find the speed in miles per hour.

A. 17.4
B. 15.4
C. 16.4
D. 19.4

ANSWER KEY

436. B	441. B	446. A
437. A	442. C	447. B
438. B	443. D	448. B
439. B	444. C	449. C
440. C	445. D	450. C

RATING

☐ 13-15 Topnotcher
☐ 9-12 Passer
☐ 6- 8 Conditional
☐ 0- 5 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 10

436. $\dfrac{\text{diff. in time}}{24} = \dfrac{\text{diff. in longitude}}{360°}$

$\dfrac{\text{diff. in time}}{24} = \dfrac{30° - 0°}{360°}$

diff. in time = 2 hours

The time in the place is 2 hours ahead of GMT because the place is at the East.
Time is 8 AM.

437. $\dfrac{\text{diff. in time}}{24} = \dfrac{\text{diff. in longitude}}{360°}$

$\dfrac{\text{diff. in time}}{24} = \dfrac{139° - 121°}{360°}$

diff. in time = 1.2 hours = 1 hour and 12 minutes

438. 360 degrees = 24 hours

$$\text{time} = 1 \text{ degree}\left(\frac{24 \text{ hrs}}{360 \text{ degrees}}\right)$$

$$\text{time} = 0.0667 \text{ hr} \times \frac{60 \text{ min}}{\text{hr}}$$

time = 4 minutes

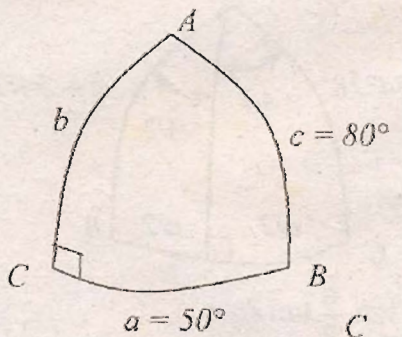
439. sin co-c = cos a cos b

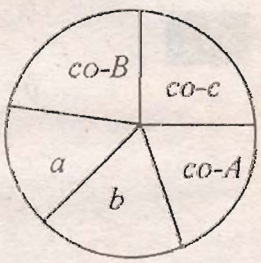
cos c = cos a cos b

$\cos b = \dfrac{\cos c}{\cos a}$

$\cos b = \dfrac{\cos 80°}{\cos 50°}$

b = 74.33°

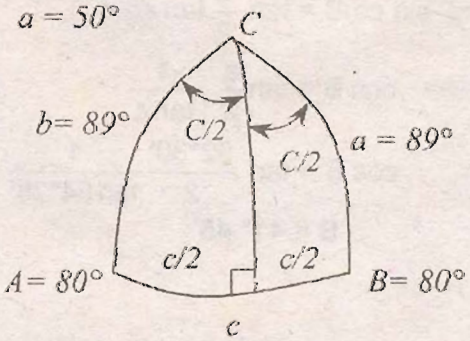




440. sin co-A = $\tan\dfrac{c}{2}$ tan co-b

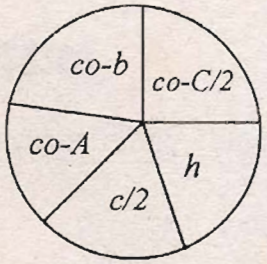
$\cos A = \tan\dfrac{c}{2}\ \dfrac{1}{\tan b}$

$\tan\dfrac{c}{2} = \cos A \tan b$



$$\tan\frac{c}{2} = \cos 80°\tan 89°$$

$$c = 168°\ 31'$$



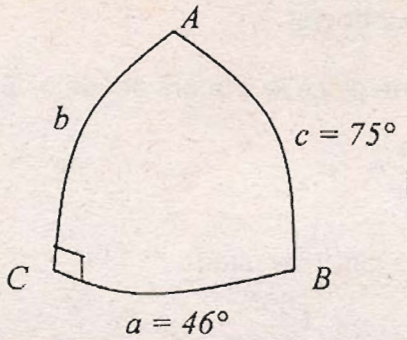
441. $\sin$ co-c = $\cos a \cos b$

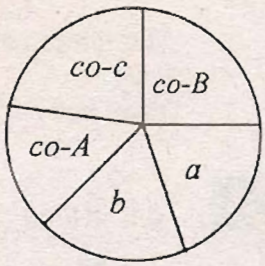
$\cos c = \cos a \cos b$

$$\cos b = \frac{\cos c}{\cos a}$$

$$\cos b = \frac{\cos 75°}{\cos 46°}$$

$b = 68°\ 07'$



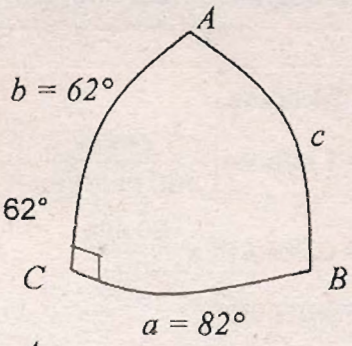


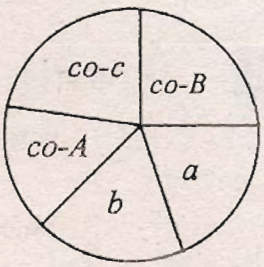
442. $\sin$ co-c = $\cos a \cos b$

$\cos c = \cos a \cos b$

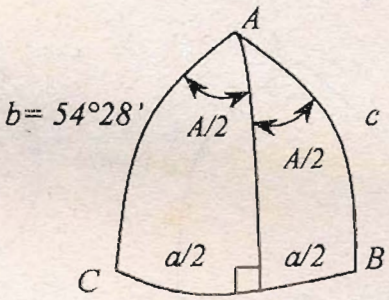
$\cos c = \cos 82° \cos 62°$

$c = 86°\ 15'$





443.

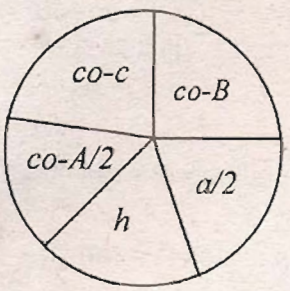


$\sin$ co-B = $\tan\frac{a}{2}\tan$ co-c

$$\cos B = \tan\frac{a}{2}\ \frac{1}{\tan c}$$

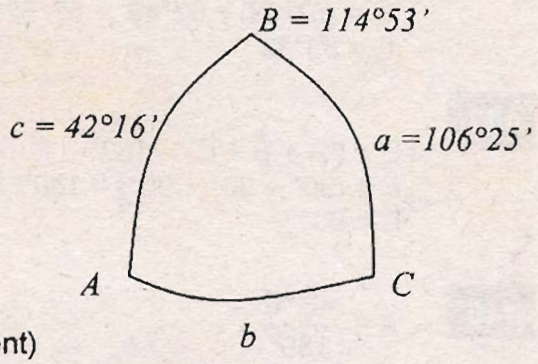
$$\cos B = \tan\frac{92°30'}{2}\ \frac{1}{\tan 54°28'}$$

$B = 41°\ 45'$



444. Using law of cosines for sides:

$\cos b = \cos a \cos c + \sin a \sin c \cos B$
$\cos b = \cos 106°25' \cos 42°16' + \sin 106°25' \sin 42°16' \cos 114°53'$
$b = 118°43'$



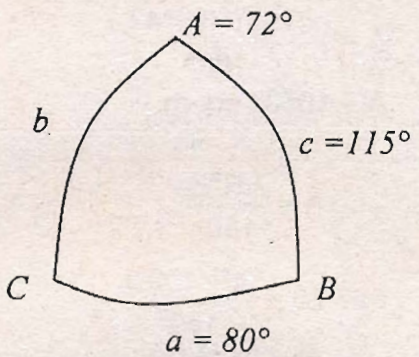
Using law of sines:

$$\frac{\sin A}{\sin 106°25'} = \frac{\sin B}{\sin b}$$
$$\frac{\sin A}{\sin 106°25'} = \frac{\sin 114°53'}{\sin 118°43'}$$
$A = 82°51'$ or
$A = 97°09'$ (its supplement)

445. Using law of sines:

$$\frac{\sin C}{\sin c} = \frac{\sin A}{\sin a}$$

$$\frac{\sin C}{\sin 115°} = \frac{\sin 72°}{\sin 80°}$$
$C = 61°04'$ or
$C = 118°56'$

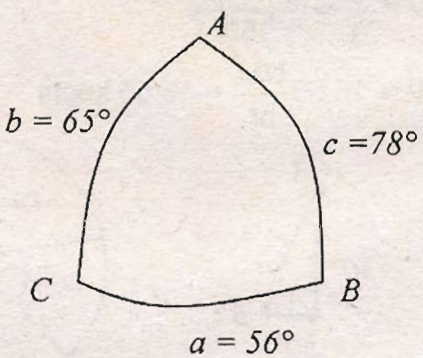


Since side c > side a
$\therefore$ Angle C > angle A
use $C = 118°56'$

446. Using law of cosines for sides:
$\cos a = \cos b \cos c + \sin b \sin c \cos A$
$\cos 56° = \cos 65° \cos 78° + \sin 65° \sin 78° \cos A$
$A = 57°53'$

Using law of sines:

$$\frac{\sin C}{\sin c} = \frac{\sin A}{\sin a}$$
$$\frac{\sin C}{\sin 78°} = \frac{\sin 57°53'}{\sin 56°}$$
$C = 87°52'$



$$\frac{\sin B}{\sin b} = \frac{\sin A}{\sin a}$$
$$\frac{\sin B}{\sin 65°} = \frac{\sin 57°53'}{\sin 56°}$$

B = 67°48'

Spherical excess, E:
E = (A + B + C) – 180
E = (57°53' + 67°48' + 87°52') – 180°
E = 33°33'

447.

E = (A + B + C) – 180°
E = (90° + 90° + 90°) – 180°
E = 90°

448. $A = \dfrac{\pi R^2 E}{180°}$

E = (A + B + C) – 180°
E = (93°40' + 64°12' + 116°51') – 180°
E = 94°43'

$$A = \frac{\pi(100)^2(94°43')}{180°}$$

A = 16531 sq. m.

449. $A = \dfrac{\pi R^2 E}{180°}$

$$327.25 = \frac{\pi R^2(30°)}{180°}$$

R = 25 km.

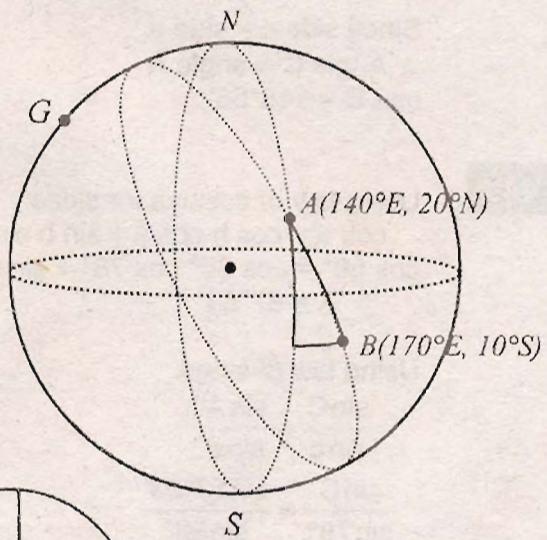
450. sin co-c = cos 30 cos 30
cos c = cos 30 cos 30
c = 41°24'35"

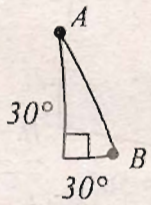
$$AB = 41°24'35"\left(\frac{60\,NM}{1°}\right)$$

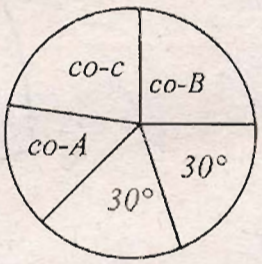
AB = 2434.58 NM

$$V = \frac{D}{t} = \frac{2484.58}{150}$$

$$V = 16.56\frac{NM}{hr} = 16.56 \text{ knots}$$







DAY 11

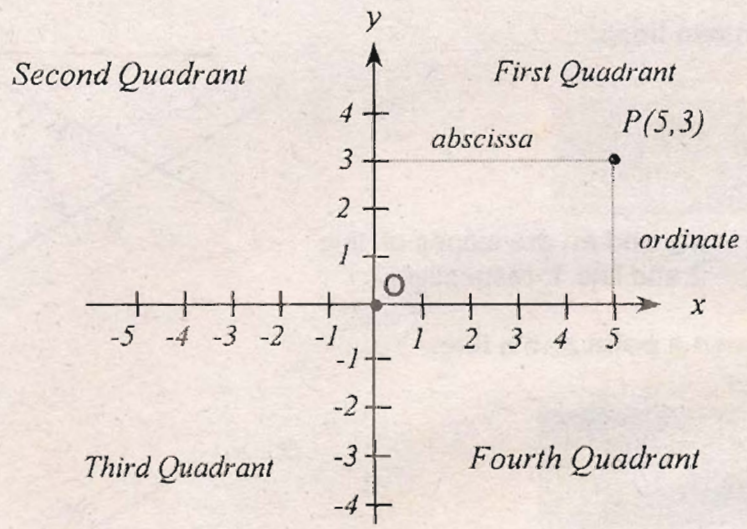
ANALYTIC GEOMETRY (POINTS, LINES & CIRCLES)

RECTANGULAR COORDINATES SYSTEM

Analytic geometry deals with geometric problems using coordinates system thereby converting it into algebraic problems.

Rene Descartes (1596 – 1650, **Cartesius** in Latin language) is regarded as the founder of analytic geometry by introducing coordinates system in 1637.

Rectangular Coordinates System (Also known as Cartesian Coordinates System)

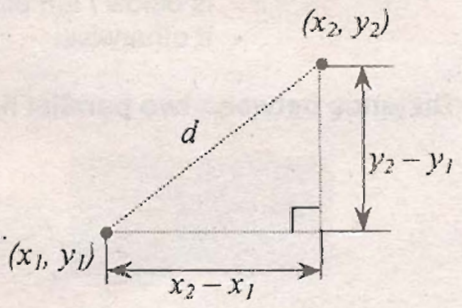


Point O is the origin and has coordinates (0,0). The x-coordinate or **abscissa** is always measured from the y-axis while the y-coordinate or **ordinate** is always measured from the x-axis. The point P has 5 and 3 as abscissa and ordinate, respectively.

Distance between two points:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

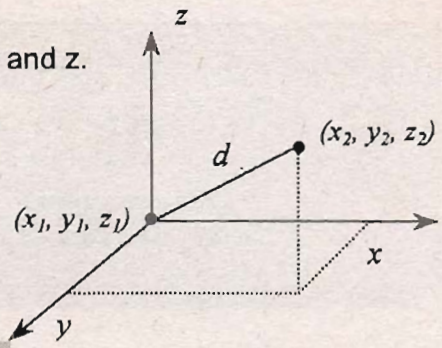
This formula is known as the distance formula.



Distance between two points in space:

In space, there are three axes, namely x, y and z.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



Slope of a line (m):

$$\text{Tan}\,\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

Since slope, m = tan θ, ∴

$$m = \frac{y_2 - y_2}{x_2 - x_1}$$

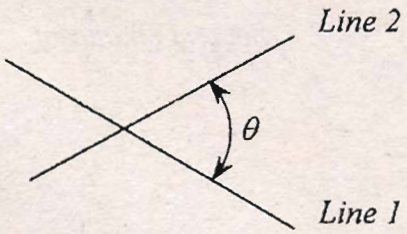
For parallel lines, $m_1 = m_2$

For perpendicular lines, $m_2 = -\frac{1}{m_1}$

Angle between two lines:

$$\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}$$

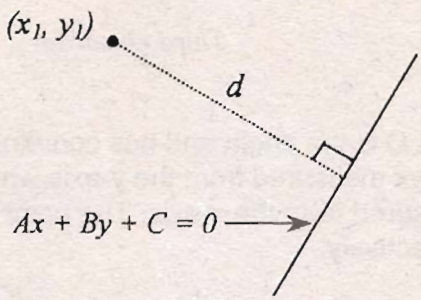
where: m_2 and m_1 are slopes of line 2 and line 1, respectively.



Distance between a point and a line:

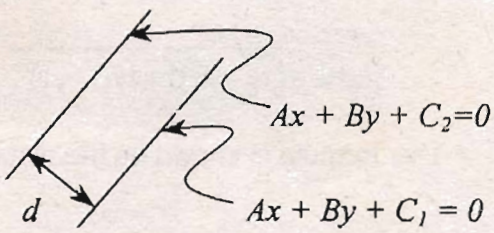
$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$$

Use: + If B is positive and the point is above / right of the line.
+ If B is negative and the point is below / left of the line.
- If otherwise.



Distance between two parallel lines:

$$d = \frac{C_1 - C_2}{\pm\sqrt{A^2 + B^2}}$$

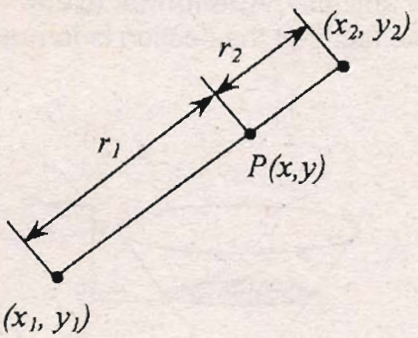


Division of line segment:

Let r_1 and r_2 be the corresponding ratio of its length to the total distance between two points.

$$x = \frac{(x_1 r_2) + (x_2 r_1)}{r_1 + r_2}$$

$$y = \frac{(y_1 r_2) + (y_2 r_1)}{r_1 + r_2}$$

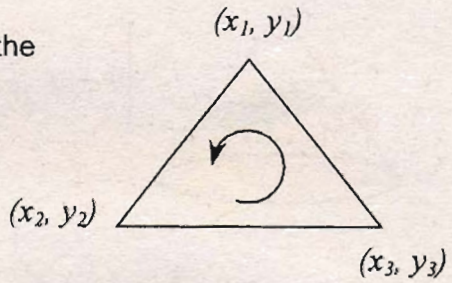


If the point P(x,y) is at the midpoint of P_1 and P_2, then

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_2 + y_1}{2}$$

Area by coordinates:

Consider a polygon with coordinates of the vertices given as shown.



$$A = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$$

$$A = \frac{1}{2}\left[(x_1y_2 + x_2y_3 + x_3y_1) - (y_1x_2 + y_2x_3 + y_3x_1)\right]$$

LINES

Equations of Lines:

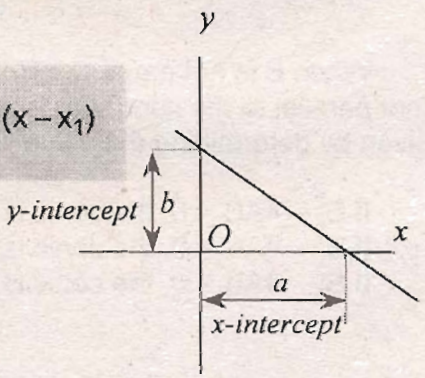
❶ General Equation: $Ax + By + C = 0$

❷ Point-Slope Form: $y - y_1 = m(x - x_1)$

❸ Slope-Intercept Form: $y = mx + b$

❹ Two-Point Form: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

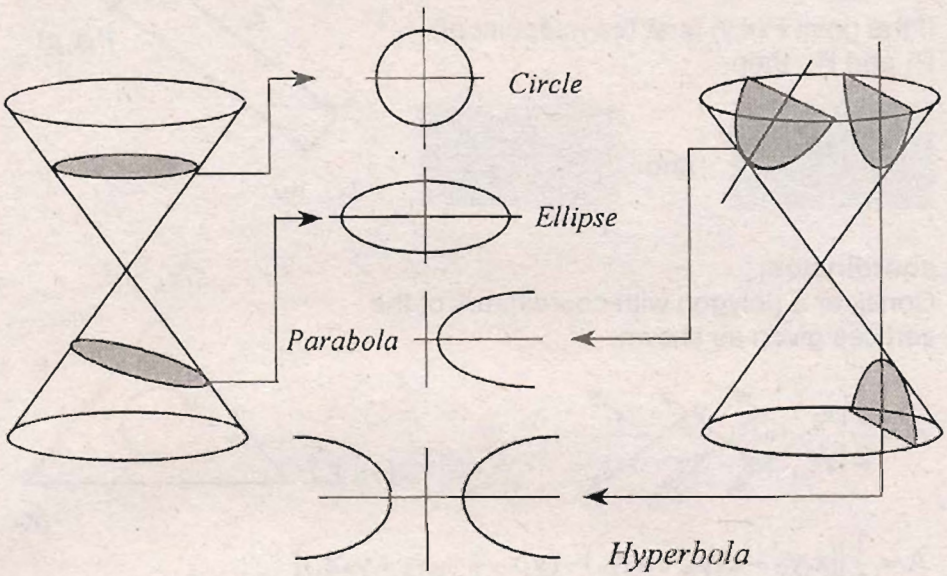
❺ Intercept Form: $\frac{x}{a} + \frac{y}{b} = 1$



CONIC SECTIONS

Conic section (or simply **Conic**) is the locus of a point which moves so that its distance from a fixed point (**focus**) is in constant ratio, **e (eccentricity)** to its distance from a fixed straight line (**directrix**).

The term "conic" was first introduced by a renowned mathematician and astronomer of antiquity, **Apollonius** (c.255 – 170 B.C.) Also, the term "conic section" was due to the fact that the section is formed by a plane made to intersect a cone



Circle is produced when the cutting plane is parallel to the base of the cone.
Ellipse is produced when the cutting plane is not parallel (or inclined) to the base of the cone.
Parabola is produced when the cutting plane is parallel to the element (or generatrix) of the cone.
Hyperbola is produced when the cutting plane is parallel to the axis of the cone.

General Equation of a Conic Section:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

When B is not equal to zero, then the principal axes of the conic are inclined (not parallel to the coordinates axes). The curve can be identified from the equation given by determining the value of the determinant, $B^2 - 4AC$.

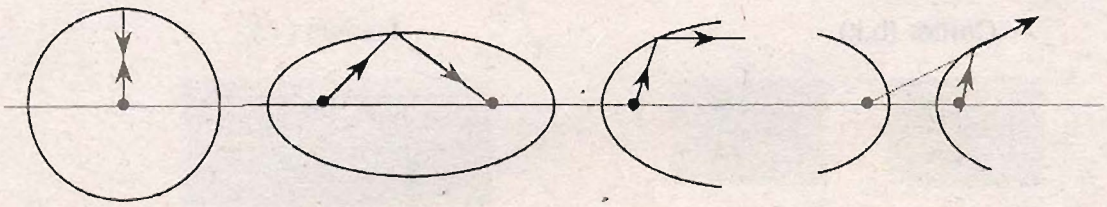
If $B^2 - 4AC < 0$, the conic is an ellipse. $\longrightarrow$ $e < 1.0$
If $B^2 - 4AC = 0$, the conic is a parabola. $\longrightarrow$ $e = 1.0$
If $B^2 - 4AC > 0$, the conic is a hyperbola. $\longrightarrow$ $e > 1.0$

When B is equal to zero, then the principal axes of the conic are parallel to the coordinates axes (x and y axes). To identify the curve, compare the coefficients of A and C.

If A = C, the conic is a circle.
If A ≠ C but the same signs, the conic is an ellipse.
If A and C have different signs, the conic is a hyperbola.
If either A or C is zero, the conic is a parabola.

The conic sections have geometric properties that can be used for some engineering application such as beams of sound and reflection of rays of light.

Circle reflects rays issued from the focus **back to the center** of the circle.
Parabola reflects rays issued from the focus as **a parallel (with respect to its axis) outgoing beam.**
Ellipse reflects rays issued **from the focus into the other focus**.
Hyperbola reflects rays issued **from the focus as if coming from the other focus**.



Circle *Ellipse* *Parabola* *Hyperbola*

CIRCLES

Circle is a locus of a point that which moves so that it is equidistant from a fixed point called center.

1. General Equation:

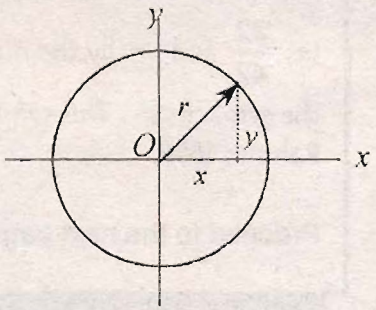
$$x^2 + y^2 + Dx + Ey + F = 0$$

If D & E = 0, center is at the origin (0,0) . If either D or E, of both D & E ≠ 0, the center is at (h,k).

2. Standard Equations:

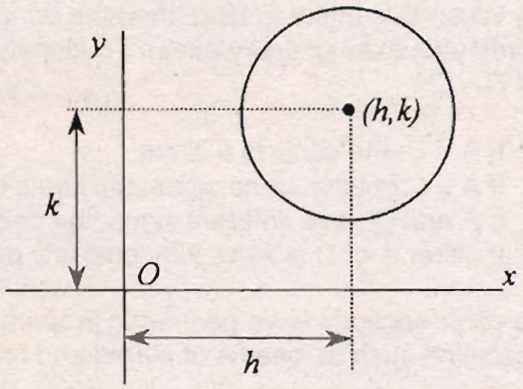
C(0,0)

$$x^2 + y^2 = r^2$$



C(h,k)

$$(x-h)^2+(y-k)^2=r^2$$



When the equation given is general equation rather than standard equation, the center **(h,k)** of the circle and its radius **(r)** can be determine by converting the general equation to standard using the process known as **completing the square.** Or using the following formulas:

General equation: $Ax^2+Cy^2+Dx+Ey+F=0$

Center (h,k)

$$h=\frac{-D}{2A} \qquad k=\frac{-E}{2A}$$

Radius (r)

$$r=\sqrt{\frac{D^2+E^2-4AF}{4A^2}}$$



Tips: 1. **Slope of line** – is defined as $\frac{\text{rise}}{\text{run}}=\frac{\Delta y}{\Delta x}$ or $\frac{y_2-y_1}{x_2-x_1}$.

Δ denotes an **increment.** When the line is parallel to the x-axis, the slope = 0. And if the line is parallel to the y-axis, its slope = ∞.

2. **Area by coordinates** is not only applied to triangles but for all polygons in general.

Did you know that... the earliest method of writing a division of two numbers was by placing the dividend above the divisor as in $\frac{32}{42}$ and later a fraction bar was introduced between the numbers, i.e. $\frac{32}{42}$ and finally the numbers themselves disappeared leaving the symbol $\div$. This symbol was introduced by Johann Heinrich Rahn in 1659.

Proceed to the next page for your 11th test. GOODLUCK! ☞

Test 11

Time element: 4.0 hours

Problem 451: ECE Board April 1999

The linear distance between –4 and 17 on the number iine is

A. 13
B. 21
C. –17
D. –13

Problem 452: EE Board April 1994

Find the distance between A (4,-3) and B (-2,5).

A. 11
B. 9
C. 10
D. 8

Problem 453:

If the distance between points (3,y) and (8,7) is 13, then y is equal to

A. 5 or –5
B. 5 or 19
C. 19
D. –5 or 19

Problem 454:

Find the coordinates of a point equidistant from (1, -6), (5, -6) and (6, -1).

A. (2, -2)
B. (3, -2)
C. (3, -3)
D. (2, -3)

Problem 455: EE Board April 1995

The line segment connecting (x,6) and (9,y) is bisected by the point (7,3). Find the values of x and y.

A. 14, 6
B. 33, 12
C. 5, 0

D. 14, 6

Problem 456:

If (-2,-4) is the midpoint of (6,-7) and (x,y), then the values of x and y are

A. x = 2, y = 1
B. x = -10, y = -1
C. x = 10, y = -1
D. x = -8, y = -1

Problem 457: ECE Board November 1998

Determine the coordinates of the point which is three-fifths of the way from the point (2,-5) to the point (-3,5).

A. (-1,1)
B. (-2,-1)
C. (-1,-2)
D. (1,-1)

Problem 458: ECE Board April 1998

The segment from (-1,4) to (2,-2) is extended three times its own length. The terminal point is

A. (11,-24)
B. (-11,-20)
C. (11,-18)
D. (11,-20)

Problem 459:

The points (a,1), (b,2) and (c,3) are collinear. Which of the following is true?

A. c – b = c – a
B. c – b = b – a
C. c – a = a – b
D. c – a = b – a

Problem 460:

If the slope of the line connecting the origin and point P is 3/4, find the abscissa of P if its ordinate is 6.

A. 2
B. 6
C. 7
D. 8

Problem 461: ECE Board April 1999

Find the inclination of the line passing through (-5,3) and (10,7).

A. 14.73
B. 14.93
C. 14.83
D. 14.63

Problem 462:

Find the angle formed by the lines $2x + y - 8 = 0$ and $x + 3y + 4 = 0$.

A. 30°
B. 35°
C. 45°
D. 60°

Problem 463:

Find the angle between the lines $3x + 2y = 6$ and $x + y = 6$.

A. 12°20'
B. 11°19'
C. 14°25'
D. 13°06'

Problem 464:

What is the acute angle between the lines $y = 3x + 2$ and $y = 4x + 9$?

A. 4.4°
B. 28.3°
C. 5.2°
D. 18.6°

Problem 465: EE Board October 1997

Find the distance of the line $3x + 4y = 5$ from the origin.

A. 4
B. 3
C. 2
D. 1

Problem 466: CE Board November 1992

The two points on the lines $2x = 3y + 4 = 0$ which are at a distance 2 from the line $3x + 4y - 6 = 0$ are

A. (-5,1) and (-5,2)
B. (64,-44) and (4,-4)
C. (8,8) and (12,12)
D. (44,-64) and (-4,4)

Problem 467: CE Board November 1992

The distance from the point (2,1) to the line $4x - 3y + 5 = 0$ is

A. 1
B. 2
C. 3
D. 4

Problem 468: CE Board November 1996

Determine the distance from (5,10) to the line $x - y = 0$.

A. 3.33
B. 3.54
C. 4.23
D. 5.45

Problem 469:

The distance from a point (1,3) to the line $4x + 3y + 12 = 0$ is

A. 4 units
B. 5 units
C. 6 units
D. 7 units

Problem 470: CE Board May 1992

Find the distance between the given lines $4x - 3y = 12$ and $4x - 3y = -8$.

A. 3
B. 4
C. 5
D. 6

Problem 471: EE Board April 1995

Find the distance between the lines, $3x + y - 12 = 0$ and $3x + y - 4 = 0$.

A. $\frac{16}{\sqrt{10}}$
B. $\frac{12}{\sqrt{10}}$
C. $\frac{4}{\sqrt{10}}$
D. $\frac{8}{\sqrt{10}}$

Problem 472: ME Board October 1996

What is the length of the line with a slope of 4/3 from a point (6,4) to the y-axis?

A. 10
B. 25
C. 50
D. 75

Problem 473: ME Board April 1998

Find the slope of the line defined by $y - x = 5$.

A. 1
B. 1/4
C. –1/2
D. 5 + x

Problem 474: CE Board November 1995

What is the slope of the line $3x + 2y + 1 = 0$?

A. 3/2
B. 2/3
C. - 3/2
D. – 2/3

Problem 475: ECE Board November 1990

In a cartesian coordinates, the vertices of a triangle are defined by the following points: (-2,0), (4,0) and (3,3). What is the area?

A. 8 sq. units
B. 9 sq. units
C. 10 sq. units
D. 11 sq. units

Problem 476: EE Board April 1994

Given three vertices of a triangle whose coordinates are A (1,1), B(3,-3) and (5,-3). Find the area of the triangle.

A. 3
B. 4
C. 5
D. 6

Problem 477: ECE Board November 1990

In a cartesian coordinates, the vertices of a square are: (1,1), (0,8), (4,5) and (-3,4). What is the area?

A. 20 sq. units
B. 30 sq. units
C. 25 sq. units

D. 35 sq. units

Problem 478: EE Board April 1997

A line passes thru (1,-3) and (-4,2). Write the equation of the line in slope-intercept form.

A. $y - 4 = x$
B. $y = -x - 2$
C. $y = x - 4$
D. $y - 2 = x$

Problem 479: EE Board October 1997

What is the x-intercept of the line passing through (1,4) and (4,1)?

A. 4.5
B. 5
C. 4
D. 6

Problem 480: ME Board April 1997

Find the equation of a straight line with a slope of 3 and a y-intercept of 1.

A. $3x + y - 1 = 0$
B. $3x - y + 1 = 0$
C. $x + 3y + 1 = 0$
D. $x - 3y - 1 = 0$

Problem 481: ECE Board April 1999

If the points (-2,3), (x,y) and (-3,5) lie on a straight line, then the equation of the line is _____.

A. $x - 2y - 1 = 0$
B. $2x + y - 1 = 0$
C. $x + 2y - 1 = 0$
D. $2x + y + 1 = 0$

Problem 482: ME Board April 1998

The equation of a line that intercepts the x-axis at $x = 4$ and the y – axis at $y = -6$ is,

A. $3x + 2y = 12$
B. $2x - 3y = 12$
C. $3x - 2y = 12$
D. $2x - 3y = 12$

Problem 483:

A line with an inclination of 45° passes through (-5/2,-9/2). What is the x-coordinate of a point on the line if its corresponding y-coordinate is 6?

A. 6
B. 7
C. 8
D. 9

Problem 484:

Find the equation of the line passing through the origin and with a slope of 6?

A. $y - 6x = 0$
B. $y = -6$
C. $x + y = -6$
D. $6x + y = 0$

Problem 485:

Find the equation of the line if the x-intercept and y-intercept are –2 and 4, respectively.

A. $y - 2x - 4 = 0$
B. $y + 2x - 4 = 0$
C. $y - 2x + 4 = 0$
D. $y + 2x + 4 = 0$

Problem 486: ECE Board April 1998

Determine B such that $3x + 2y - 7 = 0$ is perpendicular to $2x - By + 2 = 0$.

A. 5
B. 4
C. 3
D. 2

Problem 487:

The line $2x - 3y + 2 = 0$ is perpendicular to another line L_1 of unknown equation. Find the slope of L_1.

A. 3/2
B. –3/2
C. 2/3
D. –2/3

Problem 488:

A line through (-5,2) and (1,-4) is perpendicular to the line through (x,-7) and (8,7). Find x.

A. –4
B. –5

C. –6
D. –19/3

Problem 489: CE Board May 1996

What is the equation of the line that passes thru (4,0) and is parallel to the line $x - y - 2 = 0$?

A. $x - y + 4 = 0$
B. $x + y + 4 = 0$
C. $x - y - 4 = 0$
D. $x - y = 0$

Problem 490:

Find the equation of the line through point (3,1) and is perpendicular to the line $x + 5y + 5 = 0$.

A. $5x - 2y = 14$
B. $5x - y = 14$
C. $2x - 5y = 14$
D. $2x + 5y = 14$

Problem 491:

Find the equation of the perpendicular bisector of the line joining (5,0) and (-7,3)

A. $8x + 2y + 11 = 0$
B. $8x - 2y + 11 = 0$
C. $8x - y + 11 = 0$
D. $8x + y + 11 = 0$

Problem 492:

Which of the following lines is parallel to the line $3x - 2y + 6 = 0$?

A. $3x + 2y - 12 = 0$
B. $4x - 9y = 6$
C. $12x + 18y = 15$
D. $15x - 10y - 9 = 0$

Problem 493:

The equation of the line through (-3,-5) parallel to $7x + 2y - 4 = 0$ is

A. $7x + 2y + 31 = 0$
B. $7x - 2y + 30 = 0$
C. $7x + 2y - 4 = 0$
D. $2x + 7y + 30 = 0$

Problem 494:
What is the equation of the line joining the points (3,-2) and (-7,6)?

A. $2x + 3y = 0$
B. $4x - 5y = 22$
C. $4x + 5y = 2$
D. $5x + 4y = 7$

Problem 495:
What is the equation of the line passing through (-2,6) with the x-intercept half the y-intercept?

A. $x - y = 6$
B. $2x + 2y + 2 = 0$
C. $3x - y + 2 = 0$
D. $2x + y - 2 = 0$

Problem 496: CE Board May 1997
Find the slope of a line having a parametric equation of $x = 2 + t$ and $y = 5 - 3t$.

A. 2
B. 3
C. –2
D. –3

Problem 497: CE Board May 1998
Find the slope of the line having a parametric equation $y = 4t + 6$ and $x = t + 1$.

A. 1
B. 2
C. 3
D. 4

Problem 498: ECE Board April 1999
Two vertices of a triangle are (2,4) and (-2,3) and the area is 2 square units, the locus of the third vertex is

A. $4x - y = 14$
B. $4x + 4y = 14$
C. $x + 4y = 12$
D. $x - 4y = -14$

Problem 499: ECE Board April 1998

Find the area of the triangle which the line $2x - 3y + 6 = 0$ forms with the coordinate axis.

A. 3
B. 4
C. 5
D. 2

Problem 500: ECE Board November 1998

A line passes through point (2,2). Find the equation of the line if the length of the line segment intercepted by the coordinates axes is the square root of 5.

A. $2x + y - 2 = 0$
B. $2x - y - 2 = 0$
C. $2x - y + 2 = 0$
D. $2x + y + 2 = 0$

ANSWER KEY

451. B	464. A	477. C	490. B
452. C	465. D	478. B	491. B
453. D	466. B	479. B	492. D
454. C	467. B	480. B	493. A
455. C	468. B	481. D	494. C
456. B	469. B	482. C	495. D
457. A	470. B	483. C	496. D
458. D	471. D	484. A	497. D
459. B	472. A	485. B	498. D
460. D	473. A	486. C	499. A
461. B	474. C	487. B	500. B
462. C	475. B	488. C	
463. B	476. B	489. C	

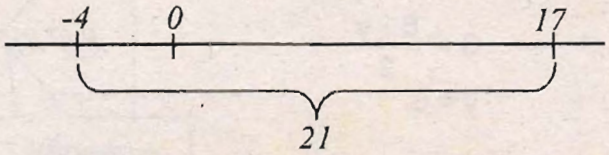
RATING

☐ 43-50 Topnotcher
☐ 33-42 Passer
☐ 25-32 Conditional
☐ 0-24 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 11

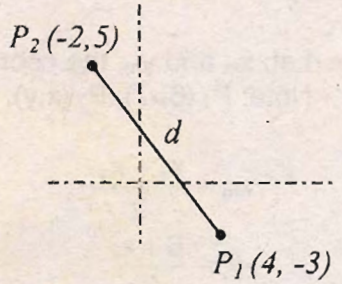
451. The linear distance from –4 to 17 is equal to 21.



452. Using distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-2-4)^2 + [5-(-3)]^2} = 10 \text{ units}$$



453. $P_1(x_1,y_1) = (3,y)$
$P_2(x_2,y_2) = (8,7)$

Using distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$(d)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$13^2 = (8-3)^2 + (7-y)^2$$
$$169 = 25 + (49 - 14y + y^2)$$
$$0 = y^2 - 14y - 95$$

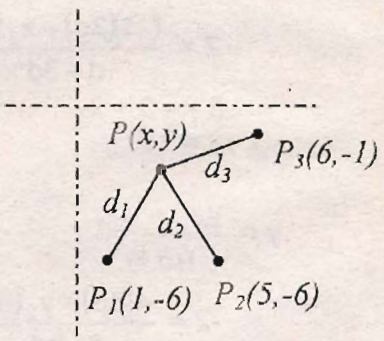
By factoring:

$$(y-19)(y+5) = 0$$
$$y = -5$$
$$y = 19$$

454.

$$d_1 = d_2$$

$$\sqrt{(1-x)^2 + (-6-y)^2} = \sqrt{(5-x)^2 + (-6-y)^2}$$
$$(1-x)^2 + (-6-y)^2 = (5-x)^2 + (-6-y)^2$$
$$(1-x)^2 = (5-x)^2$$
$$1 - 2x + x^2 = 25 - 10x + x^2$$
$$8x = 24$$
$$x = 3$$



$$d_2 = d_3$$

$$\sqrt{(5-x)^2 + (-6-y)^2} = \sqrt{(6-x)^2 + (-1-y)^2}$$
$$(5-x)^2 + (-6-y)^2 = (6-x)^2 + (-1-y)^2$$

Substitute x = 3:

$$(5-3)^2 + 36 + 12y + y^2 = (6-3)^2 + 1 + 2y + y^2$$
$$40 + 12y = 10 + 2y$$
$$10y = -30$$
$$y = -3$$

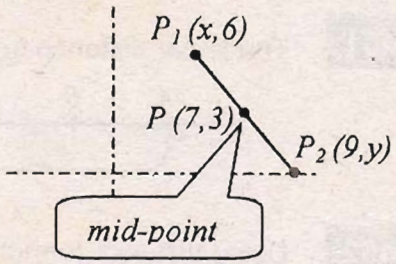
Thus the point is at (3, -3)

455. Let: x_m and y_m, the coordinates of the midpoint

$$x_m = \frac{x_1 + x_2}{2} \qquad y_m = \frac{y_1 + y_2}{2}$$

$$7 = \frac{x + 9}{2} \qquad 3 = \frac{6 + y}{2}$$

$$x = 5 \qquad y = 0$$



456. Let: x_m and y_m, the coordinates of the midpoint

Note: P_1 (6,-7); P_2 (x,y); P(-2, -4)

$$x_m = \frac{x_1 + x_2}{2} \qquad y_m = \frac{y_1 + y_2}{2}$$

$$-2 = \frac{6 + x}{2} \qquad -4 = \frac{-7 + y}{2}$$

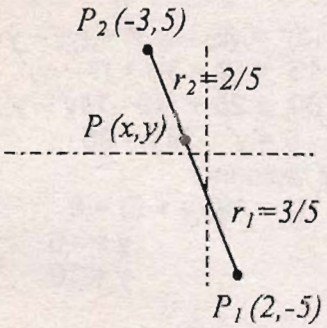
$$x = -10 \qquad y = -1$$

457. $$x = \frac{x_1 r_2 + x_2 r_1}{r_1 + r_2} = \frac{2(2/5) + (-3)(3/5)}{(3/5) + (2/5)}$$

$$x = -1$$

$$y = \frac{y_1 r_2 + y_2 r_1}{r_1 + r_2} = \frac{-5(2/5) + 5(3/5)}{(3/5) + (2/5)}$$

$$y = 1$$



458. $$x = \frac{x_1 r_2 + x_2 r_1}{r_1 + r_2}$$

$$2 = \frac{(-1)(3d) + x_2(d)}{d + 3d} = \frac{-3d + dx_2}{4d}$$

$$8 = -3 + x_2$$

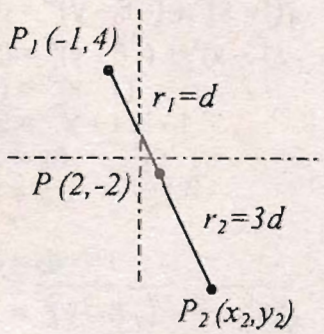
$$x_2 = 11$$

$$y = \frac{y_1 r_2 + y_2 r_1}{r_1 + r_2}$$

$$-2 = \frac{4(3d) + y_2(d)}{d + 3d} = \frac{12d + dy_2}{4d}$$

$$-8 = 12 + y_2$$

$$y_2 = -20$$



459. Given two points, the slope of the line is,

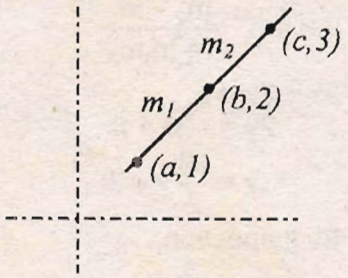
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Since collinear, $m_1 = m_2$.

$$\frac{2-1}{b-a}=\frac{3-2}{c-b}$$

$$\frac{1}{b-a}=\frac{1}{c-b}$$

$$c-b=b-a$$



460. $m=\dfrac{y_2-y_1}{x_2-x_1}$ ☞ ❶

P_1 (0, 0), thus $x_1 = 0$ and $y_1 = 0$
P_2 (x, 6), thus $x_2 = x$ and $y_2 = 6$
$m = 3/4$

Substitute in (1):

$$\frac{3}{4}=\frac{6-0}{x-0}$$

$$3x = 24$$

$$x = 8$$

461. Let: θ = angle of inclination
Note: P_1 (-5,3); P_2 (10,7)

$$\tan\theta=\frac{y_2-y_1}{x_2-x_1}=\frac{7-3}{10-(-5)}=0.2666$$

$$\theta = 14.93°$$

462. Let: θ = angle between the two lines
m_1 = slope of line 1
m_2 = slope of line 2

$$\theta=\tan^{-1}\frac{m_2-m_1}{1+m_1m_2}$$ ☞ ❶

$2x + y - 8 = 0$
$y = -2x + 8$

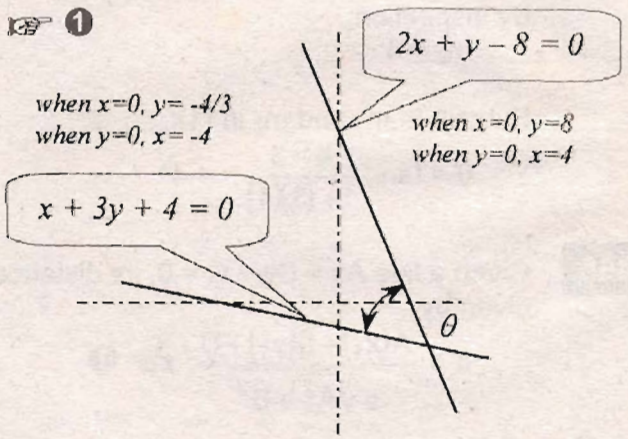
By inspection;
$m_1 = -2$

$x + 3y + 4 = 0$
$3y = -x - 4$

$$y=-\frac{1}{3}x-\frac{4}{3}$$

By inspection;

$$m_2=-\frac{1}{3}$$



Substitute m_1 and m_2 in (1):

$$\theta=\tan^{-1}\frac{(-1/3)-(-2)}{1+(-2)(-1/3)}=45°$$

463. $\theta = \tan^{-1}\dfrac{m_2 - m_1}{1 + m_1 m_2}$ ☞ ❶

$3x + 2y = 6$

$2y = -3x + 6$

$y = \dfrac{-3}{2}x + 3$

By inspection;

$m_1 = -\dfrac{3}{2}$

$x + y = 6$

$y = -x + 6$

By inspection;

$m_2 = -1$

Substitute m_1 and m_2 in (1):

$$\theta = \tan^{-1}\frac{-1-(-3/2)}{1+(-3/2)(-1)} = 11.3099°$$

$$\theta = 11° \text{ and } (0.3099° \times \frac{60'}{1°}) = 11°\ 19'$$

464. $\theta = \tan^{-1}\dfrac{m_2 - m_1}{1 + m_1 m_2}$ ☞ ❶

$y = 3x + 2$

By inspection;

$m_1 = 3$

$y = 4x + 9$

By inspection;

$m_2 = 4$

Substitute m_1 and m_2 in (1):

$$\theta = \tan^{-1}\frac{4-3}{1+(3)(4)} = 4.4°$$

465. Given a line $Ax + By + C = 0$, its distance (d) to a given point (x_1, y_1) is given by:

$$d = \frac{A(x_1) + B(y_1) + C}{\pm\sqrt{A^2 + B^2}}$$ ☞ ❶

The given line has the equation, $3x + 4y - 5 = 0$.

Thus, $A = 3$, $B = 4$ and $C = -5$

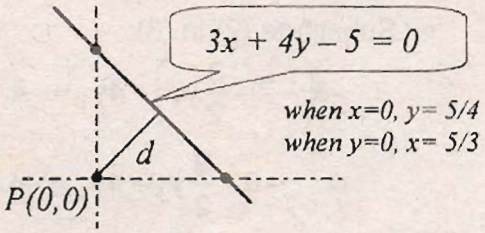
The given point is at P(0,0).

Thus, $x_1 = 0$ and $y_1 = 0$

Substitute:

$$d = \frac{3(0)+4(0)+(-5)}{\sqrt{(3)^2+(4)^2}}$$

$$d = \frac{-5}{5} = -1$$



Note: The sign of d (+ or -) denotes only the position of the point with respect to the line. And since in this problem, the point is below the line, d is negative.

466.

$$+d_1 = \frac{A(x_1)+B(y_1)+C}{\sqrt{A^2+B^2}}$$

$$2 = \frac{3(x_1)+4(y_1)+(-6)}{\sqrt{3^2+4^2}}$$

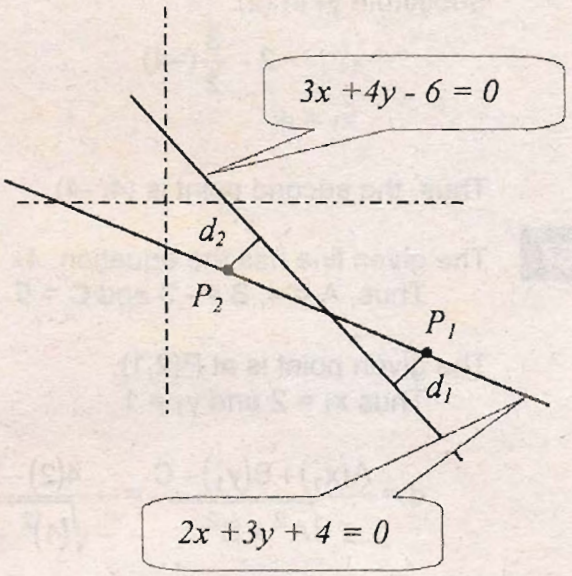
$$10 = 3x_1 + 4y_1 - 6$$

$$3x_1 + 4y_1 = 16 \quad ☞ ❶$$

$$2x_1 + 3y_1 = -4$$

$$2x_1 = -4 - 3y_1$$

$$x_1 = -2 - \frac{3}{2}y_1 \quad ☞ ❷$$



Substitute (2) in (1):

$$3(-2 - \frac{3}{2}y_1) + 4y_1 = 16$$

$$-6 - \frac{9}{2}y_1 + 4y_1 = 16$$

$$y_1 = -44$$

Substitute y_1 in (2):

$$x_1 = -2 - \frac{3}{2}(-44)$$

$$x_1 = 64$$

Thus, the first point is at (64,- 44)

$$-d_2 = \frac{A(x_1)+B(y_1)+C}{\sqrt{A^2+B^2}}$$

$$-2 = \frac{3(x_1)+4(y_1)+(-6)}{\sqrt{3^2+4^2}}$$

$$-10 = 3x_1 + 4y_1 - 6$$

$$3x_1 + 4y_1 = -4 \quad ☞ ❸$$

Substitute (2) in (3):

$$3\left(-2-\frac{3}{2}y_1\right)+4y_1=-4$$

$$-6-\frac{9}{2}y_1+4y_1=-4$$

$$-\frac{1}{2}y_1=2$$

$$y_1=-4$$

Substitute y_1 in (2):

$$x_1=-2-\frac{3}{2}(-4)$$

$$x_1=4$$

Thus, the second point is (4, -4)

467. The given line has the equation, $4x - 3y + 5 = 0$.
Thus, A = 4, B = - 3 and C = 5

The given point is at P(2,1).
Thus $x_1 = 2$ and $y_1 = 1$

$$d=\frac{A(x_1)+B(y_1)+C}{\pm\sqrt{A^2+B^2}}=\frac{4(2)-3(1)+5}{-\sqrt{(4)^2+(-3)^2}}=(-)\ 2\ \text{units}$$

468. The given line has the equation, $x - y = 0$.
Thus, A = 1, B = -1 and C = 0

The given point is at P(5,10).
Thus, $x_1 = 5$ and $y_1 = 10$

$$d=\frac{A(x_1)+B(y_1)+C}{\sqrt{A^2+B^2}}=\frac{1(5)+(-1)(10)+0}{-\sqrt{(1)^2+(-1)^2}}=3.54\ \text{units}$$

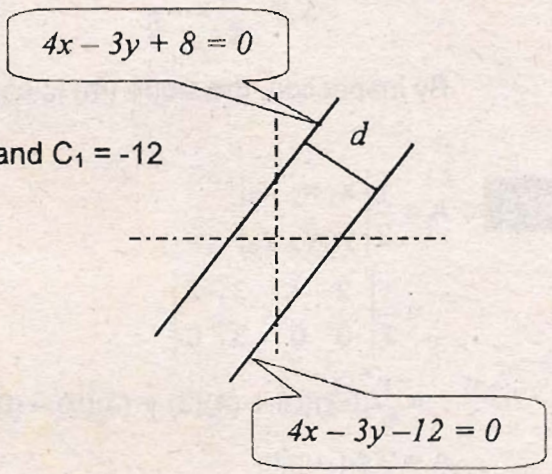
469. The given line has the equation, $4x - 3y + 12 = 0$.
Thus, A = 4, B = - 3 and C= 12

The given point is at P(1,3).
Thus, $x_1 = 1$ and $y_1 = 3$

$$d=\frac{A(x_1)+B(y_1)+C}{+\sqrt{A^2+B^2}}=\frac{4(1)+3(3)+12}{+\sqrt{(4)^2+(-3)^2}}=5\ \text{units}$$

470. Given two parallel lines; $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ the distance between these two lines is given by:

$$d = \frac{C_2 - C_1}{\sqrt{A^2 + B^2}} \quad \text{(1)}$$



$4x - 3y - 12 = 0$

By inspection, $A = 4$, $B = -3$ and $C_1 = -12$

$4x - 3y + 8 = 0$

By inspection, $C_2 = 8$

Substitute A, B, C_1 and C_2 in (1):

$$d = \frac{8 - (-12)}{\sqrt{(4)^2 + (-3)^2}} = 4 \text{ units}$$

471. The first line has the equation, $3x + y - 12 = 0$.

Thus, $A = 3$, $B = 1$ and $C_1 = -12$

The second line has the equation, $3x + y - 4 = 0$.

Thus, $C_2 = -4$

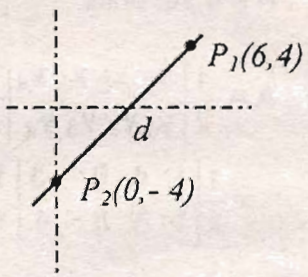
$$d = \frac{C_2 - C_1}{\sqrt{A^2 + B^2}} = \frac{-4 - (-12)}{\sqrt{(3)^2 + (1)^2}} = \frac{8}{\sqrt{10}} \text{ units}$$

472. Using point slope form:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{4}{3}(x - 6)$$

Thus at $x = 0$; $y = -4$



Using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(6 - 0)^2 + [4 - (-4)]^2} = 10 \text{ units}$$

473. Using the point-slope form: $y = mx + b$

$y - x = 5$

$y = x + 5$

By inspection, the slope (m) is equal to 1.

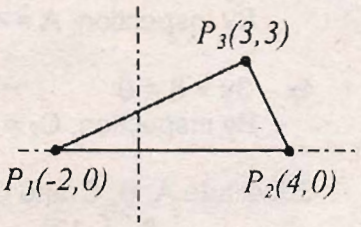
474. $3x + 2y + 1 = 0$

$$2y = -3x - 1$$

$$y = \frac{-3}{2}x - \frac{1}{2}$$

By inspection, the slope (m) is equal to $\frac{-3}{2}$

475.



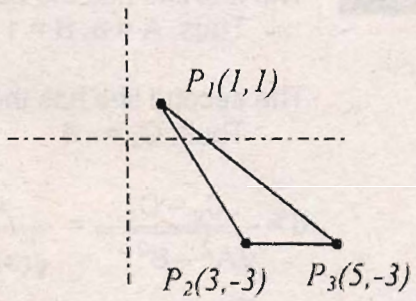
$$A = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -2 & 4 & 3 & -2 \\ 0 & 0 & 3 & 0 \end{vmatrix}$$

$$= \frac{1}{2}[(-2)(0) + (4)(3) + (3)(0) - (0)(4) - (0)(3) - (3)(-2)]$$

A = 9 sq. units

476.



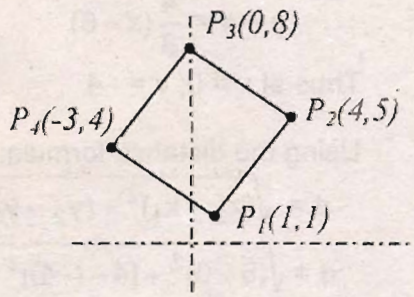
$$A = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 3 & 5 & 1 \\ 1 & -3 & -3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[(1)(-3) + (3)(-3) + (5)(1) - (1)(3) - (-3)(5) - (-3)(1)]$$

A = 4 sq. units

477.



$$A = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 4 & 0 & -3 & 1 \\ 1 & 5 & 8 & 4 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[(1)(5) + (4)(8) + (0)(4) + (-3)(1) - (1)(4) - (5)(0) - (8)(-3) - (4)(1)]$$

A = 25 sq. units

478. P_1 (1, -3), thus $x_1 = 1$ and $y_1 = -3$

P_2 (-4, 2), thus $x_2 = -4$ and $y_2 = 2$

Using two point form:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

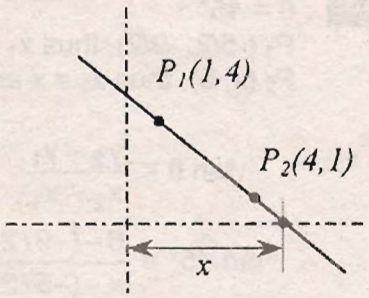
$$y - (-3) = \frac{3 - (-3)}{-4 - 1}(x - 1)$$

$$y + 3 = \frac{5}{-5}(x-1)$$

$$y + 3 = -x + 1$$

$$y = -x - 2$$

479. P_1 (1, 4), thus $x_1 = 1$ and $y_1 = 4$
P_2 (4, 1), thus $x_2 = 4$ and $y_2 = 1$



Using two point form:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 4 = \frac{1-4}{4-1}(x-1)$$

$$y + 4 = -x + 1$$

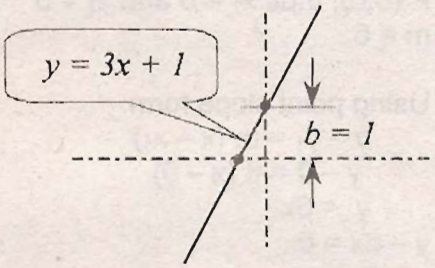
$$x + y = 5$$

Substitute y = 0 to solve for the x-intercept:

$$x + 0 = 5$$

$$x = 5$$

480. Using point slope form: y = mx + b
where:
m = slope
b = y-intercept



Thus,

$$y = 3x + 1$$

$$3x - y + 1 = 0$$

481. P_1 (-2, 3), thus $x_1 = -2$ and $y_1 = 3$
P_2 (-3, 5), thus $x_2 = -3$ and $y_2 = 5$

Using two point form:

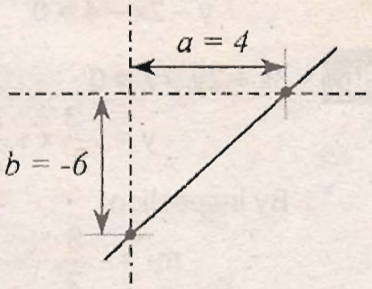
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 3 = \frac{5-3}{-3-(-2)}[x-(-2)]$$

$$y - 3 = -2(x+2)$$

$$2x + y + 1 = 0$$

482. Using the intercept form: $\frac{x}{a} + \frac{y}{b} = 1$



As given, a = 4 and b = - 6

$$\frac{x}{4} + \frac{y}{-6} = 1$$

$$\left(\frac{x}{4}+\frac{y}{-6}=1\right)24$$

$$6x - 4y = 24$$

$$3x - 2y = 12$$

483. $\theta = 45°$

P_1 (-5/2, -9/2), thus $x_1 = -5/2$ and $y_1 = -9/2$

P_2 (x, 6), thus $x_2 = x$ and $y_2 = 6$

$$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\tan 45° = \frac{6-(-9/2)}{x-(-5/2)}$$

$$1 = \frac{21/2}{x+5/2}$$

$$x + \frac{5}{2} = \frac{21}{2}$$

$$x = 8$$

484. P (0,0), thus $x_1 = 0$ and $y_1 = 0$

$m = 6$

Using point slope form:

$$y - y_1 = m\,(x - x_1)$$

$$y - 0 = 6\,(x - 0)$$

$$y = 6x$$

$$y - 6x = 0$$

485. $$\frac{x}{a}+\frac{y}{b}=1$$

where: a = x-intercept

b = y-intercept

$$\frac{x}{-2}+\frac{y}{4}=1$$

$$\left[-\frac{x}{2}+\frac{y}{4}=1\right]4$$

$$-2x + y = 4$$

$$y - 2x - 4 = 0$$

486. $3x + 2y - 7 = 0$

$$y = \frac{-3}{2}x+\frac{7}{2}$$

By inspection:

$$m_1 = -\frac{3}{2}$$

$2x - By + 2 = 0$

$$y = \frac{2}{B}x + \frac{2}{B}$$

By inspection:

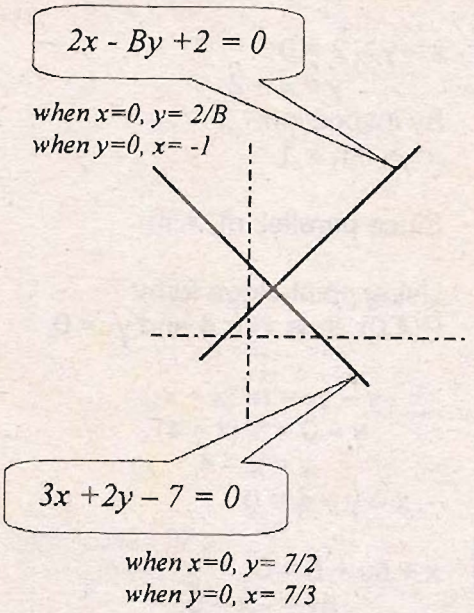
$$m_2 = \frac{2}{B}$$

Since perpendicular, $m_2 = -\frac{1}{m_1}$:

$$\frac{2}{B} = -\frac{1}{-3/2} = \frac{2}{3}$$

$2B = 6$

$B = 3$



487. $2x - 3y + 2 = 0$

$-3y = -2x - 2$

$$y = \frac{2}{3}x + \frac{2}{3}$$

By inspection:

$$m_1 = \frac{2}{3}$$

Since perpendicular, $m_2 = -\frac{1}{m_1}$:

$$m_2 = -\frac{1}{m_1} = -\frac{1}{2/3} = -\frac{3}{2}$$

488. $P_1(-5, 2)$, thus $x_1 = -5$ and $y_1 = 2$

$P_2(1, -4)$, thus $x_2 = 1$ and $y_2 = -4$

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-2}{1-(-5)} = -1$$

Since perpendicular, $m_2 = -\frac{1}{m_1}$:

$$m_2 = -\frac{1}{m_1} = -\frac{1}{(-1)} = 1$$

$P_3(x, -7)$, thus $x_3 = x$ and $y_3 = -7$

$P_4(8, 7)$, thus $x_4 = 8$ and $y_4 = 7$

$$m_2 = \frac{y_4 - y_3}{x_4 - x_3}$$

$$1 = \frac{7-(-7)}{8-x}$$

$8 - x = 14$

$x = -6$

489. $x - y - 2 = 0$

$y = x - 2$

By inspection:

$m_1 = 1$

Since parallel: $m_2 = m_1$

Using point-slope form:
P(4,0), thus $x_1 = 4$ and $y_1 = 0$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 1(x - 4)$$
$$y = x - 4$$
$$x - y - 4 = 0$$

490. $x + 5y + 5 = 0$

$5y = -x - 5$

$$y = \frac{-1}{5}x - 1$$

By inspection:

$$m_1 = -\frac{1}{5}$$

Since perpendicular: $m_2 = -\dfrac{1}{m_1}$

$$m_2 = -\frac{1}{m_1} = -\frac{1}{-1/5} = 5$$

Using point-slope form:
P(3,1), thus $x_1 = 3$ and $y_1 = 1$

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 5(x - 3)$$
$$y - 1 = 5x - 15$$
$$5x - y = 14$$

491. $P_1(5, 0)$, thus $x_1 = 5$ and $y_1 = 0$
$P_2(-7, 3)$, thus $x_2 = -7$ and $y_2 = 3$

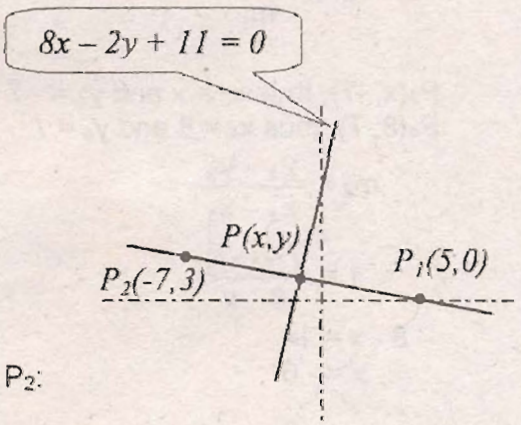
$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 0}{-7 - 5} = -\frac{1}{4}$$

Since perpendicular: $m_2 = -\dfrac{1}{m_1}$

$$m_2 = -\frac{1}{m_1} = -\frac{1}{-1/4} = 4$$

Solving for the midpoint of P_1 and P_2:

$$x = \frac{x_1 + x_2}{2} = \frac{5 + (-7)}{2} = -1$$



$$y = \frac{y_2 + y_1}{2} = \frac{0+3}{2} = \frac{3}{2}$$

Using point-slope form:

$$y - y_1 = m\,(x - x_1)$$

$$y - \frac{3}{2} = 4[x - (-1)]$$

$$\left(y - \frac{3}{2} = 4x + 4\right)2$$

$$2y - 3 = 8x + 8$$

$$8x - 2y + 11 = 0$$

492. $3x - 2y + 6 = 0$

$$2y = 3x + 6$$

$$y = \frac{3}{2}x + 3\text{, thus } m_1 = \frac{3}{2}$$

Since parallel: $m_2 = m_1$

Solving for slopes of the given lines.

a. $3x + 2y - 12 = 0$

$$2y = -3x + 12$$

$$y = \frac{-3}{2}x + 6\text{, thus } m = \frac{-3}{2}$$

b. $4x - 9y = 6$

$$9y = 4x - 6$$

$$y = \frac{4}{9}x - \frac{2}{3}\text{, thus } m = \frac{4}{9}$$

c. $12x + 18y = 15$

$$18y = -12x + 15$$

$$y = -\frac{2}{3}x + \frac{5}{6}\text{, thus } m = \frac{2}{3}$$

d. $15x - 10y - 9 = 0$

$$10y = 15x - 9$$

$$y = \frac{3}{2}x - \frac{9}{10}$$

$$m = \frac{3}{2}$$ ☞ *Answer!*

493. $7x + 2y - 4 = 0$

$$2y = -7x + 4$$

$$y = \frac{-7}{2}x + 2$$

By inspection:

$$m_1 = -\frac{7}{2}$$

Since parallel: $m_2 = m_1$

Using point-slope form:
P(-3,-5), thus $x_1 = -3$ and $y_1 = -5$

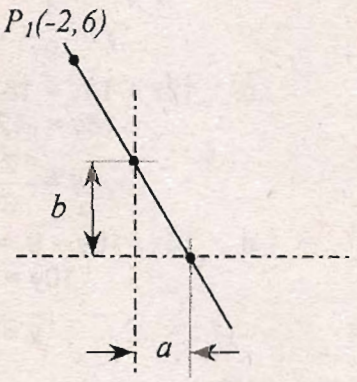
$$y - y_1 = m(x - x_1)$$
$$y - (-5) = -\frac{7}{2}[x - (-3)]$$
$$2y + 10 = -7x - 21$$
$$7x + 2y + 31 = 0$$

494. $P_1(3, -2)$, thus $x_1 = 3$ and $y_1 = -2$
$P_2(-7,6)$, thus $x_2 = -7$ and $y_2 = 6$

Using two point form:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$y - (-2) = \frac{6 - (-2)}{-7 - 3}(x - 3)$$
$$y + 2 = \frac{8}{-10}(x - 3)$$
$$-10y - 20 = 8x - 24$$
$$8x + 10y - 4 = 0$$
$$4x + 5y = 2$$

495.
$$a = \frac{1}{2}b$$
$$b = 2a \quad ☞ ❶$$



Using the intercept form:
$$\frac{x}{a} + \frac{y}{b} = 1 \quad ☞ ❷$$
Substitute: x = -2, y = 6 in (2):
$$\frac{-2}{a} + \frac{6}{b} = 1 \quad ☞ ❸$$
Substitute (1) in (3):
$$\frac{-2}{a} + \frac{6}{2a} = 1$$
$$\frac{1}{a} = 1$$
$$a = 1$$
$$b = 2$$
Substitute a = 1 and b = 2 in (2):
$$\left(\frac{x}{1} + \frac{y}{2} = 1\right)2$$
$$2x + y = 2$$
$$2x + y - 2 = 0$$

496. $x = 2 + t$

$t = x - 2$ ☞ ❶

$y = 5 - 3t$ ☞ ❷

Substitute (1) in (2):

$y = 5 - 3(x - 2)$

$.= 5 - 3x + 6$

$y = -3x + 11$

By inspection:

$m = -3$

497. $y = 4t + 6$ ☞ ❶

$x = t + 1$

$t = x - 1$ ☞ ❷

Substitute (2) in (1):

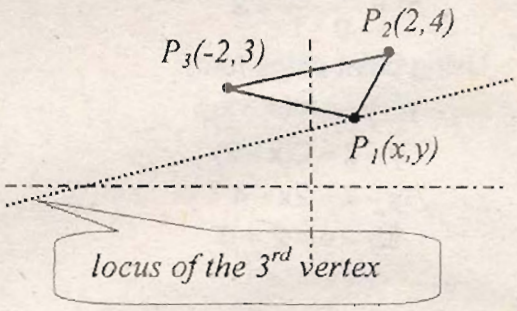
$y = 4(x - 1) + 6$

$= 4x - 4 + 6$

$y = 4x + 2$

By inspection:

$m = 4$



498. $A = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$

$2 = \frac{1}{2}\begin{vmatrix} x & 2 & -2 & x \\ y & 4 & 3 & y \end{vmatrix}$

$4 = (4)(x) + (2)(3) + (-2)(y) - (y)(2) - (4)(-2) - (3)(x)$

$4 = 4x + 6 - 2y - 2y + 8 - 3x$

$4 = x - 4y + 14$

$x - 4y = -10$

Suggested answer is choice d.

499. $2x - 3y + 6 = 0$

at $x = 0$,

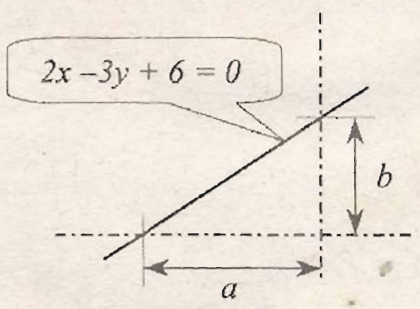
$2(0) - 3y + 6 = 0$

$y = 2$

at $y = 0$,

$2x - 3(0) + 6 = 0$

$x = -3$



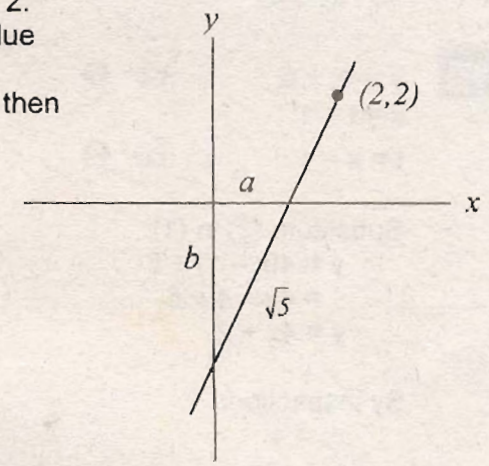
Thus, the x-intercept (a) is 3 and the y-intercept (b) is 2.

$$2x - 3(0) + 6 = 0$$
$$x = -3$$

Thus, the x-intercept (a) is 3 and the y-intercept (b) is 2.

$$A = \frac{1}{2}ab$$
$$A = \frac{1}{2}(3)(2) = 3 \text{ square units}$$

500. The value of "a" should be less than 2. Assuming a whole number for its value in order to come up with an integer coefficients as found in the choices, then "a" must be equal to 1.



If a is equal to 1, then b is equal to –2

Solving for slope: m

$$m = \frac{-2-0}{0-1} = 2$$

Using point-slope form:

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 2(x - 2)$$
$$y - 2 = 2x - 4$$
$$2x - y - 2 = 0$$

DAY 12

ANALYTIC GEOMETRY
PARABOLA, ELLIPSE & HYPERBOLA

PARABOLA

Parabola is a locus of a point which moves so that it is always **equidistant** to a fixed point called *focus* and to a fixed straight line called *directrix*.

I. Equations:

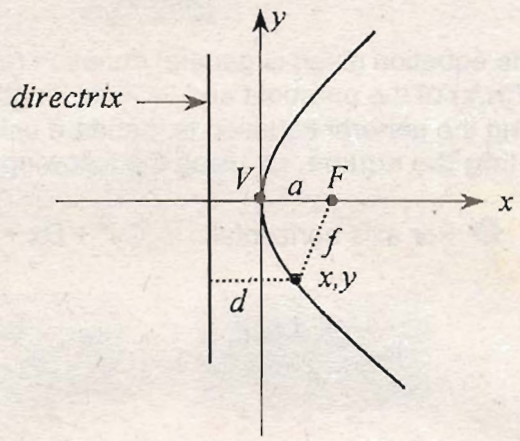
General equations:

Axis parallel to the y-axis:

$$Ax^2 + Dx + Ey + F = 0$$

Axis parallel to the x-axis:

$$Cy^2 + Dx + Ey + F = 0$$



Note: a = distance from V to F
f = focal distance

Standard equations:

Vertex (V) at origin (0,0)

A. Axis along x-axis:

Opens to the right	Opens to the left
$y^2 = 4ax$	$y^2 = -4ax$

B. Axis along y-axis:

Opens up	Opens down
$x^2 = 4ay$	$x^2 = -4ay$

Vertex (V) at (h,k)

A. Axis parallel to the x-axis:

Opens to the right	Opens to the left
$(y-k)^2 = 4a(x-h)$	$(y-k)^2 = -4a(x-h)$

B. Axis parallel to the y-axis:

Opens upward	Opens downward
$(x-h)^2 = 4a(y-k)$	$(x-h)^2 = -4a(y-k)$

II. **Eccentricity:** The ratio of the distance to the focus to the distance to the directrix.

$$e = \frac{f}{d}$$ Since f = d, $e = 1$

III. **Latus rectum:** A line that passes through the focus and perpendicular to the axis of the conic.

$$LR = 4a$$

When the equation given is general equation rather than standard equation, the vertex **V(h,k)** of the parabola and its focal length /radius **(a)** can be calculated by converting the general equation to standard using the process known as **completing the square.** Or using the following formulas:

❶ For axis horizontal: $Cy^2 + Dx + Ey + F = 0$

$$h = \frac{E^2 - 4CF}{4CD} \qquad k = \frac{-E}{2C} \qquad a = \frac{-D}{4C}$$

❷ For axis vertical: $Ax^2 + Dx + Ey + F = 0$

$$h = \frac{-D}{2A} \qquad k = \frac{D^2 - 4AF}{4AE} \qquad a = \frac{-E}{4A}$$

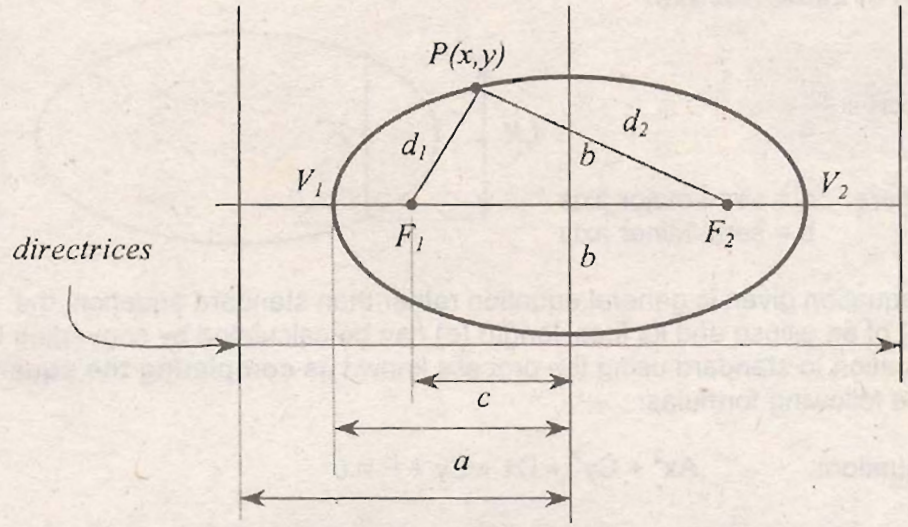
ELLIPSE

Ellipse is a locus of a point which moves so that the **sum** of its distance to the fixed points (*foci*) is constant and is equal to the length of the major axis (2a).

I. **Equations:**

General Equation:

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$



Note: $d_1 + d_2 = 2a$. The major axis = 2a, is the distance from V_1 to V_2. Also, the relationship between a, b and c is

$$b^2 + c^2 = a^2$$ a is always greater than b.

If D & E = 0, center is at the origin (0,0) . If either D or E, of both D & E ≠ 0, the center is at (h,k).

Standard Equations:

C(0,0)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \longrightarrow \textit{Major axis is horizontal}$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \longrightarrow \textit{Major axis is vertical}$$

C(h,k)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \longrightarrow \textit{Major axis is horizontal}$$

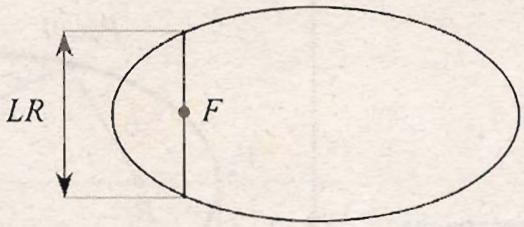
$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \longrightarrow \textit{Major axis is vertical}$$

II. **Eccentricity: (e < 1.0)**

$$e = \frac{c}{a} \quad \text{or} \quad e = \frac{a}{D}$$

III. **Length of Latus Rectum:**

$$LR = \frac{2b^2}{a}$$



where: a = semi-major axis
b = semi-minor axis

When the equation given is general equation rather than standard equation, the center (h,k) of an ellipse and its focal length (c) can be calculated by converting the general equation to standard using the process known as **completing the square.** Or using the following formulas:

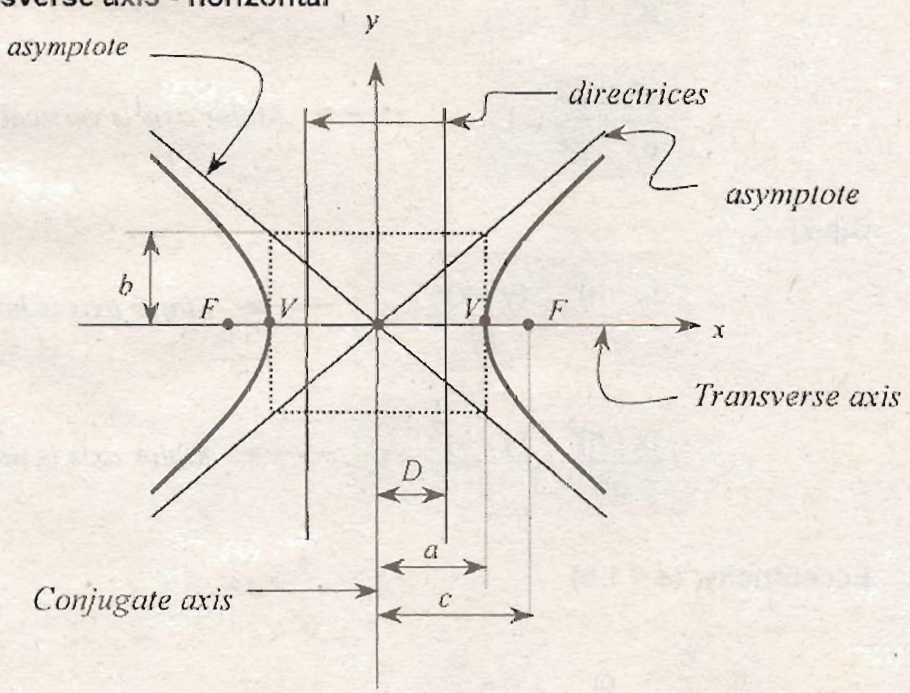
General equation: $Ax^2 + Cy^2 + Dx + Ey + F = 0$

$$h = \frac{-D}{2A} \qquad k = \frac{-E}{2C} \qquad c = \sqrt{a^2 - b^2}$$

HYPERBOLA

Hyperbola is a locus of a point which moves so that the **difference** of the distances to the fixed points (*foci*) is constant and is equal to the length of the transverse axis (2a).

I. **Equations:**
General equations:
A. Transverse axis - horizontal



$$Ax^2 - Cy^2 + Dx + Ey + F = 0$$

Note: **Transverse axis** is the axis that passes through the foci, vertices and the center of the hyperbola while the **conjugate axis** is the one that is perpendicular to the transverse axis.

Length of the transverse axis = $2a$ or $2\sqrt{C}$

Length of the conjugate axis = $2b$ or $2\sqrt{A}$

where: A and C are the numerical coefficients (absolute value) of x^2 and y^2, respectively.

Also, the relationship between a, b and c is $a^2 + b^2 = c^2$

B. **Transverse axis – vertical:**

$$Cy^2 - Ax^2 + Dx + Ey + F = 0$$

Length of the transverse axis = $2a$ or $2\sqrt{A}$

Length of the conjugate axis = $2b$ or $2\sqrt{C}$

where: A and C are the numerical coefficients (absolute value) of x^2 and y^2, respectively.

Also, the relationship between a, b and c is $a^2 + b^2 = c^2$

Standard equations:
C(0,0)

Transverse axis horizontal:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Transverse axis vertical:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

C(h,k)

Transverse axis horizontal:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Transverse axis vertical:

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

II. **Eccentricity: (e > 1.0)**

$$e = \frac{c}{a} \quad \text{or} \quad e = \frac{a}{D}$$

III. **Length of Latus Rectum:**

$$LR = \frac{2b^2}{a}$$

where: a = semi-major axis
b = semi-minor axis

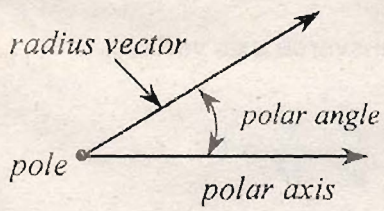
When the equation given is general equation rather than standard equation, the center **(h,k)** of a hyperbola can be calculated by converting the general equation to standard using the process known as **completing the square**. Or using the following formulas:

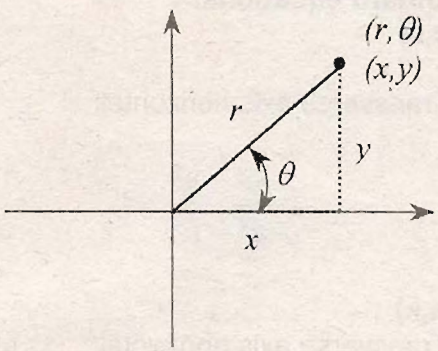
$$h = \frac{-D}{2A} \qquad k = \frac{-E}{2C}$$

POLAR COORDINATES

Polar coordinates refers to the coordinates of a point in a system of coordinates where the position of a point is determined by the length of ray segment (the **radius vector**) from a fixed origin (the **pole**) and the angle (the **polar angle**) the ray (the **vector**) makes with a fixed line (the **polar axis**).

Polar angle is sometimes called the **vectoral angle**, the **argument**, the **amplitude**, or the **azimuth** of a point.



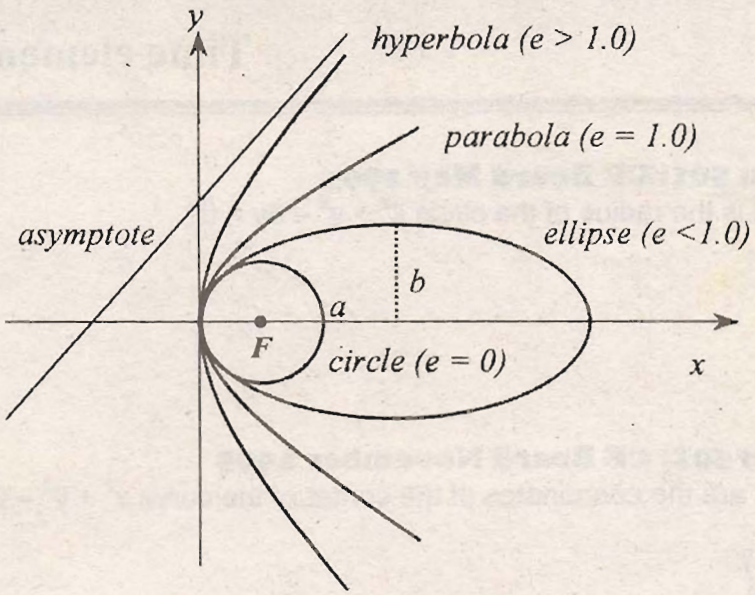


Relationship between polar coordinates and rectangular coordinates:

$$x = r\cos\theta \qquad y = r\sin\theta \qquad r = \sqrt{x^2 + y^2} \qquad \theta = \arctan\frac{y}{x}$$



Tip: Common forms of conic sections:



Did you know that... the oldest known example of numeration using place value is not the Roman nor the Arabic numeration but the Babylonian or Mesopotamians sexagesimal system of numeration which dates back to the 2nd millenium B.C. ! This system of numeration still survive today (i.e 1 hour = 60 minutes and 1 minute = 60 seconds. and 1 degree = 60 minutes)

Proceed to the next page for your 12th test. GOODLUCK ! ☞

Test 12

Time element: 4.0 hours

Problem 501: CE Board May 1995

What is the radius of the circle $x^2 + y^2 - 6y = 0$?

A. 2
B. 3
C. 4
D. 5

Problem 502: CE Board November 1995

What are the coordinates of the center of the curve $x^2 + y^2 - 2x - 4y - 31 = 0$?

A. (-1,-1)
B. (-2.-2)
C. (1,2)
D. (2,1)

Problem 503:

A circle whose equation is $x^2 + y^2 + 4x + 6y - 23 = 0$ has its center at

A. (2,3)
B. (3,2)
C. (-3,2)
D. (-2,-3)

Problem 504: ME Board April 1998

What is the radius of a circle with the ff. equation: $x^2 - 6x + y^2 - 4y - 12 = 0$

A. 3.46
B. 7
C. 5
D. 6

Problem 505: ECE Board April 1998

The diameter of a circle described by $9x^2 + 9y^2 = 16$ is

A. 4/3
B. 16/9
C. 8/3
D. 4

Problem 506: CE Board May 1996

How far from the y-axis is the center of the curve $2x^2 + 2y^2 + 10x - 6y - 55 = 0$?

A. –2.5
B. – 3.0
C. –2.75
D. –3.25

Problem 507:

What is the distance between the centers of the circles $x^2 + y^2 + 2x + 4y - 3 = 0$ and $x^2 + y^2 - 8x - 6y + 7 = 0$?

A. 7.07
B. 7.77
C. 8.07
D. 7.87

Problem 508: CE Board November 1993

The shortest distance from A (3,8) to the circle $x^2 + y^2 + 4x - 6y = 12$ is equal to

A. 2.1
B. 2.3
C. 2.5
D. 2.7

Problem 509: ME Board October 1996

The equation $x^2 + y^2 - 4x + 2y - 20 = 0$ describes:

A. A circle of radius 5 centered at the origin.
B. An ellipse centered at (2,-1).
C. A sphere centered at the origin.
D. A circle of radius 5 centered at (2,-1). *

Problem 510: EE Board April 1997

The center of a circle is at (1,1) and one point on its circumference is (-1,-3). Find the other end of the diameter through (-1,-3).

A. (2,4)
B. (3,5)
C. (3,6)
D. (1,3)

Problem 511:

Find the area (in square units) of the circle whose equation is $x^2 + y^2 = 6x - 8y$.

A. 20π
B. 22π
C. 25π
D. 27π

Problem 512:

Determine the equation of the circle whose radius is 5, center on the line x = 2 and tangent to the line 3x – 4y + 11 = 0.

A. $(x-2)^2 + (y-2)^2 = 5$
B. $(x-2)^2 + (y+2)^2 = 25$
C. $(x-2)^2 + (y+2)^2 = 5$
D. $(x-2)^2 + (y-2)^2 = 25$

Problem 513:

Find the equation of the circle with the center at (-4,-5) and tangent to the line 2x + 7y – 10 = 0.

A. $x^2 + y^2 + 8x - 10y - 12 = 0$
B. $x^2 + y^2 + 8x - 10y + 12 = 0$
C. $x^2 + y^2 + 8x + 10y - 12 = 0$
D. $x^2 + y^2 - 8x + 10y + 12 = 0$

Problem 514: ECE Board April 1998

Find the value of k for which the equation $x^2 + y^2 + 4x - 2y - k = 0$ represents a point circle.

A. 5
B. 6
C. –6
D. –5

Problem 515: ECE Board April 1999

$3x^2 + 2x - 5y + 7 = 0$. Determine the curve.

A. Parabola
B. Ellipse
C. Circle
D. Hyperbola

Problem 516: CE Board May 1993, CE Board November 1993, ECE Board April 1994

The focus of the parabola $y^2 = 16x$ is at

A. (4,0)
B. (0,4)
C. (3,0)
D. (0,3)

Problem 517: CE Board November 1994

Where is the vertex of the parabola $x^2 = 4(y - 2)$?

A. (2,0)
B. (0,2)

C. (3,0)
D. (0,3)

Problem 518: ECE Board April 1994, ECE Board April 1999
Find the equation of the directrix of the parabola $y^2 = 16x$.

A. $x = 2$
B. $x = -2$
C. $x = 4$
D. $x = -4$

Problem 519:
Given the equation of a parabola $3x + 2y^2 - 4y + 7 = 0$. Locate its vertex.

A. (5/3, 1)
B. (5/3, -1)
C. (-5/3, -1)
D. (-5/3, 1)

Problem 520: ME Board April 1997
In the equation $y = -x^2 + x + 1$, where is the curve facing?

A. Upward
B. Facing left
C. Facing right
D. Downward

Problem 521: CE Board May 1995
What is the length of the length of the latus rectum of the curve $x^2 = 20y$?

A. $\sqrt{20}$
B. 20
C. 5
D. $\sqrt{5}$

Problem 522: EE Board October 1997
Find the location of the focus of the parabola $y^2 + 4x - 4y - 8 = 0$.

A. (2.5, -2)
B. (3,1)
C. (2,2)
D. (-2.5,-2)

Problem 523: ECE Board April 1998
Find the equation of the axis of symmetry of the function $y = 2x^2 - 7x + 5$.

A. $7x + 4 = 0$
B. $4x + 7 = 0$
C. $4x - 7 = 0$

D. $x - 2 = 0$

Problem 524:

A parabola has its focus at (7,-4) and directrix y = 2. Find its equation.

A. $x^2 + 12y - 14x + 61 = 0$
B. $x^2 - 14y + 12x + 61 = 0$
C. $x^2 - 12x + 14y + 61 = 0$
D. None of the above

Problem 525:

A parabola has its axis parallel to the x-axis, vertex at (-1,7) and one end of the latus rectum at (-15/4, 3/2). Find its equation.

A. $y^2 - 11y + 11x - 60 = 0$
B. $y^2 - 11y + 14x - 60 = 0$
C. $y^2 - 14y + 11x + 60 = 0$
D. None of the above

Problem 526: ECE Board November 1997

Compute the focal length and the length of the latus rectum of the parabola $y^2 + 8x - 6y + 25 = 0$.

A. 2, 8
B. 4, 16
C. 16, 64
D. 1, 4

Problem 527:

Given a parabola $(y - 2)^2 = - 8(x - 1)$. What is the equation of its directrix?

A. x = -3
B. x = 3
C. y = -3
D. y = 3

Problem 528: ME Board October 1997

The general equation of a conic section is given by the following equation: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. A curve maybe identified as an ellipse by which of the following conditions?

A. $B^2 - 4AC < 0$
B. $B^2 - 4AC = 0$
C. $B^2 - 4AC > 0$
D. $B^2 - 4AC = 1$

Problem 529: CE Board November 1994

What is the area enclosed by the curve $9x^2 + 25y^2 - 225 = 0$?

A. 47.1
B. 50.2
C. 63.8
D. 72.3

Problem 530: ECE Board April 1998

Point P(x,y) moves with a distance from point (0,1) one-half of its distance from line y = 4. The equation of its locus is

A. $2x^2 - 4y^2 = 5$
B. $4x^2 + 3y^2 = 12$
C. $2x^2 + 5y^3 = 3$
D. $x^2 + 2y^2 = 4$

Problem 531:

The lengths of the major and minor axes of an ellipse are 10 m and 8 m, respectively. Find the distance between the foci.

A. 3
B. 4
C. 5
D. 6

Problem 532:

The equation $25x^2 + 16y^2 - 150x + 128y + 81 = 0$ has its center at

A. (3,-4)
B. (3,4)
C. (4,-3)
D. (3,5)

Problem 533: EE Board October 1997

Find the major axis of the ellipse $x^2 + 4y^2 - 2x - 8y + 1 = 0$.

A. 2
B. 10
C. 4
D. 6

Problem 534: CE Board May 1993

The length of the latus rectum for the ellipse $\frac{x^2}{64}+\frac{y^2}{16}=1$ is equal to

A. 2
B. 3
C. 4
D. 5

Problem 535:

An ellipse with an eccentricity of 0.65 and has one of its foci 2 units from the center. The length of the latus rectum is nearest to

A. 3.5 units
B. 3.8 units
C. 4.2 units
D. 3.2 units

Problem 536:

An earth satellite has an apogee of 40,000 km and a perigee of 6,600 km. Assuming the radius of the earth as 6,400 km, what will be the eccentricity of the elliptical path described by the satellite with the center of the earth at one of the foci?

A. 0.46
B. 0.49
C. 0.52
D. 0.56

Problem 537: ECE Board April 1998

The major axis of the elliptical path in which the earth moves around the sun is approximately 186,000,000 miles and the eccentricity of the ellipse is 1/60. Determine the apogee of the earth.

A. 93,000,000 miles
B. 91,450,000 miles
C. 94,335,100 miles
D. 94,550,000 miles

Problem 538: CE Board November 1992

The earth's orbit is an ellipse with the sun at one of the foci. If the farthest distance of the sun from the earth is 105.50 million km and the nearest distance of the sun from the earth is 78.25 million km, find the eccentricity of the ellipse.

A. 0.15
B. 0.25
C. 0.35
D. 0.45

Problem 539:

An ellipse with center at the origin has a length of major axis 20 units. If the distance from center of ellipse to its focus is 5, what is the equation of its directrix?

A. $x = 18$
B. $x = 20$
C. $x = 15$
D. $x = 16$

Problem 540:

What is the length of the latus rectum of $4x^2 + 9y^2 + 8x - 32 = 0$?

A. 2.5
B. 2.7
C. 2.3
D. 2.9

Problem 541: EE Board October 1993

$4x^2 - y^2 = 16$ is the equation of a/an

A. parabola
B. hyperbola
C. circle
D. ellipse

Problem 542: EE Board October 1993

Find the eccentricity of the curve $9x^2 - 4y^2 - 36x + 8y = 4$

A. 1.80
B. 1.92
C. 1.86
D. 1.76

Problem 543: CE Board November 1995

How far from the x-axis is the focus F of the hyperbola $x^2 - 2y^2 + 4x + 4y + 4 = 0$?

A. 4.5
B. 3.4
C. 2.7
D. 2.1

Problem 544: EE Board October 1994

The semi-transverse axis of the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is

A. 2
B. 3
C. 4
D. 5

Problem 545: CE Board May 1996

What is the equation of the asymptote of the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$?

A. $2x - 3y = 0$
B. $3x - 2y = 0$
C. $2x - y = 0$
D. $2x + y = 0$

Problem 546: EE Board April 1994

Find the equation of the hyperbola whose asymptotes are $y = \pm 2x$ and which passes through (5/2, 3).

A. $4x^2 + y^2 + 16 = 0$
B. $4x^2 + y^2 - 16 = 0$
C. $x^2 - 4y^2 - 16 = 0$
D. $4x^2 - y^2 = 16$

Problem 547:

Find the equation of the hyperbola with vertices at (-4,2) and (0,2) and foci at (-5,2) and (1,2).

A. $5x^2 - 4y^2 + 20x + 16y - 16 = 0$
B. $5x^2 - 4y^2 + 20x - 16y - 16 = 0$
C. $5x^2 - 4y^2 - 20x + 16y + 16 = 0$
D. $5x^2 + 4y^2 - 20x + 16y - 16 = 0$

Problem 548:

Find the distance between $P_1(6,-2,-3)$ and $P_2(5,1,-4)$.

A. 11
B. $\sqrt{11}$
C. 12
D. $\sqrt{12}$

Problem 549:

The point of intersection of the planes $x + 5y - 2z = 9$; $3x - 2y + z = 3$ and $x + y + z = 2$ is at

A. (2,1-1)
B. (2,0,-1)
C. (-1,1,-1)
D. (-1,2,1)

Problem 550: ME Board April 1997

What is the radius of the sphere center at the origin that passes the point 8,1,6?

A. 10
B. 9
C. $\sqrt{101}$
D. 10.5

Problem 551:

The equation of a sphere with center at (-3,2,4) and of radius 6 units is

A. $x^2 + y^2 + z^2 + 6x - 4y - 8z = 36$
B. $x^2 + y^2 + z^2 + 6x - 4y - 8z = 7$
C. $x^2 + y^2 + z^2 + 6x - 4y + 8z = 6$
D. $x^2 + y^2 + z^2 + 6x - 4y + 8z = 36$

Problem 552: EE Board April 1997

Find the polar equation of the circle, if its center is at (4,0) and the radius 4.

A. $r - 8\cos\theta = 0$
B. $r - 6\cos\theta = 0$
C. $r - 12\cos\theta = 0$
D. $r - 4\cos\theta = 0$

Problem 553: ME Board October 1996

What are the x and y coordinates of the focus of the conic section described by the following equation? (Angle θ corresponds to a right triangle with adjacent side x, opposite side y and the hypotenuse r.)

$$r\sin^2\theta = \cos\theta$$

A. (1/4, 0)
B. $(0, \pi/2)$
C. (0,0)
D. (-1/2,0)

Problem 554:

Find the polar equation of the circle of radius 3 units and center at (3,0).

A. $r = 3\cos\theta$
B. $r = 3\sin\theta$
C. $r = 6\cos\theta$
D. $r = 9\sin\theta$

Problem 555: EE Board October 1997

Given the polar equation r = 5 sin θ. Determine the rectangular coordinates (x,y) of a point in the curve when θ is 30°.

A. (2.17, 1.25)
B. (3.08, 1.5)
C. (2.51, 4.12)
D. (6,3)

ANSWER KEY

501. B	516. A	531. D	546. D
502. C	517. B	532. A	547. A
503. D	518. D	533. C	548. B
504. C	519. D	534. C	549. A
505. C	520. D	535. A	550. C
506. A	521. B	536. D	551. B
507. A	522. C	537. D	552. A
508. A	523. C	538. A	553. A
509. D	524. A	539. B	554. C
510. B	525. C	540. B	555. A
511. C	526. A	541. B	
512. B	527. B	542. A	
513. C	528. A	543. C	
514. D	529. A	544. B	
515. A	530. B	545. A	

RATING

☐ 46-55 Topnotcher
☐ 33-45 Passer
☐ 27-32 Conditional
☐ 0-26 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 12

501. $x^2 + y^2 - 6y = 0$

By completing square:

$x^2 + y^2 - 6y + (3)^2 = (3)^2$

$x^2 + (y - 3)^2 = (3)^2$

Standard equation of a circle with center at (h,k):

$(x - h)^2 + (y - k)^2 = r^2$

By inspection, $r = 3$

502. $x^2 + y^2 - 2x - 4y - 31 = 0$

$x^2 - 2x + y^2 - 4y = 31$

By completing square:

$x^2 - 2x + (1)^2 + y^2 - 4y + (2)^2 = 31 + (1)^1 + (2)^2$

$(x - 1)^2 + (y - 2)^2 = 36$

Standard equation of a circle with center at (h,k):

$(x - h)^2 + (y - k)^2 = r^2$

By inspection:

h = 1 and k = 2, thus the center is at (1,2).

503. $x^2 + y^2 + 4x + 6y - 23 = 0$

$x^2 + 4x + y^2 + 6y = 23$

By completing square:

$x^2 + 4x + (2)^2 + y^2 + 6y + (3)^2 = 23 + (2)^2 + (3)^2$

$(x + 2)^2 + (y + 3)^2 = 36$

Standard equation of a circle with center at (h,k):

$(x - h)^2 + (y - k)^2 = r^2$

By inspection:

h = - 2 and k = - 3, thus the center is at (-2, -3).

504. $x^2 - 6x + y^2 - 4y - 12 = 0$

$x^2 - 6x + y^2 - 4y = 12$

By completing square:

$x^2 - 6x + + (3)^2 + y2 - 4y + (2)^2 = 12 + (3)^2 + (2)^2$

$(x - 3)^2 + (y - 2)^2 = 25 = (5)^2$

Standard equation of a circle with center at (h,k):

$(x - h)^2 + (y - k)^2 = r^2$

By inspection:

r = 5

505. $9x^2 + 9y^2 = 16$

$$x^2 + y^2 = \frac{16}{9} = \left(\frac{4}{3}\right)^2$$

Standard equation of a circle with center at (0,0):

$x^2 + y^2 = r^2$

By inspection:

$$r = \frac{4}{3} \text{ and } d = 2r = \frac{8}{3}$$

506. $\left[2x^2 + 2y^2 + 10x - 6y - 55 = 0\right]\frac{1}{2}$

$$x^2 + y^2 + 5x - 3y = 27.5$$

By completing square:

$$x^2 + 5x + (5/2)^2 + y^2 - 3y + (3/2)^2 = 27.5 + (5/2)^2 + (3/2)^2$$

$$(x + 2.5)^2 + (y - 1.5)^2 = 36$$

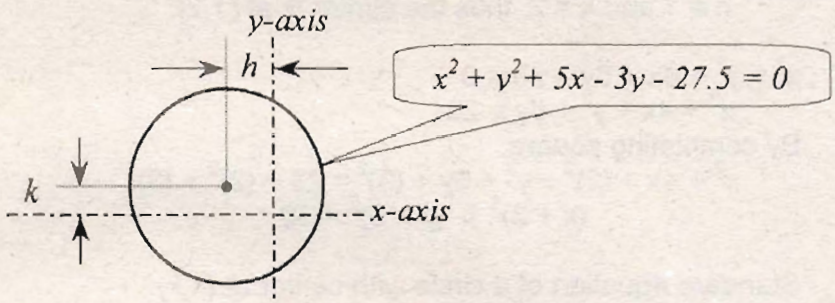
Standard equation of a circle with center at (h,k):

$$(x - h)^2 + (y - k)^2 = r^2$$

By inspection:

$$h = -2.5 \text{ and } k = 1.5$$

Note: The distance of the center of the circle from the y-axis is equal to h. Thus, the answer is (–) 2.5 unit length.



507. $x^2 + y^2 + 2x + 4y - 3 = 0$

$$x^2 + y^2 + 2x + 4y = 3$$

By completing square:

$$(x + 1)^2 + (y + 2)^2 = 3 + (1)^2 + (2)^2$$

$$(x + 1)^2 + (y + 2)^2 = 8$$

By inspection, the center of the first circle is at $C_1(-1, -2)$.

$$x^2 + y^2 + 8x - 6y + 7 = 0$$

$$x^2 + y^2 + 8x - 6y = -7$$

By completing square:

$$(x - 4)^2 + (y - 3)^2 = -7 + (4)^2 + (3)^2$$

$$(x - 4)^2 + (y - 3)^2 = 8$$

By inspection, the center of the second circle is at $C_2(4, 3)$.

Using distance formula to solve for the distance between C_1 & C_2:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{[4 - (-1)]^2 + [3 - (-2)]^2} = 7.07$$

508. $x^2 + y^2 + 4x - 6y = 12$

By completing square:

$(x + 1)^2 + (y - 3)^2 = 12 + (2)^2 + (3)^2$

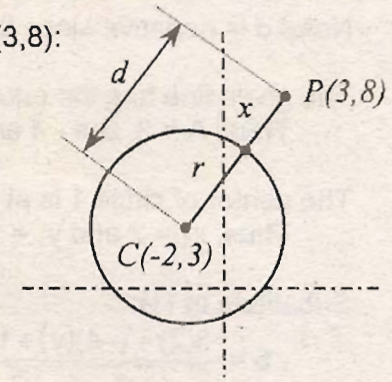
$(x + 2)^2 + (y - 3)^2 = 25 = (5)^2$

By inspection, the center is at C(-2, 3) and the radius, r = 5.

Solving for distance between (-2,3) and (3,8):

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{[3 - (-2)]^2 + (8 - 3)^2} = 7.1$$



Let: x = shortest distance

$x = d - r$

$x = 7.1 - 5 = 2.1$

509. $x^2 + y^2 - 4x + 2y - 20 = 0$

$x^2 + y^2 - 4x + 2y = 20$

By completing square:

$(x - 1)^2 + (y - 1)^2 = 20 + (2)^2 + (1)^2$

$(x - 2)^2 + (y + 1)^2 = 25 = (5)^2$

By inspection, the center is at C(2, -1) and the radius, r = 5.

510. Using midpoint formula:

$$x = \frac{x_1 + x_2}{2}$$

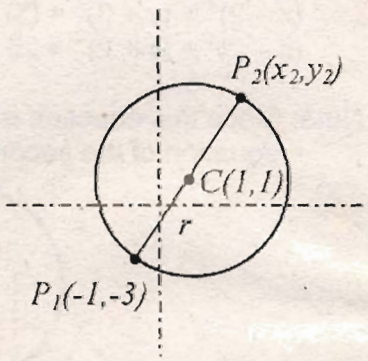
$$1 = \frac{-1 + x_2}{2}$$

$x_2 = 3$

$$y = \frac{y_1 + y_2}{2}$$

$$1 = \frac{-3 + y_2}{2}$$

$y_2 = 5$



Thus, the point is (3,5)

511. $x^2 + y^2 - 6x + 8y = 0$

By completing square:

$(x - 3)^2 + (y + 4)^2 = (3)^2 + (4)^2$

$(x - 3)^2 + (y + 4)^2 = 25 = (5)^2$

By inspection, the radius is 5.

Area = $\pi r^2 = \pi(5)^2 = 25\,\pi$ square units

512. Note: There are two possible circles
Try circle 1:
Using distance formula:

$$-d = \frac{A(x_1) + B(y_1) + C}{\pm\sqrt{A^2 + B^2}} \qquad \text{☞ ❶}$$

Note: d is negative since the point is below the line.

The given line has the equation, $3x - 4y + 11 = 0$.
Thus, A = 3, B = - 4 and C = 11

The center of circle 1 is at $C_1 = (2,y)$
Thus, $x_1 = 2$ and $y_1 = y$

Substitute in (1):

$$-5 = \frac{3(2) + (-4)(y) + 11}{-\sqrt{(3)^2 + (-4)^2}}$$

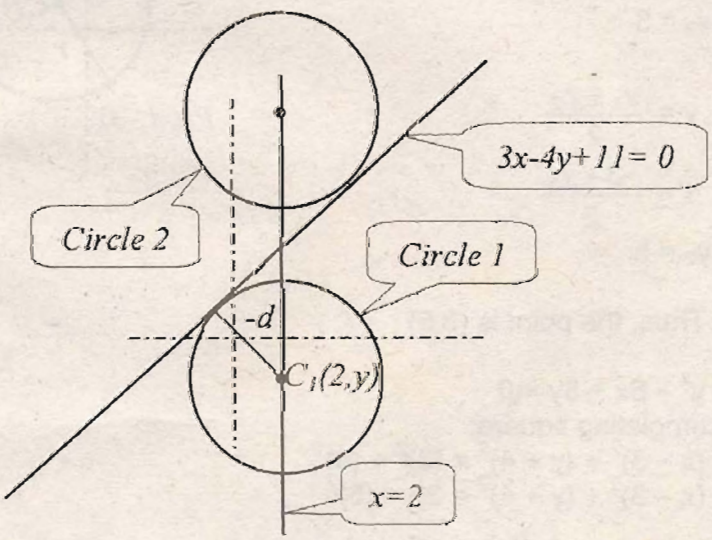
$$-5 = \frac{6 - 4y + 11}{-5}$$

$$y = -2$$

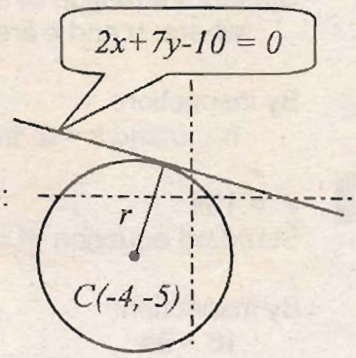
Solving for the equation of the circle with center at (2, -2):

$$(x - h)^2 + (y - k)^2 = (r)^2$$
$$(x - 2)^2 + (y + 2)^2 = (5)^2$$
$$(x - 2)^2 + (y + 2)^2 = 25$$

Note: Since this equation is in the choices, there is no need to get the equation of the second circle.



513. The line has an equation, $2x + 7y - 10 = 0$.
Thus, A = 2, B = 7 and C = -10



The center of the circle is at (-4, -5).
Thus, $x_1 = -4$ and $y_1 = -5$

Solving for the distance of point C to the line:

$$r = \frac{A(x_1) + B(y_1) + C}{\sqrt{A^2 + B^2}}$$

$$r = \frac{2(-4) + 7(-5) - 10}{+\sqrt{(2)^2 + (7)^2}} = (-)\ 7.28$$

Solving for the equation of the circle with center at (-4, -5):

$$(x - h)^2 + (y - k)^2 = r^2$$

514.
$$(x + 4)^2 + (y + 5)^2 = (7.28)^2$$
$$x^2 + 8x + 16 + y^2 + 10y + 25 = 53$$
$$x^2 + y^2 + 8x + 10y - 12 = 0$$

$x^2 + y^2 + 4x - 2y - k = 0$
$x^2 + 4x^2 + y^2 - 2y = k$
By completing square:

$$(x + 2)^2 + (y - 1)^2 = k + (2)^2 + (1)^2$$
$$= k + 5$$
$$(x + 2)^2 + (y - 1)^2 = \left(\sqrt{k + 5}\right)^2$$

By inspection:

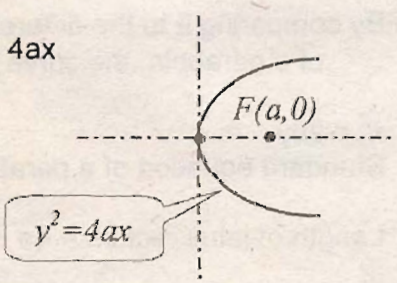
$$r = \sqrt{k + 5}$$

Note: For a point circle, r = 0.

$$r = \sqrt{k + 5}$$
$$k = -5$$

515. Since only x out from the two variables (x & y) has a *second* degree exponent, thus the equation is a parabola.

516. $y^2 = 16x$
Standard equation of a parabola: $y^2 = 4ax$



By inspection:
$4a = 16$
$a = 4$

Thus, the focus is at (4,0)

517. $x^2 = 4(y - 2)$

Standard equation of a parabola: $(x - h)^2 = 4 (y - k)$

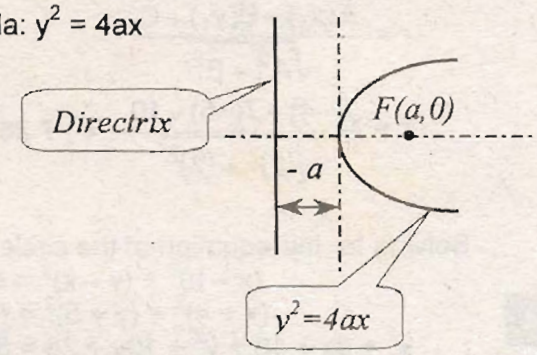
where: h and k are the coordinates of the vertex

By inspection:

h = 0 and k = 2, thus the vertex is at (0,2)

518. $y^2 = 16x$

Standard equation of a parabola: $y^2 = 4ax$



By inspection:

$16 = 4a$

$a = 4$

Equation of directrix:

$x = - a$

$x = - 4$

519. $3x + 2y^2 - 4y + 7 = 0$

$$y^2 - 2y + \frac{3}{2}x + \frac{7}{2} = 0$$

By completing square:

$$(y - 1)^2 = -\frac{3}{2}x - \frac{7}{2} + (1)^2$$

$$= -\frac{3}{2}x - \frac{5}{2}$$

$$(y - 1)^2 = -\frac{3}{2}\left(x + \frac{5}{3}\right)$$

Standard equation of a parabola: $(y - h)^2 = - 4a (x - k)$

By inspection:

h = - 5/3 and k = 1, thus the vertex is at (-5/3, 1)

520. $y = - x^2 + x + 1$

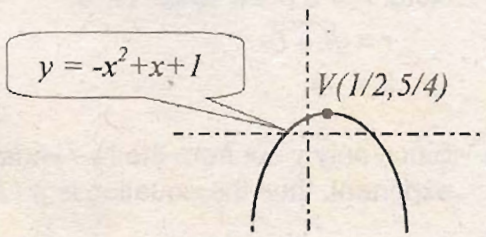
$x^2 - x = 1 - y$

By completing square:

$(x - 1/2)^2 = 1 - y + (1/2)^2$

$= - y + 5/4$

$(x - 1/2)^2 = - 1(y - 5/4)$



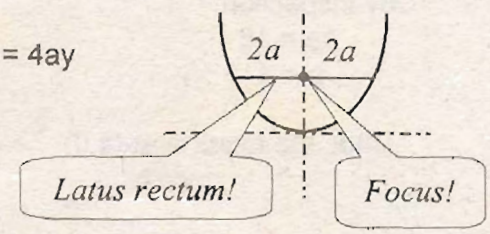
By comparing it to the different standard equation of a parabola, the curve is *facing downward*.

521. $x^2 = 20y$

Standard equation of a parabola: $x^2 = 4ay$

Length of latus rectum = 4a

By inspection, 4a = 20



522. $y^2 + 4x - 4y - 8 = 0$

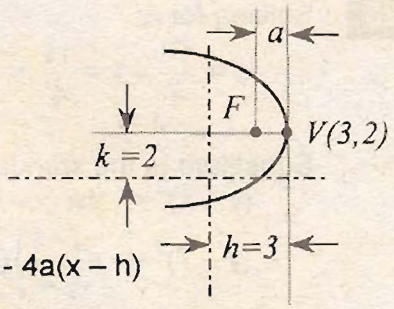
$y^2 - 4y = -4x + 8$

By completing square:

$(y-2)^2 = -4x + 8 + (2)^2$

$= -4x + 12$

$(y-2)^2 = -4(x-3)^2$



Standard equation of a parabola: $(y-k)^2 = -4a(x-h)$

By inspection:

$4a = 4$

$a = 1$

Thus, the focus is at (2,2).

523. $y = 2x^2 - 7x + 5$

$2x^2 - 7x = y - 5$

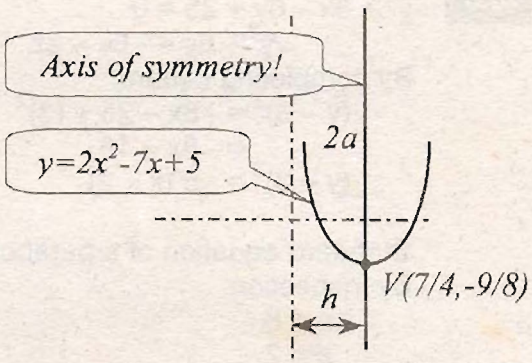
$$x^2 - \frac{7}{2}x = \frac{y}{2} - \frac{5}{2}$$

By completing square:

$$\left(x - \frac{7}{4}\right)^2 = \frac{y}{2} - \frac{5}{2} + \left(\frac{7}{4}\right)^2$$

$$\left(x - \frac{7}{4}\right)^2 = \frac{y}{2} + \frac{9}{16}$$

$$\left(x - \frac{7}{4}\right)^2 = \frac{1}{2}\left(y + \frac{9}{8}\right)$$



Standard equation of a parabola: $(x-h)^2 = 4a(y-k)$

By inspection:

h = 7/4 and k = -9/8, thus the vertex is at (7/4, -9/8)

Refer to the figure, the axis of symmetry is,

$x = 7/4$

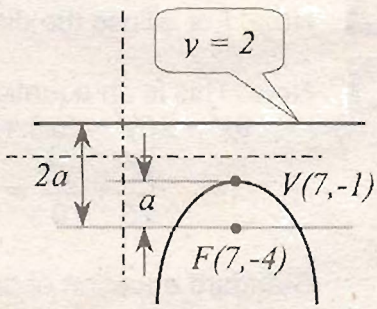
$4x - 7 = 0$

524. By inspection:

$2a = 6$

$a = 3$

The coordinates of the vertex is at (7,-1).



Substitute to the standard equation:

$(x-h)^2 = -4a(y-k)$

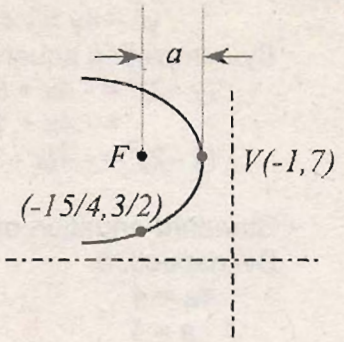
$(x-7)^2 = -4(3)(y+1)$

$x^2 - 14x + 49 = -12y - 12$

$x^2 - 14x + 12y + 61 = 0$

525. Solving for a:

$$a = \frac{15}{4} - 1 = \frac{11}{4}$$



Substitute to the standard equation:

$$(y - k)^2 = -4a(x - h)$$

$$(y - 7)^2 = -4\left(\frac{11}{4}\right)(x + 1)$$

$$(y - 7)^2 = -11(x + 1)$$

$$y^2 - 14y + 49 = -11x - 11$$

$$y^2 - 14y + 11x + 60 = 0$$

526. $y^2 + 8x - 6y + 25 = 0$

$$y^2 - 6y = -8x - 25$$

By completing square:

$$(y - 3)^2 = -8x - 25 + (3)^2$$

$$= -8x - 16$$

$$(y - 3)^2 = -8(x + 2)$$

Standard equation of a parabola: $(y - k)^2 = -4a(x - h)$

By inspection:

$4a = 8$

$a = 2$

Focal length = a = 2

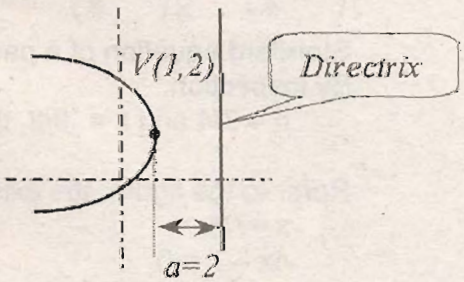
Length of latus rectum = 4a = 8

527. $(y - 2)^2 = -8(x - 1)$

By inspection:

$4a = 8$

$a = 2$



Refer to the figure:

$x = 3$ ☞ *Equation of directrix*

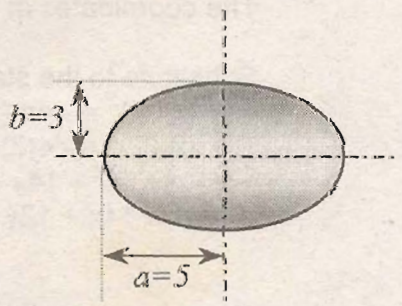
528. Note: For ellipse the discriminant is less than zero ($B^2 - 4AC < 0$).

529. Note: This is an equation of an ellipse

$$9x^2 + 25y^2 - 225 = 0$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Standard equation of an ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



By inspection:

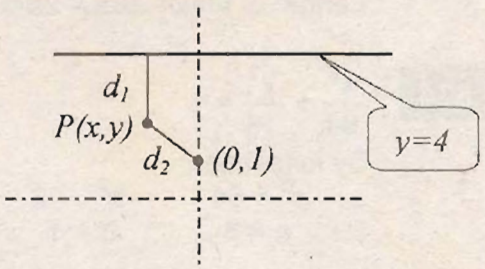
$a = 5$ and $b = 3$

Area = πab
Area = $\pi(5)(3) = 47.12$ square units

530.

$$d_2 = \frac{1}{2}d_1$$

$$\sqrt{(x-0)^2 + (y-1)^2} = \frac{1}{2}(4-y)$$



Squaring both sides:

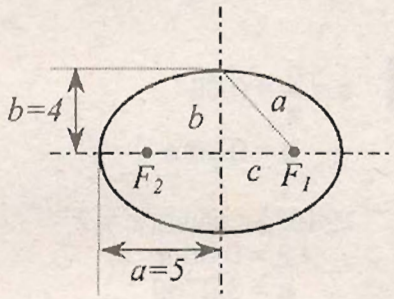
$$x^2 + (y-1) = \frac{1}{4}(4-y)^2$$

$$x^2 + y^2 - 2y + 1 = \frac{1}{4}(16 - 8y + y^2)$$

$$4x^2 + 4y^2 - 8y + 4 = 16 - 8y + y^2$$
$$4x^2 + 3y^2 = 12$$

531.

$2a = 10$ $\quad$ $2b = 8$
$a = 5$ $\quad$ $b = 4$



Solving for c:

$$c = \sqrt{a^2 - b^2}$$

$$c = \sqrt{(5)^2 - (4)^2} = 3$$

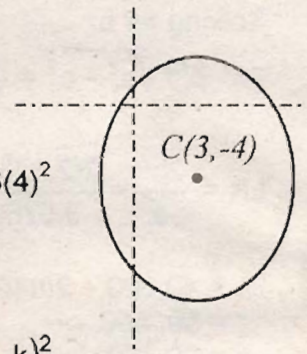
Distance between foci = 2c = 2(3) = 6

532.

$$25x^2 + 16y^2 - 150x + 128y + 81 = 0$$
$$25(x^2 - 6x) + 16(y^2 + 8y) = -81$$

By completing square:

$$25(x-3)^2 + 16(y+4)^2 = -81 + 25(3)^2 + 16(4)^2$$
$$25(x-3)^2 + 16(y+4)^2 = 400$$
$$\frac{(x-3)^2}{16} + \frac{(y+4)^2}{25} = 1$$



Standard equation of an ellipse: $\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$

By inspection:

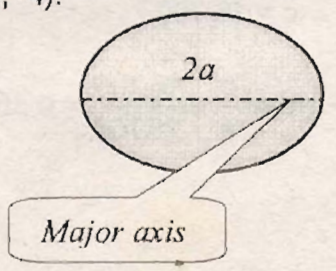
h = 3 and k = - 4, thus the center is at (3, -4).

533.

$$x^2 + 4y^2 - 2x - 8y + 1 = 0$$
$$x^2 - 2x + 4(y^2 + 2y) = -1$$

By completing square:

$$(x-1)^2 + 4(y-1)^2 = -1 + (1)^2 + 4(1)^2$$
$$(x-1)^2 + 4(y-1)^2 = 4$$
$$\frac{(x-1)^2}{4} + (y-1)^2 = 1$$



Standard equation of an ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

By inspection:

$a^2 = 4$

$a = 2$

Length of major axis = $2a = 2(2) = 4$

534. $\frac{x^2}{64} + \frac{y^2}{16} = 1$

By inspection:

$a^2 = 64$ $\quad b^2 = 16$

$a = 8$ $\quad b = 4$

Length of latus rectum:

$$LR = \frac{2b^2}{a} = \frac{2(4)^2}{8} = 4$$

535. $e = \frac{c}{a} = 0.65$

$c = 0.65\,a$

Note: As given, $c = 2$

$2 = 0.65\,a$

$a = 3.0769$

Solving for b:

$$b = \sqrt{a^2 - c^2} = \sqrt{(3.0769)^2 - (2)^2} = 2.338$$

$$LR = \frac{2b^2}{a} = \frac{2(2.338)^2}{3.0769} = 3.55$$

536. $2a = 40{,}000 + 2(6400) + 6{,}000$

$= 59{,}400$

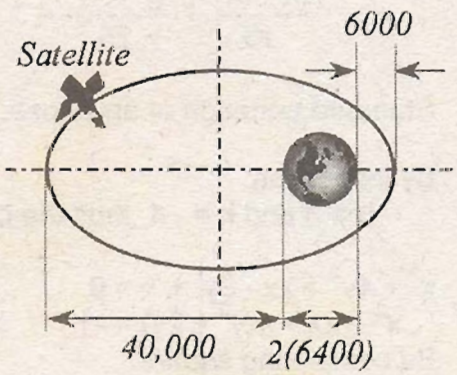
$a = 29{,}700$

$c = a - (6{,}600 + 6400)$

$= 29{,}700 - (6{,}600 + 6400)$

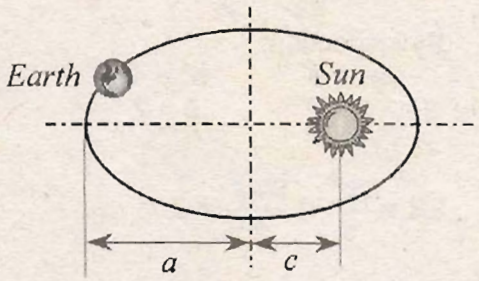
$c = 16{,}700$

$$e = \frac{c}{a} = \frac{16{,}700}{29{,}700} = 0.56$$



537. $2a = 186{,}000$

$a = 93{,}000{,}000$



$e = \frac{c}{a}$

$c = ea$

$= \frac{1}{60}(93{,}000{,}000)$

$c = 1{,}550{,}000$

Assuming the radius of earth and sun to be very small compared to a:

Apogee = a + c

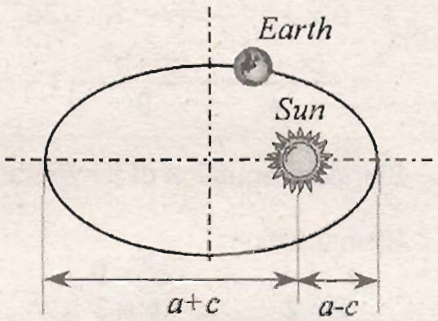
= 93,000,000 + 1,550,000

= 94,550,000 miles

538. $a + c = 105.5$ ☞ ❶

$a - c = 78.25$

$a = 78.25 + c$ ☞ ❷



Substitute (2) in (1):

$(78.25 + c) + c = 105.5$

$2c = 27.25$

$c = 13.625$

Substitute c in (2):

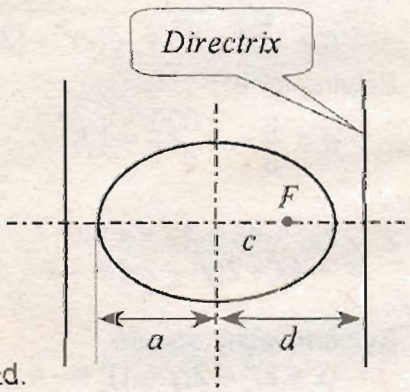
$a = 78.25 + 13.625$

$a = 91.875$

$e = \frac{c}{a} = \frac{13.625}{91.875} = 0.15$

539. $2a = 20$

$a = 10$



$e = \frac{c}{a} = \frac{a}{d}$

$a^2 = cd$

$(10)^2 = 5d$

$d = 20$

The equation of the directrix is $x = \pm d$.

Thus, $x = \pm 20$. ☞ *Answer*

540. $4x^2 + 9y^2 + 8x - 32 = 0$

$4(x^2 + 2x) + 9y^2 = 32$

By completing square:

$4(x + 1)^2 + 9y^2 = 32 + 4(1)^2$

$4(x + 1)^2 + 9y^2 = 36$

$$\frac{(x+1)^2}{9}+\frac{y^2}{4}=1$$

By inspection:

$a^2 = 9$ $\quad$ $b^2 = 4$

$a = 3$ $\quad$ $b = 2$

$$LR = \frac{2b^2}{a} = \frac{2(4)^2}{3} = 2.7$$

541. $4x^2 - y^2 = 16$

Note: Since the coefficients of x^2 and y^2 are *opposite in sign*, then the conic section represented by the given equation is a *hyperbola*.

542. $9x^2 - 4y^2 - 36x + 8y = 4$

$9(x^2 - 4x) - 4(y^2 - 2y) = 4$

By completing square:

$9(x-2)^2 - 4(y-1)^2 = 4 + 9(2)^2 - 4(1)^2$

$9(x-2)^2 - 4(y-1)^2 = 36$

$$\frac{(x-2)^2}{4}+\frac{(y-1)^2}{9}=1$$

Standard equation of a hyperbola: $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$

By inspection:

$a^2 = 4$ $\quad$ $b^2 = 9$

$a = 2$ $\quad$ $b = 3$

Solving for c:

$$c = \sqrt{a^2+b^2} = \sqrt{(2)^2+(3)^2} = 3.605$$

Solving for e:

$$e = \frac{c}{a} = \frac{3.605}{2} = 1.8$$

543. $x^2 - 2y^2 + 4x + 4y + 4 = 0$

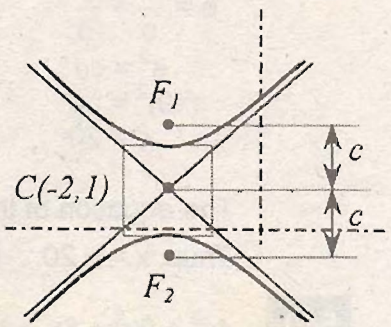
$x^2 + 4x - 2(y^2 - 2y) = -4$

By completing square:

$(x+2)^2 - 2(y-1)^2 = -4 + (2)^2 - 2(1)^2$

$(x+2)^2 - 2(y-1)^2 = -2$

$$(y-1)^2 - \frac{(x+2)^2}{2} = 1$$



Standard equation of a hyperbola: $\dfrac{(y-k)^2}{a^2}-\dfrac{(x-h)^2}{b^2}=1$

By inspection:

$a^2 = 1$, $b^2 = 2$, $h = -2$ and $k = 1$

Solving for c:

$$c = \sqrt{a^2 + b^2} = \sqrt{1+2} = 1.73$$

Refer to the figure:

Distance of focus to x – axis = 1 + c = 1 + 1.73 = 2.73 or
= c - 1 = 1.73 – 1 = 0.73

544. $\frac{x^2}{9} - \frac{y^2}{4} = 1$

Standard equation of a hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

By inspection:

$a^2 = 9$ $\quad b^2 = 4$

$a = 3$ $\quad b = 2$

Length of semi-transverse axis = a = 3

545. $\frac{x^2}{9} - \frac{y^2}{4} = 1$

Standard equation of a hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

By inspection:

$a^2 = 9$ $\quad b^2 = 4$

$a = 3$ $\quad b = 2$

Equation of asymptote: $y = \pm \frac{b}{a} x$

$$y = \pm \frac{2}{3} x$$

$2x - 3y = 0$ or $2x + 3y = 0$

546. Given equation of the asymptote: $y = \pm 2x$

Standard equation of the asymptote of a hyperbola with center at (0,0):

$$y = \pm \frac{b}{a} x$$

By inspection:

$$\frac{b}{a} = 2$$

$$b = 2a$$

Standard equation of a hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Substitute b = 2a and the coordinates of point (5/2, 3) to the std. equation:

$$\frac{(5/2)^2}{a^2} - \frac{(3)^2}{(2a)^2} = 1$$

$$\frac{25}{4a^2} - \frac{9}{4a^2} = 1$$

$$\frac{16}{4a^2} = 1$$

$$16 = 4a^2$$

$$a^2 = 4$$

$$a = 2$$

$$b = 2(2) = 4$$

$$b^2 = 16$$

Thus, the equation is,

$$\left(\frac{x^2}{4} - \frac{y^2}{16} = 1\right)16$$

$$4x^2 - y^2 = 16$$

547. By inspection the coordinates of the center is at (-2,2).

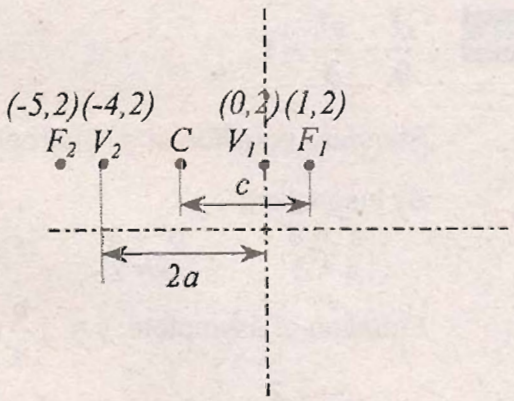
$$2a = 4$$

$$a = 2$$

$$c = 3$$

$$b = \sqrt{c^2 - a^2}$$

$$b = \sqrt{(3)^2 - (2)^2} = \sqrt{5}$$



Thus, $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$$\frac{(x+2)^2}{(2)^2} - \frac{(y-2)^2}{\left(\sqrt{5}\right)^2} = 1$$

$$\left(\frac{(x+2)^2}{4} - \frac{(y-2)^2}{5} = 1\right)20$$

$$5(x+2)^2 - 4(y-2)^2 = 20$$

$$5(x^2 + 4x + 4) - 4(y^2 - 4y + 4) = 20$$

$$5x^2 + 20x + 20 - 4y^2 + 16y - 16 = 20$$

$$5x^2 - 4y^2 + 20x + 16y - 16 = 0$$

548. $P_1(6, -2, -3)$, thus $x_1 = 6$, $y_1 = -2$ and $z_1 = -3$

$P_2(5, 1, -4)$, thus $x_2 = 5$, $y_2 = 1$ and $z_2 = -4$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{(5-6)^2 + (1+2)^2 + (-4+3)^2}$$

$$d = \sqrt{11}$$

549. $x + 5y - 2z = 9$ ☞ ❶

$3x - 2y + z = 3$ ☞ ❷

$x + y + z = 2$ ☞ ❸

Subtract (3) from (1):

$(x + 5y - 2z) - (x + y + z) = 9 - 2$

$4y - 3z = 7$ ☞ ❹

Multiply (3) by 3:

$3x + 3y + 3z = 6$ ☞ ❺

Subtract (2) from (5):

$(3x + 3y + 3z) - (3x - 2y + z) = 6 - 3$

$5y + 2z = 3$

$z = \frac{3}{2} - \frac{-5}{2}y$ ☞ ❻

Substitute (6) in (4):

$$4y - 3\left[\frac{3}{2} - \frac{5}{2}y\right] = 7$$

$$4y - \frac{9}{2} + \frac{15}{2}y = 7$$

$y = 1$

Substitute y in (6):

$$z = \frac{3}{2} - \frac{5}{2}(1)$$

$z = -1$

Substitute z and y in (3):

$x + (1) + (-1) = 2$

$x = 2$

Thus the point is (2, 1, -1) ☞ *Answer*

550. Standard equation of a sphere with center at (0,0): $x^2 + y^2 + z^2 = r^2$

Substitute the coordinates of the given point to the std. equation:

$r^2 = x^2 + y^2 + z^2$

$= (8)^2 + (1)^2 + (6)^2$

$r^2 = 101$

$r = \sqrt{101}$

551. $(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$

where:

(h, k, l) = coordinates of the center

As given the center is at (-3, 2, 4), thus h = - 3, k = 2 and l = 4.

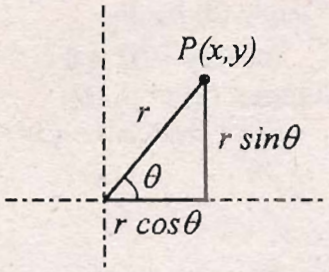
Substitute:

$$(x + 3)^2 + (y - 2)^2 + (z - 4)^2 = 6^2$$
$$x^2 + 6x + 9 + y^2 - 4y + 4 + z^2 - 8t + 16 = 36$$
$$x^2 + y^2 + z^2 + 6x - 4y - 8z = 36 - 9 - 4 - 16$$
$$x^2 + y^2 + z^2 + 6x - 4y - 8z = 7$$

552. $(x - h)^2 + (y - k)^2 = r^2$

Substitute coordinates of the center and radius:

$$(x - 4)^2 + (y - 0)^2 = 4^2$$
$$x^2 - 8x + 16 + y^2 = 16$$
$$x^2 - 8x + y^2 = 0$$



Note: $x = r \cos \theta$

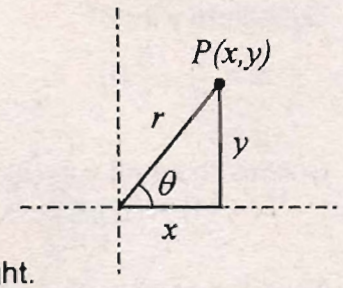
$y = r \sin \theta$

Substitute:

$$(r \cos \theta)^2 - 8(r \cos \theta) + (r \sin \theta)^2 = 0$$
$$r^2 \cos^2 \theta - 8 r \cos \theta + r^2 \sin^2 \theta = 0$$
$$r^2 (\cos^2 \theta + \sin^2 \theta) = 8r \cos \theta$$
$$r^2 = 8r \cos \theta$$
$$r = 8 \cos \theta$$
$$r - 8 \cos \theta = 0$$

553. $r \sin^2 \theta = \cos \theta$

$$r \left(\frac{y}{r}\right)^2 = \frac{x}{r}$$
$$y^2 = x$$



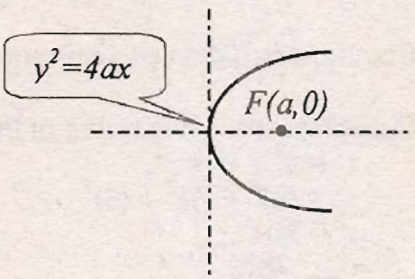
Note: This equation is a parabola with vertex at (0,0) and opening to the right.

Standard equation: $y^2 = 4ax$

By inspection:

$4a = 1$

$a = 1/4$



Thus, the focus is at (1/4, 0).

554. $(x - h)^2 + (y - k)^2 = r^2$

Substitute coordinates of center and radius:

$$(x - 3)^2 + (y - 0)^2 = (3)^2$$
$$x^2 - 6x + 9 + y^2 = 9$$
$$x^2 - 6x + y^2 = 0$$

Note: $x = r \cos \theta$

$y = r \sin \theta$

Substitute:

$$(r\cos\theta)^2 - 6(r\cos\theta) + (r\sin\theta)^2 = 0$$
$$r^2\cos^2\theta - 6r\cos\theta + r^2\sin^2\theta = 0$$
$$r^2(\cos^2\theta + \sin^2\theta) = 6r\cos\theta$$
$$r^2 = 6r\cos\theta$$
$$r = 6\cos\theta$$

555. $x = r\cos\theta$

Substitute $r = 5\sin\theta$ and $\theta = 30°$

$$x = (5\sin\theta)(\cos\theta)$$
$$= (5\sin 30°)(\cos 30°)$$
$$x = 2.17$$

$y = r\sin\theta$

Substitute $r = 5\sin\theta$ and $\theta = 30°$

$$y = (5\sin\theta)(\sin\theta)$$
$$= 5\sin^2\theta$$
$$= 5\sin^2 30°$$
$$y = 1.25$$

Thus, the point is at (2.17, 1.25).

DAY 13

DIFFERENTIAL CALCULUS (LIMITS & DERIVATIVES)

The term "Calculus" was derived from a Latin word "**calx**" which means "**stone**" and from a Greek word "**chalis**" which means "**limestone**".

In 1684, a German mathematician and philosopher **Gottfried Wilhelm von Leibniz** published his early work on calculus, while an English astronomer, physicist and mathematician **Isaac Newton** made an early study on the subject in 1665 but did not published his work until 1704. These two mathematicians are now considered as the **founders of today's calculus.**

The subject Calculus is divided into four areas namely, **differential calculus, integral calculus, differential equations** and **calculus of variation.**

Differential Calculus is a branch of mathematics which deals with derivatives and limits.

DERIVATIVES

A. Algebraic Functions:

1. $\dfrac{dc}{dx} = 0$

2. $\dfrac{d}{dx}(u+v) = \dfrac{du}{dx} + \dfrac{dv}{dx}$

3. $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

4. $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

5. $\dfrac{d}{dx}u^n = nu^{n-1}\dfrac{du}{dx}$

6. $\dfrac{d}{dx}\sqrt{u} = \dfrac{\dfrac{du}{dx}}{2\sqrt{u}}$

7. $\dfrac{d}{dx}\left(\dfrac{u}{c}\right) = \dfrac{1}{c}\dfrac{du}{dx}$

8. $\dfrac{d}{dx}\dfrac{c}{u} = \dfrac{c\dfrac{du}{dx}}{u^2}$

B. Exponential Functions:

9. $\dfrac{d}{dx}(a^u) = a^u \ln a \dfrac{du}{dx}$

10. $\dfrac{d}{dx}(e^u) = e^u \dfrac{du}{dx}$

C. Logarithmic Functions:

11. $\frac{d}{dx}(\log_a u) = \frac{\log_a e \frac{du}{dx}}{u}$

12. $\frac{d}{dx}(\log_{10} u) = \frac{\log_{10} e \frac{du}{dx}}{u}$

13. $\frac{d}{dx}(\ln u) = \frac{\frac{du}{dx}}{u}$

D. Trigonometric Functions:

14. $\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$

15. $\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$

16. $\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx}$

17. $\frac{d}{dx}(\cot u) = -\csc^2 u \frac{du}{dx}$

18. $\frac{d}{dx}(\sec u) = \sec u \tan u \frac{du}{dx}$

19. $\frac{d}{dx}(\csc u) = -\csc u \cot u \frac{du}{dx}$

E. Inverse Trigonometric Functions:

20. $\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$

21. $\frac{d}{dx}(\cos^{-1} u) = \frac{-1}{\sqrt{1-u^2}} \frac{du}{dx}$

22. $\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} \frac{du}{dx}$

23. $\frac{d}{dx}(\cot^{-1} u) = \frac{-1}{1+u^2} \frac{du}{dx}$

24. $\frac{d}{dx}(\sec^{-1} u) = \frac{1}{u\sqrt{u^2-1}} \frac{du}{dx}$

25. $\frac{d}{dx}(\csc^{-1} u) = \frac{-1}{u\sqrt{u^2-1}} \frac{du}{dx}$

F. Hyperbolic Functions:

26. $\frac{d}{dx}(\sinh u) = \cosh u \frac{du}{dx}$

27. $\frac{d}{dx}(\cosh u) = \sinh u \frac{du}{dx}$

28. $\frac{d}{dx}(\tanh u) = \operatorname{sech}^2 u \frac{du}{dx}$

26. $\frac{d}{dx}(\coth u) = -\operatorname{csch}^2 u \frac{du}{dx}$

27. $\frac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u \tanh u \frac{du}{dx}$

28. $\frac{d}{dx}(\operatorname{csch} u) = -\operatorname{csch} u \coth u \frac{du}{dx}$



Tips: Critical and stationary points

Critical point – a point a in the domain of f(x) if f'(a) = 0 or if f(x) is not differentiable at x = a.

Stationary points – critical points at which the derivative does not exist (and so equals zero).

Did you know that... the most proved theorem in Mathematics is the Pythagorean Theorem which has more than 370 different proofs! All the 370 different proofs are found in the book entitled "The Pythagorean Proposition" which was published in 1940

Proceed to the next page for your 13th test. GOODLUCK ! ☞

Test 13

Time element: 4.0 hours

Problem 556: CE Board November 1997

Evaluate: $\lim_{x \to 1} \dfrac{x^2 - 1}{x^2 + 3x - 4}$

A. 1/5
B. 2/5
C. 3/5
D. 4/5

Problem 557: ECE Board April 1998

Evaluate: $\lim_{x \to 4} \dfrac{(x - 4)}{(x^2 - x - 12)}$

A. Undefined
B. 0
C. Infinity
D. 1/7

Problem 558: ME Board April 1998

Evaluate the $\lim_{x \to 4} \dfrac{x^2 - 16}{x - 4}$

A. 0
B. 1
C. 8
D. 16

Problem 559: ECE Board April 1993

Evaluate: $M = \lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$

A. 0
B. 2
C. 4
D. 6

Problem 560: EE Board April 1995

Evaluate: $\lim_{x \to 0} \dfrac{1-\cos x}{x^2}$

A. 0
B. 1/2
C. 2
D. –1/2

Problem 561: ME Board October 1997

Compute the following limit:: Limit: $\lim_{x \to \infty} \dfrac{x+4}{x-4}$

A. 1
B. 0
C. 2
D. Infinite

Problem 562: EE Board October 1994

Evaluate: $\lim_{x \to \infty} \dfrac{3x^4 - 2x^2 + 7}{5x^3 + x - 3}$

A. Undefined
B. 3/5
C. Infinity
D. Zero

Problem 563: ECE Board November 1991

Evaluate: $\lim_{x \to 4} (x^2 + 3x - 4)$

A. 24
B. 26
C. 28
D. 30

Problem 564: ECE Board November 1994

Evaluate: $\lim_{x \to 1} (2-x)^{\tan\frac{\pi x}{2}}$

A. $e^{2\pi}$
B. $e^{2/\pi}$
C. 0
D. ∞

Problem 565: EE Board October 1997

Differentiate $y = e^x \cos x^2$.

A. $-e^x \sin x^2$
B. $e^x (\cos x^2 - 2x \sin x^2)$
C. $e^x \cos x^2 - 2x \sin x^2$
D. $-2xe^x \sin x$

Problem 566: EE Board October 1997

Differentiate $y = \sec (x^2 + 2)$.

A. $2x \cos (x^2 + 2)$
B. $-\cos (x^2 + 2) \cot (x^2 + 2)$
C. $2x \sec (x^2 + 2) \tan (x^2 + 2)$
D. $\cos (x^2 + 2)$

Problem 567: CE Board November 1994

What is the derivative with respect to x of $(x + 1)^3 - x^3$?

A. $3x + 6$
B. $3x - 3$
C. $6x - 3$
D. $6x + 3$

Problem 568: EE Board October 1997

Differentiate $y = \log_{10} (x^2 + 1)^2$

A. $4x (x^2 + 1)$
B. $\dfrac{4x \log_{10} e}{x^2 + 1}$
C. $\log e(x)(x^2 + 1)$
D. $2x(x^2 + 1)$

Problem 569: EE Board October 1997

Differentiate $(x^2 + 2)^{1/2}$.

A. $\dfrac{(x^2 + 2)^{1/2}}{2}$
B. $\dfrac{x}{(x^2 + 2)^{1/2}}$
C. $\dfrac{2x}{(x^2 + 2)^{1/2}}$
D. $(x^2 + 2)^{3/2}$

Problem 570: EE Board October 1997

If $y = (t^2 + 2)^2$ and $t = x^{1/2}$, determine $\frac{dy}{dx}$.

A. $\frac{3}{2}$

B. $\frac{2x^2 + 2x}{3}$

C. $2(x + 2)$

D. $x^{5/2} + x^{1/2}$

Problem 571: ME Board April 1997

What is the first derivative of the expression $(xy)^x = e$?

A. 0

B. $\frac{x}{y}$

C. $-y\,\frac{(1+\ln xy)}{x}$

D. $-y\,\frac{(1-\ln xy)}{x^2}$

Problem 572: ME Board April 1998

Find the derivative with respect to x the function $\sqrt{2-3x^2}$

A. $\frac{-2x^2}{\sqrt{2-3x^2}}$

B. $\frac{-3x}{\sqrt{2-3x^2}}$

C. $\frac{-3x^2}{\sqrt{2-3x^2}}$

D. $\frac{3x}{\sqrt{2-3x^2}}$

Problem 573: EE Board April 1995

Find y' if y = arc sin cos x

A. – 1
B. – 2
C. 1
D. 2

Problem 574: CE Board May 1997

Find the derivative of arc cos 4x.

A. $\frac{-4}{(1-16x^2)^{0.5}}$

B. $\frac{4}{(1-16x^2)^{0.5}}$

C. $\frac{-4}{(1-4x^2)^{0.5}}$

D. $\frac{4}{(1-4x^2)^{0.5}}$

Problem 575: CE Board November 1996

Find the derivative of $\frac{(x+1)^3}{x}$.

A. $\frac{(x+1)^2}{x} - \frac{(x+1)^3}{x}$

B. $\frac{4(x+1)^2}{x} - \frac{2(x+1)^3}{x}$

C. $\frac{2(x+1)^3}{x} - \frac{(x+1)^3}{x^3}$

D. $\frac{3(x+1)^2}{x} - \frac{(x+1)^3}{x^2}$

Problem 576: ECE Board November 1991

Differentiate the equation: $y = \frac{x^2}{x+1}$

A. $\frac{x^2+2x}{(x+1)^2}$

B. $\frac{x}{x+1}$

C. $2x$

D. $\frac{2x^2}{x+1}$

Problem 577: CE Board November 1995

The derivative with respect to x of $2\cos^2(x^2+2)$ is

A. $2\sin(x^2+2)\cos(x^2+2)$

B. $-2\sin(x^2+2)\cos(x^2+2)$

C. $8x \sin (x^2 + 2) \cos (x^2 + 2)$
D. $-8x \sin (x^2 + 2) \cos (x^2 + 2)$

Problem 578: CE Board November 1993

Find the second derivative of y by implicit differentiation from the equation $4x^2 + 8y^2 = 36$.

A. $64\ x^2$
B. $-\frac{9}{4}y^3$
C. $32\ xy$
D. $-\frac{16}{9}y^3$

Problem 579: ME Board April 1998

Find the partial derivatives with respect to x of the function $xy^2 - 5y + 6$.

A. $y^2 - 5$
B. y^2
C. $xy - 5y$
D. $2xy$

Problem 580: ME Board October 1997

Find the second derivative of $x^3 - 5x^2 + x = 0$.

A. $10x - 5$
B. $6x - 10$
C. $3x + 10$
D. $3x^2 - 5x$

Problem 581: ME Board April 1998

Given the function f(x) = x to the 3rd power – 6x + 2. Find the first derivative at x = 2.

A. 6
B. 7
C. $3x^2 - 5$
D. 8

Problem 582: CE Board May 1996

Find the slope of the ellipse $x^2 + 4y^2 - 10x - 16y + 5 = 0$ at the point where $y = 2 + 8^{0.5}$ and x = 7.

A. – 0.1463
B. – 0.1538
C. – 0.1654
D. – 0.1768

Problem 583: EE Board October 1997

If y = 4 cos x + sin 2x, what is the slope of the curve when x = 2 radians?

A. –2.21
B. –4.94
C. –3.25
D. 2.21

Problem 584: ECE Board November 1991

Find the slope of the line tangent to the curve $y = x^3 - 2x + 1$ at x = 1.

A. 1
B. 1/2
C. 1/3
D. 1/4

Problem 585: ECE Board November 1991

Give the slope of the curve at the point (1,1): $y = \frac{x^3}{4} - 2x + 1$

A. 1/4
B. –1/4
C. 1 1/4
D. –1 1/4

Problem 586: ECE Board November 1998

Find the slope of $x^2y = 8$ at the point (2,2).

A. 2
B. –1
C. –1/2
D. –2

Problem 587: CE Board May 1998

Find the slope of the curve $x^2 + y^2 - 6x + 10y + 5 = 0$ at point (1,0).

A. 1/5
B. 2/5
C. 1/4
D. 2

Problem 588: CE Board May 1996

Find the slope of the tangent to the curve, $y = 2x - x^2 + x^3$ at (0,2).

A. 1
B. 2
C. 3
D. 4

Problem 589: ECE Board April 1999

Find the coordinates of the vertex of the parabola $y = x^2 - 4x + 1$ by making use of the fact that at the vertex, the slope of the tangent is zero.

A. (2,-3)
B. (3,-2)
C. (-1,-3)
D. (-2,-3)

Problem 590: ECE Board April 1999

Find the equation of the normal to $x^2 + y^2 = 5$ at the point (2,1).

A. $y = 2x$
B. $x = 2y$
C. $2x + 3y = 3$
D. $x + y = 1$

Problem 591: CE Board May 1995

What is the equation of the normal to the curve $x^2 + y^2 = 25$ at (4,3)?

A. $5x + 3y = 0$
B. $3x - 4y = 0$
C. $3x + 4y = 0$
D. $5x - 3y = 0$

Problem 592: EE Board April 1997

Locate the points of inflection of the curve $y = f(x) = x^2 e^x$.

A. $-2 \pm \sqrt{3}$
B. $2 \pm \sqrt{2}$
C. $-2 \pm \sqrt{2}$
D. $2 \pm \sqrt{3}$

Problem 593: ECE Board November 1991

In the curve $2 + 12x - x^3$, find the critical points.

A. (2,18) & (-2,-14)
B. (2,18) & (2,-14)
C. (-2,18) & (2,-14)
D. (-2,18) & (-2,14)

Problem 594: CE Board November 1997

Find the radius of curvature of a parabola $y^2 - 4x = 0$ at point (4,4).

A. 22.36 units
B. 25.78 units
C. 20.33 units

D. 15.42 units

Problem 595: ECE Board November 1996

Find the radius of curvature at any point in the curve $y + \ln \cos x = 0$.

A. cos x
B. 1.5707
C. sec x
D. 1

ANSWER KEY

556. B	566. C	576. A	586. D
557. D	567. D	577. D	587. B
558. C	568. B	578. B	588. B
559. C	569. B	579. B	589. A
560. B	570. C	580. B	590. B
561. A	571. C	581. A	591. B
562. C	572. B	582. D	592. C
563. A	573. A	583. B	593. A
564. B	574. A	584. A	594. A
565. B	575. D	585. D	595. C

RATING

- ☐ 34-40 Topnotcher
- ☐ 24-33 Passer
- ☐ 20-23 Conditional
- ☐ 0-19 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 13

556. $\text{Limit} = \dfrac{x^2-1}{x^2+3x-4} = \dfrac{(1)^2-1}{(1)^1+3(1)-4} = \dfrac{0}{0}$, indeterminate

Note: Using L'Hospital's rule, differentiate separately the numerator and denominator and substitute the value of limit to the variable

$$\text{Limit} = \frac{2x}{2x+3}$$

Substitute x = 1:

$$\text{Limit} = \frac{2(1)}{2(1)+3} = \frac{2}{5}$$

557. $\text{Limit} = \dfrac{x-4}{x^2-x-12} = \dfrac{4-4}{(4)^2-4-12} = \dfrac{0}{0}$, indeterminate

Apply L'Hospital's rule:

$$\text{Limit} = \frac{1}{2x-1}$$

Substitute x = 4:

$$\text{Limit} = \frac{1}{2(4)-1} = \frac{1}{7}$$

558. $\text{Limit} = \dfrac{x^2-16}{x-4} = \dfrac{(4)^2-16}{4-4} = \dfrac{0}{0}$, indeterminate

Apply L'Hospital's rule:

$$\text{Limit} = \frac{2x}{1} = 2x$$

Substitute x = 4:

$$\text{Limit} = 2(4) = 8$$

559. $\text{Limit} = \dfrac{x^2-4}{x-2} = \dfrac{(2)^2-4}{2-2} = \dfrac{0}{0}$, indeterminate

Apply L'Hospital's rule:

$$\text{Limit} = \frac{2x}{1} = 2x$$

Substitute x = 2:

$$\text{Limit} = 2(2) = 4$$

560. $\text{Limit} = \dfrac{1-\cos x}{x^2} = \dfrac{1-\cos 0}{(0)^2} = \dfrac{0}{0}$, indeterminate

Apply L'Hospital's rule:

$$\text{Limit} = \frac{\sin x}{2x} = \frac{\sin 0}{2(0)} = \frac{0}{0}, \text{ indeterminate}$$

Apply L'Hospital's rule again:

$$\text{Limit} = \frac{\cos x}{2}$$

Substitute x = 0:

$$\text{Limit} = \frac{\cos 0°}{2} = \frac{1}{2}$$

561. $\text{Limit} = \dfrac{x+4}{x-4} = \dfrac{\infty+4}{\infty-4} = \dfrac{\infty}{\infty}$, indeterminate

Apply L'Hospital's rule:

$$\text{Limit} = \frac{1}{1} = 1$$

562. $\text{Limit} = \dfrac{3x^4 - 2x^2 + 7}{5x^3 + x - 3} = \dfrac{3(\infty)^4 - 2(\infty)^2 + 7}{5(\infty)^3 + \infty - 3} = \dfrac{\infty}{\infty}$, indeterminate

Apply L'Hospital's rule:

$$\text{Limit} = \frac{12x^3 - 4x}{15x^2 + 1} = \frac{12(\infty)^3 - 4(\infty)}{15(\infty)^2 + 1} = \frac{\infty}{\infty}, \text{ indeterminate}$$

Apply again L'Hospital's rule:

$$\text{Limit} = \frac{36x^2 - 4}{30x} = \frac{36(\infty)^2 - 4}{30(\infty)} = \frac{\infty}{\infty}, \text{ indeterminate}$$

Apply again L'Hospital's rule:

$$\text{Limit} = \frac{72x}{30}$$

Substitute x = ∞:

$$\text{Limit} = \frac{72(\infty)}{30} = \infty$$ ☞ *Answer!*

563. Substitute x = 4 to the given equation:

$\text{Limit} = x^2 + 3x - 4$

$= (4)^2 + 3(4) - 4$

$\text{Limit} = 24$

564. $\text{Limit} = (2-x)^{\tan\frac{\pi x}{2}} = (2-1)^{\tan\frac{\pi(1)}{2}} = 1^{\infty}$, indeterminate

Take ln on both sides:

$$\ln \text{Limit} = \tan\frac{\pi x}{2}\ln(2-x) = \frac{\ln(2-x)}{\dfrac{1}{\tan\frac{\pi x}{2}}} = \frac{\ln(2-x)}{\cot\frac{\pi x}{2}}$$

Note: $d \ln u = \dfrac{du}{u}$ and $d \tan u = -\csc^2 u\, du$

Apply L'Hospital's rule:

$$\text{ln Limit} = \frac{\frac{-1}{2-x}}{\csc^2\frac{\pi x}{2}\left(\frac{\pi}{2}\right)} = \frac{\frac{-1}{2-1}}{-\csc^2\frac{\pi(1)}{2}\left(\frac{\pi}{2}\right)} = \frac{-1}{-\frac{\pi}{2}} = \frac{2}{\pi}$$

Take exponential on both sides:

$$e^{\text{ln Limit}} = e^{2/\pi}$$

$$\text{Limit} = e^{2/\pi}$$

565. $y = e^x \cos x^2$

Note: $d(uv) = udv + vdu$
where:

$u = e^x$; $du = e^x$; $v = \cos x^2$; $dv = -2x \sin x^2$

$y' = e^x(-2x \sin x^2) + \cos x^2\, e^x = e^x \cos x^2 - e^x 2x \sin x^2$
$y' = e^x(\cos x^2 - 2x \sin x^2)$

566. $y = \sec(x^2 + 2)$

Note: $d \sec u = \sec u \tan u\, du$
where:

$u = x^2 + 2$; $du = 2x$

$y' = \sec(x^2 + 2) \tan(x^2 + 2)(2x) = 2x \sec(x^2 + 2) \tan(x^2 + 2)$

567. $y = (x + 1)^3 - x^3$

Note: $du^n = nu^{n-1}\, du$

$y' = 3(x + 1)^2(1) - 3(x)^2(1) = 3(x + 1)^2 - 3x^2$
$= 3(x^{2+} + 2x + 1) - 3x^2 = 3x^2 + 6x + 3 - 3x^2$
$y' = 6x + 3$

568. $y = \log_{10}(x^2 + 1)^2$

Note: $d \log_{10} u = \log_{10} e\left(\frac{du}{u}\right)$

where:

$u = (x^2 + 1)^2$; $du = 2(x^2 + 1)(2x) = 4x(x^2 + 1)$

$$y' = \log_{10} e\left[\frac{4x(x^2+1)}{(x^2+1)^2}\right] = \frac{4x \log_{10} e}{x^2+1}$$

569. $y = (x^2 + 2)^{1/2}$

Note: $du^n = nu^{n-1}\, du$

where:

$n = 1/2;\ u = x^2 + 2;\ du = 2x$

$$y' = \frac{1}{2}\left(x^2+2\right)^{1/2-1}(2x) = x\left(x^2+2\right)^{-1/2} = \frac{x}{\left(x^2+2\right)^{1/2}}$$

570.

$t = x^{1/2}$ ☞ ❶

$y = (t^2 + 2)^2$ ☞ ❷

Substitute (1) in (2):

$y = [(x^{1/2})^2 + 2]^2 = (x + 2)^2$

$y' = 2(x + 2)(1) = 2(x + 2)$

571.

$(xy)^x = e$

Take in on both sides

$\ln (xy)^x = \ln e$

$x \ln xy = 1$

Note: $d(uv) = udv + vdu$

Differentiating both sides

$$x\left[\frac{xy'+y}{xy}\right] + \ln xy(1) = 0$$

$$(xy' + y) + y \ln xy = 0$$

$$xy' = -y - y \ln xy$$

$$= -y\left[1+\ln xy\right]$$

$$y' = \frac{-y}{x}(1+\ln xy)$$

572. $\sqrt{2-3x^2} = (2-3x^2)^{1/2}$

Note: $d u^n = nu^{n-1}\, du$

where: $u = 2 - 3x^2;\ du = -6x;\ n = 1/2$

$$y' = 1/2(2-3x^2)^{1/2-1}(-6x) = -3x(2-3x^2)^{-1/2}$$

$$y' = \frac{-3x}{\sqrt{2-3x^2}}$$

573. $y = \sin^{-1}\cos x$

Note: $d \sin^{-1} u = \dfrac{du}{\sqrt{1-u^2}}$

where: $u = \cos x;\ u^2 = \cos^2 x;\ du = -\sin x$

$$y' = \frac{-\sin x}{\sqrt{1-\cos^2 x}} = \frac{-\sin x}{\sqrt{\sin^2 x}} = \frac{-\sin x}{\sin x} = -1$$

574. $y = \cos^{-1} 4x$

Note: $d \cos^{-1} u = \dfrac{-du}{\sqrt{1-u^2}}$

where:

$u = 4x;\ u^2 = 16x^2;\ du = 4$

$$y' = \frac{-4}{\sqrt{1-16x^2}} = \frac{-4}{(1-16x^2)^{0.5}}$$

575. $y = \dfrac{(x+1)^3}{x}$

Note: $d\left(\dfrac{u}{v}\right) = \dfrac{vdu - udv}{v^2}$

where:

$u = (x+)^3;\ du = 3(x+1)^2;\ v = x;\ v^2 = x^2;\ dv = 1$

$$y' = \frac{x(3)(x+1)^2 - (x+1)^3(1)}{x^2} = \frac{3(x+1)^2}{x} - \frac{(x+1)^3}{x^2}$$

576. $y = \dfrac{x^2}{x+1}$

Note: $d\left(\dfrac{u}{v}\right) = \dfrac{vdu - udv}{v^2}$

where: $u = x^2;\ du = 2x;\ v = x + 1;\ v^2 = (x+1)^2;\ dv = 1$

$$y' = \frac{(x+1)(2x) - x^2(1)}{(x+1)^2} = \frac{2x^2 + 2x - x^2}{(x+1)^2} = \frac{x^2 + 2x}{(x+1)^2}$$

577. $y = 2\cos^2 (x^2 + 2)$

Note: $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$$\cos^2 (x^2 + 2) = \frac{1+\cos 2(x^2+2)}{2} = \frac{1+\cos(2x^2+4)}{2}$$

$$y = 2\left[\frac{1+\cos 2(2x^2+4)}{2}\right]$$

$$y = 1 + \cos (2x^2 + 4)$$

Note: $d \cos u = -\sin u\, du$

where:

$u = 2x^2 + 4;\ du = 4x$

$y' = -\sin (2x^2 + 4)\ (4x) = -4x \sin 2(x^2 + 2)$

Note: $\sin 2\theta = 2\cos\theta\sin\theta$

$$y' = -4x\left[2\cos(x^2+2)\sin(x^2+2)\right] = -8x\cos(x^2+2)\sin(x^2+2)$$

578. $4x^2 + 8y^2 = 36$

$x^2 + 2y^2 = 9$ ☞ ❶

Differentiate both sides:

$2x + 4yy' = 0$

$$y' = \frac{-2x}{4y} = \frac{-x}{2y}$$ ☞ ❷

Take second derivative:

$$y'' = -\frac{1}{2}\left[\frac{y(1) - xy'}{y^2}\right]$$ ☞ ❸

Substitute (2) in (3):

$$y'' = -\frac{1}{2}\left[\frac{y - x\left(\frac{-x}{2y}\right)}{y^2}\right] = -\frac{1}{2}\left[\frac{y + \frac{x^2}{2y}}{y^2}\right] = -\frac{1}{2y^2}\left[\frac{2y^2 + x^2}{2y}\right]$$

Substitute (1) in y'':

$$y'' = -\frac{9}{4y^3}$$

579. $\frac{\partial(xy^2 - 5y + 6)}{\partial x} = y^2(1) = y^2$

Note: $\frac{d(-5y+6)}{dx} = 0$

580. $y = x^3 - 5x^2 + x = 0$

$y' = 3x^2 - 10x$

$y'' = 6x - 10$

581. $f(x) = x^3 - 6x + 2$

$f'(x) = 3x^2 - 6$

$f'(2) = 3(2)^2 - 6 = 6$

582. $x^2 + 4y^2 - 10x - 16y = -5$

Note: slope = y'

Differentiate:

$2x + 8yy' - 10 - 16y' = 0$

$$y'(8y - 16) = 10 - 2x$$

$$y' = \frac{10 - 2x}{8y - 16}$$

At $y = 2 + 8^{0.5} = 4.828$, $x = 7$

Substitute:

$$y' = \frac{10 - 2(7)}{8(4.828) - 16} = -0.1768$$

583. $y = 4\cos x + \sin 2x$

Differentiate:

$$y' = 4(-\sin x) + \cos 2x\,(2) = 2\cos 2x - 4\sin x$$

At $x = 2$ rad.

$$y' = 2\cos 2(2)\left[\frac{180°}{\pi}\right] - 4\sin\left[2\left(\frac{180°}{\pi}\right)\right]$$

$$y' = 2\cos 229.183° - 4\sin 114.591° = -4.94$$

584. $y = x^3 - 2x + 1$

Let: slope = y'

Differentiate:

$$y' = 3x^2 - 2$$

Substitute $x = 1$:

$$y' = 3(1)^2 - 2 = 1$$

Note: Since the line is tangent to the curve at $x = 1$, then the slope of the line is the same as the slope of the curve at the given point.

Thus, the slope of the line is equal to 1.

585. $y = \frac{x^3}{4} - 2x + 1$

Note: slope = y'

Differentiate:

$$y' = \frac{1}{4}(3x^2) - 2$$

Substitute $x = 1$:

$$y' = \frac{3}{4}(1)^2 - 2 = -1\frac{1}{4}$$

586. $x^2 y = 8$

$$y = \frac{8}{x^2}$$

Note: slope = y'

Differentiate:

$$y' = \frac{-2x(8)}{x^4} = \frac{-16}{x^3}$$

Substitute: x = 2

$$y' = \frac{-16}{(2)^3} = \frac{-16}{8} = -2$$

587. $x^2 + y^2 - 6x + 10y + 5 = 0$

Note: slope = y'

Differentiate:

$$2x + 2yy' - 6 + 10y' + 0 = 0$$

$$y'(2y + 10) = 6 - 2x$$

$$y' = \frac{6 - 2x}{2y + 10}$$

Substitute x = 1 and y = 0:

$$y' = \frac{6 - 2(1)}{2(0) + 10} = \frac{4}{10} = \frac{2}{5}$$

588. $y = 2x - x^2 + x^3$

Let: slope = y'

Differentiate:

$$y' = 2 - 2x + 3x^2$$

Substitute x = 0:

$$y' = 2 - 290) + 3(0)^2 = 2$$

Note: Since the line is tangent to the curve at (0,2), then the slope of the line is the same as the slope of the curve at the given point.

Thus, the slope of the line is equal to 2.

589. $y = x^2 - 4x + 1$

Let: slope = y'

Differentiate:

$$y' = 2x - 4$$

Substitute y' = 0 as given:

$$0 = 2x - 4$$

$$x = 2$$

Substitute x = 2 to the given equation:

$$y = (2)^2 - 4(2) + 1 = -3$$

Thus, the vertex is at (2, -3)

590. $x^2 + y^2 = 1$

Let: m_1 = slope of the given curve

Differentiate:

$$2x + 2yy' = 0$$

$$y' = -\frac{x}{y}$$

Substitute x = 2 and y = 1:

$$m_1 = -\frac{2}{1} = -2$$

Note: Since the line is normal to the curve at the given point, the slope (m_2) of the line is equal the negative reciprocal of the slope of the given curve.

$$m_2 = -\frac{1}{m_1} = -\frac{1}{-2} = \frac{1}{2}$$

Using point slope form:

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{2}(x - 2)$$

$$2y - 2 = x - 2$$

$$x = 2y$$

591. $x^2 + y^2 = 25$

Let: m_1 = slope of the given curve

m_2 = slope of the normal line

Differentiate:

$$2x + 2yy' = 0$$

$$y' = -\frac{x}{y}$$

Substitute x = 4 and y = 3:

$$m_1 = -\frac{4}{3}$$

$$m_2 = -\frac{1}{m_1} = -\frac{1}{-4/3} = \frac{3}{4}$$

Using point slope form:

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{3}{4}(x - 4)$$

$$4y - 12 = 3x - 12$$

$$3x - 4y = 0$$

592. $y = x^2e^x$

Note: $d(uv) = udv + vdu$

$d e^u = e^u du$

$$y' = x^2(e^x) + e^x(2x)$$

$$y'' = x^2(e^x) + e^x(2x) + e^x(2) + 2x(e^x)$$

$$y'' = x^2e^x + 4xe^x + 2e^x$$

At point of inflection, y" = 0

$$0 = x^2e^x + 4xe^x + 2e^x$$

$$0 = x^2 + 4x + 2$$

By quadratic formula:

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(2)}}{2(1)} = \frac{-4 \pm \sqrt{8}}{2}$$

$$x = -2 \pm \sqrt{2} = -2 \pm 1.41$$

Substitute the values of x to the given equation to solve for y:

At $x = -2 + 1.41 = -0.59$; $y = (-0.59)^2(e^{-0.59}) = 0.19$

At $x = -2 - 1.41 = -3.41$; $y = (-3.41)^2(e^{-3.41}) = 0.38$

Note: From the choices, only values of the x-coordinates are given. Thus the suggested answer is choice "c".

593. $y = 2 + 12x - x^3$

Note: Critical points are points wherein the slope of the curve is zero.

$y' = 12 - 3x^2$ = slope

$0 = 12 - 3x^2$

$x^2 = 4$

$x = \pm 2$

Substitute the values of x to the general equation:

At $x = 2$, $y = 2 + 12(2) - (2)^3 = 18$

At $x = -2$, $y = 2 + 12(-2) - (-2)^3 = -14$

Thus, the points are (2, 18) and (-2, -14).

594. Note: Radius of curvature $(R) = \dfrac{\left[1 + (y')^2\right]^{3/2}}{|y''|}$

$y^2 - 4x = 0$

$2yy' - 4 = 0$

$$y' = \frac{4}{2y} = \frac{2}{y}$$

$$y'' = \frac{y(0) - 2y'}{y^2} = -\frac{2y'}{y^2}$$

$$y''y^2 = -2\left[\frac{2}{y}\right]$$

$$y'' = -\frac{4}{y^3}$$

Substitute y = 4, y' and y'' to solve for R:

$$R = \frac{\left[1 + (2/y)^2\right]^{3/2}}{\left|-4/y^3\right|} = \frac{\left[1 + (2/4)^2\right]^{3/2}}{\left|-4/(4)^3\right|}$$

R = 22.36 unit length

595. $R = \dfrac{\left[1+(y')^2\right]^{3/2}}{|y''|}$ ☞ ❶

$y + \ln \cos x = 0$
$y = -\ln \cos x$

Note: $d \ln u = \dfrac{du}{u}$

where: $u = \cos x$; $du = -\sin x$

$y' = -\dfrac{-\sin x}{\cos x} = \tan x$
$y'' = \sec^2 x$

Substitute in (1):

$$R = \frac{\left[1+(\tan x)^2\right]^{3/2}}{|\sec^2 x|} = \frac{\left(\sec^2 x\right)^{3/2}}{\sec^2 x}$$

$$R = \frac{\sec^3 x}{\sec^2 x} = \sec x$$

DAY 14

DIFFERENTIAL CALCULUS
(MAXIMA-MINIMA & TIME RATES)

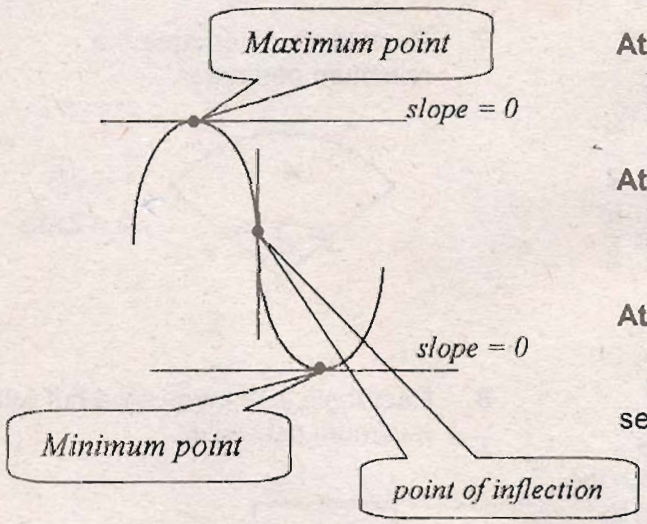
MAXIMA / MINIMA

In solving a problem under maxima / minima, the following steps are to be considered:

1. Draw a figure when necessary.
2. Identify what / which to maximize or minimize.
3. Formulate equation
4. Reduce to one variable.
5. **Differentiate**
6. Equate to zero

This is where the formulas will be used. Memorize all formulas!

Note: When the first derivative **(slope)** is equated to zero, it results to either maximum point or minimum point.



At maximum point,
y' = 0
y" is negative (concave down)

At minimum point,
y' = 0
y" is positive (concave up)

At point of inflection,
y" = 0
where y' and y" are the first and second derivatives respectively

TIME RATES

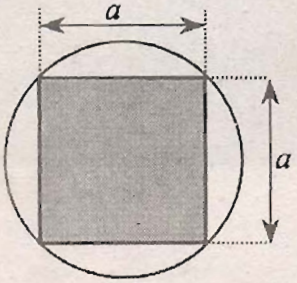
In solving a problem under time rates, the following steps are to be considered:

1. Draw a figure when necessary.
2. Formulate equation.
3. Differentiate with respect to time.
4. Substitute the boundary condition(s) to the equation.

Important: Substitute the given values only after differentiating.

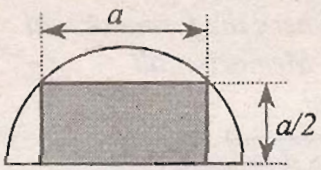
RELATIONSHIPS BETWEEN THE VARIABLES AND THE MAXIMA/MINIMA VALUES

1. Largest rectangle inscribed in a circle.

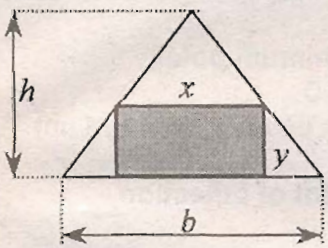


The maximum rectangle is a **square**.

2. Largest rectangle that can be inscribed in semicircle.

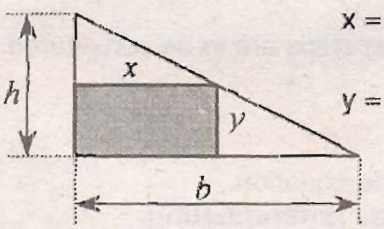


3. Largest rectangle that can be inscribed in a triangle with one side lying on the base of the triangle.



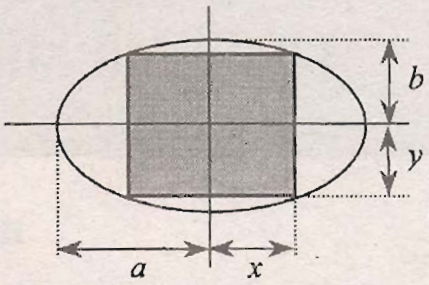
$$x = \frac{b}{2} \qquad y = \frac{h}{2}$$

4 Largest rectangle that can be inscribed in a right triangle with the sides of the rectangle parallel to the legs of the triangle.



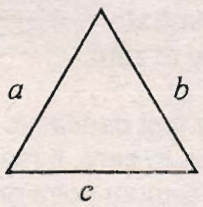
$$x = \frac{b}{2} \qquad y = \frac{h}{2}$$

5. Largest rectangle that can be inscribed in an ellipse.



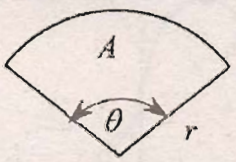
$$x = \frac{a}{\sqrt{2}} \qquad y = \frac{b}{\sqrt{2}}$$

6. Largest area of a triangle with given perimeter.



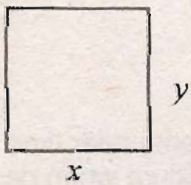
$$a = b = c = \frac{P}{3}$$

7. Sector with given area but minimum perimeter.



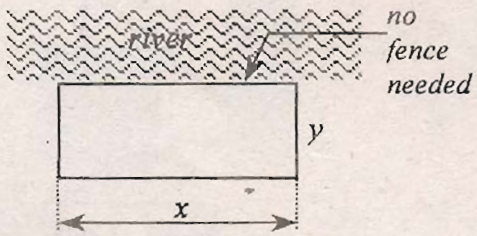
$$r = \sqrt{A}$$
$$\theta = 2 \text{ rad}$$

8. Rectangle with given area but with minimum perimeter.



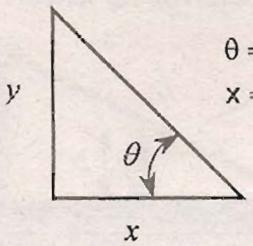
$$x = y$$

9. Rectangle with given area and minimum perimeter to be fenced along 3 sides only.



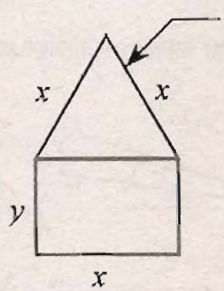
x = 2y

10. Right triangle with maximum perimeter or maximum area.



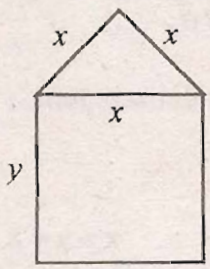
θ = 45°
x = y

11. Maximum area with perimeter (P) given.



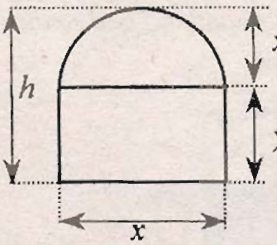
$$x = \frac{P}{4.268}$$

12. Maximum light admittance for a rectangular window surmounted with an isosceles triangle.



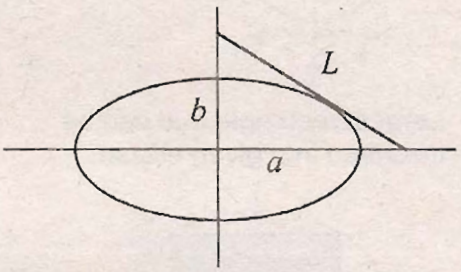
x = y

13. Maximum light admittance for a Norman window.



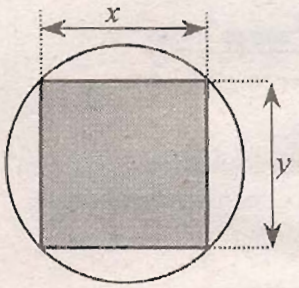
x = 2y
h = x

14. Maximum length of line segment tangent to an ellipse.



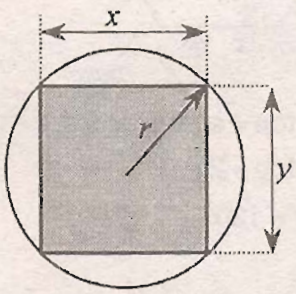
L = a + b

15. Rectangle of maximum perimeter inscribed in a circle of radius r.



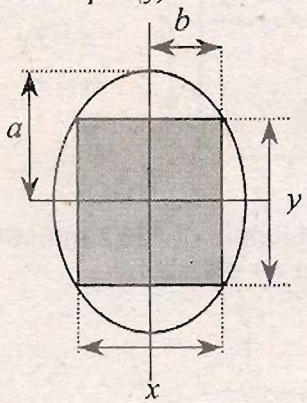
x = y

16. Stiffest beam that can be cut from a circular section of radius r.
"Stiffness is proportional to the product of breadth (x) and cube of width (y)."



$y = x\sqrt{3}$

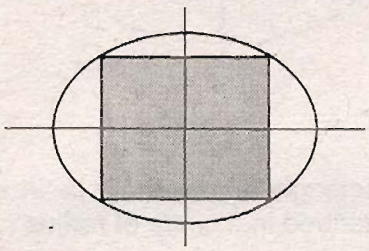
17. Strongest beam that can be cut from an elliptical section.
"Strength is proportional to the product of breadth (x) and square of the depth (y)"



$$x = 2b\sqrt{\frac{1}{3}}$$

$$y = 2a\sqrt{\frac{2}{3}}$$

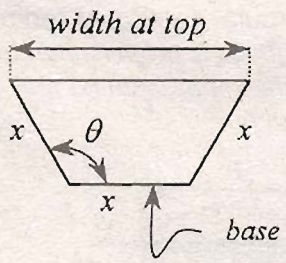
18. Largest rectangle that can be inscribed in a given ellipse.



$$\frac{A_{ellipse}}{A_{rectangle}} = \frac{\pi}{2}$$

19. Most efficient trapezoidal section.
"Maximum capacity with minimum perimeter"

It is ½ of a regular hexagon.

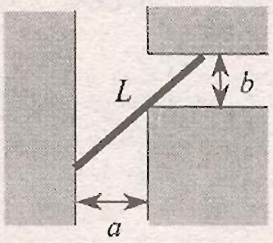


width at top = sum of sides

width at top = 2x

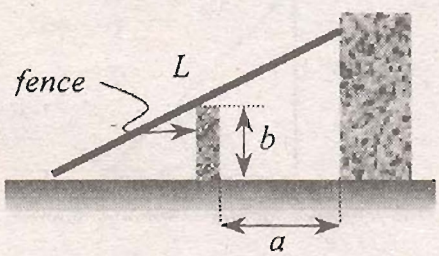
θ = 120°

20. Length of rigid beam that can pass a perpendicular hallways.



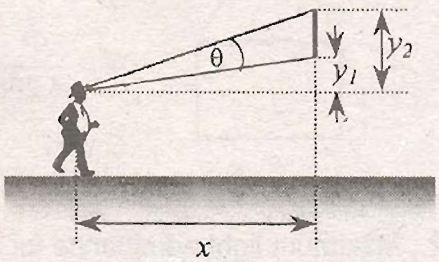
$$L = \sqrt[3]{\left(a^{2/3} + b^{2/3}\right)^2}$$

21. Minimum length of ladder/rod to be extended from ground to a wall with an intervening fence.



$$L = \sqrt[3]{\left(a^{2/3} + b^{2/3}\right)^2}$$

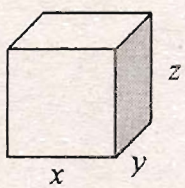
22. Best possible view of a picture or clock.



Best view means θ is maximized,

$$x = \sqrt{y_1 y_2}$$

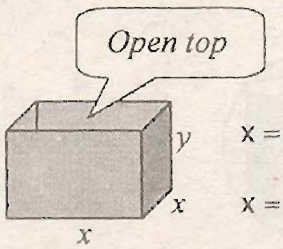
23. Parallelepiped with maximum volume.



x = y = z

∴ a cube

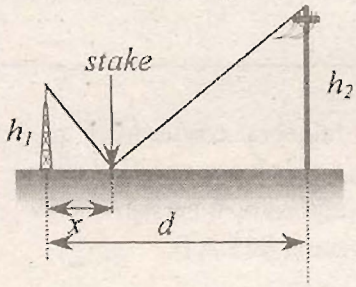
24. Open square container with maximum volume.



$x = 2y$

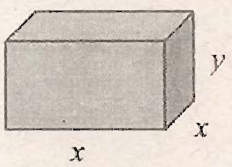
$x = \sqrt{\frac{\text{surface area}}{3}}$

25. Location of single stake at ground level to minimize length of wire.



$$x = \frac{dh_1}{h_1 + h_2}$$

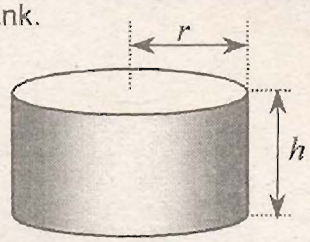
26. Least amount of material to be used for a square base rectangular parallelepiped.



$x = 2y$

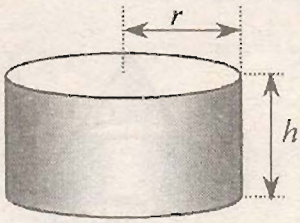
$x = \sqrt[3]{2(\text{Volume})}$

27. Least amount of material to be used for an open top cylindrical tank.



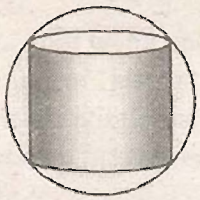
$r = h$

28. Minimum cost for a given volume, V.



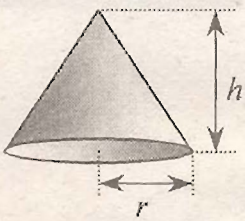
$$r = \sqrt[3]{\frac{V}{2\pi}}$$

29. Ratio of the weight of heaviest cylinder, W_c to the weight of the circumscribing sphere, W_s.



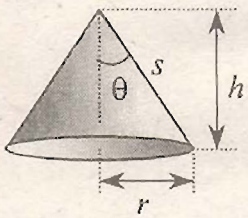
$$\frac{W_c}{W_s} = \frac{1}{\sqrt{3}}$$

30. Least amount of material for a given volume.



$$r = \frac{h}{\sqrt{2}}$$

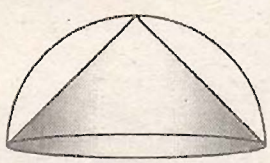
31. Maximum volume of cone with a given slant height.



$$h = \frac{s}{\sqrt{3}}$$

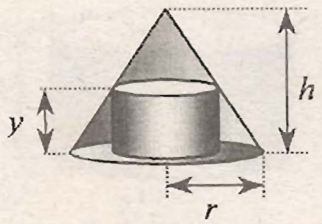
$\theta = \tan^{-1}\sqrt{2}$

32. Volume of largest cone, V_c that can be inscribed in a hemisphere.



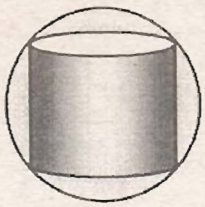
$$V_c = \frac{1}{2}V_h$$

33. Largest cylinder that can be inscribed in a cone.



$$y = \frac{h}{3}$$

34. Maximum volume of right circular cylinder inscribed in a sphere of radius r.



$$V_c = \frac{4}{\sqrt{27}}\pi r^3$$



Did you know that... that the term "algebra" comes from an Arabic term "al-jabr" meaning to transpose terms from one side of an equation to the other! This was introduced by a Persian mathematician, al-Khowarizmi in around 825 A.D.

Proceed to the next page for your 14th test. GOODLUCK ! ☞

Test 14

Time element: 3.0 hours

Problem 596: ECE Board April 1999

Find the minimum distance from the point (4,2) to the parabola $y^2 = 8x$.

A. $4\sqrt{3}$
B. $2\sqrt{2}$
C. $\sqrt{3}$
D. $2\sqrt{3}$

Problem 597: EE Board April 1990

The sum of two positive numbers is 50. What are the numbers if their product is to be the largest possible.

A. 24 & 26
B. 28 & 22
C. 25 & 25
D. 20 & 30

Problem 598: EE Board March 1998

A triangle has variable sides x,y,z subject to the constraint such that the perimeter is fixed to 18 cm. What is the maximum possible area for the triangle?

A. 15.59 cm^2
B. 18.71 cm^2
C. 17.15 cm^2
D. 14.03 cm^2

Problem 599: EE Board October 1997

A farmer has enough money to build only 100 meters of fence. What are the dimensions of the field he can enclose the maximum area?

A. 25 m x 25 m
B. 15 m x 35 m
C. 20 m x 30 m
D. 22.5 m x 27.5 m

Problem 600: CE Board May 1997

Find the minimum amount of tin sheet that can be made into a closed cylinder having a volume of 108 cu. inches in square inches.

A. 125.50
B. 127.50
C. 129.50
D. 123.50

Problem 601: ME Board April 1998

A box is to be constructed from a piece of zinc 20 sq.in by cutting equal squares from each corner and turning up the zinc to form the side. What is the volume of the largest box that can be so constructed?

A. 599.95 cu in.
B. 592.59 cu in.
C. 579.50 cu in.
D. 622.49 cu in.

Problem 602: EE Board April 1997

A poster is to contain 300 (cm square) of printed matter with margins of 10 cm at the top and bottom and 5 cm at each side. Find the overall dimensions if the total area of the poster is minimum.

A. 27.76 cm, 47.8 cm
B. 20.45 cm, 35.6 cm
C. 22.24 cm, 44.5 cm
D. 25.55 cm, 46.7 cm

Problem 603: CE Board November 1996

A norman window is in the shape of a rectangle surmounted by a semi-circle. What is the ratio of the width of the rectangle to the total height so that it will yield a window admitting the most light for a given perimeter?

A. 1
B. 1/2
C. 2
D. 2/3

Problem 604: CE Board May 1998

Determine the diameter of a closed cylindrical tank having a volume of 11.3 cu. m to obtain minimum surface area.

A. 1.22
B. 1.64
C. 2.44
D. 2.68

Problem 605: EE Board April 1997

The cost of fuel in running a locomotive is proportional to the square of the speed and is $ 25 per hour for a speed of 25 miles per hour. Other costs amount to $ 100 per hour, regardless of the speed. What is the speed which will make the cost per mile a minimum?

A. 40
B. 55
C. 50
D. 45

Problem 606: ME Board April 1996

The cost C of a product is a function of the quantity x of the product : $C(x) = x^2 - 4000x + 50$. Find the quantity for which the cost is minimum.

A. 1000
B. 1500
C. 2000
D. 3000

Problem 607:

An open top rectangular tank with square bases is to have a volume of 10 cu. m. The materials for its bottom is to cost P 15 per square meter and that for the sides, P6 per square meter. Find the most economical dimensions for the tank.

A. 1.5m x 1.5m x 4.4m
B. 2m x 2m x 2.5m
C. 4m x 4m x 0.6m
D. 3m x 3m x 1.1m

Problem 608: ME Board October 1996

What is the maximum profit when the profit-versus-production function is as given below? P is profit and x is unit of production.

$$P = 200{,}00 - x - \left(\frac{1.1}{x+1}\right)^8$$

A. 285,000
B. 200,000
C. 250,000
D. 305,000

Problem 609: EE Board October 1993

A boatman is at A which is 4.5 km from the nearest point B on a straight shore BM. He wishes to reach in minimum time a point C situated on the shore 9 km from B. How far from C should he land if he can row at the rate of 6 kph and can walk at the rate of 7.5 kph?

A. 4.15 km
B. 3.0 km

C. 3.25 km
D. 4.0 km

Problem 610: EE Board March 1998

A fencing is limited to 20 ft length. What is the maximum rectangular area that can be fenced in using two perpendicular corner sides of an existing wall?

A. 120
B. 100
C. 140
D. 190

Problem 611: EE Board October 1992

The cost per hour of running a motor boat is proportional to the cube of the speed. At what speed will the boat run against a current of 8 km/hr in order to go a given distance most economically?

A. 10 kph
B. 13 kph
C. 11 kph
D. 12 kph

Problem 612: ECE Board November 1998

Given a cone of diameter x and altitude of h. What percent is the volume of the largest cylinder which can be inscribed in the cone to the volume of the cone?

A. 44 %
B. 46 %
C. 56 %
D. 65 %

Problem 613: EE Board October 1993

At any distance x from the source of light, the intensity of illumination varies directly as the intensity of the source and inversely as the square of x. Suppose that there is a light at A, and another at B, the one at B having an intensity 8 times that of A. The distance AB is 4 m. At what point from A on the line AB will the intensity of illumination be least?

A. 2.15 m
B. 1.33 m
C. 1.50 m
D. 1.92 m

Problem 614: CE Board May 1995

A wall "h" meters high is 2 m away from the building. The shortest ladder that can reach the building with one end resting on the ground outside the wall is 6 m. How high is the wall in meters?

A. 2.34
B. 2.24

C. 2.44
D. 2.14

Problem 615: EE Board April 1997

The coordinates (x,y) in feet of a moving particle P are given by x = cost – 1 and y = 2 sin t + 1, where t is the time in seconds. At what extreme rates in fps is P moving along the curve?

A. 3 and 2
B. 3 and 1
C. 2 and 0.5
D. 2 and 1

Problem 616: ECE Board April 1998

A statue 3 m high is standing on a base of 4 m high. If an observer's eye is 1.5 m above the ground, how far should he stand from the base in order that the angle subtended by the statue is a maximum.

A. 3.41 m
B. 3.51 m
C. 3.71 m
D. 4.41 m

Problem 617:

A man walks across a bridge at the rate of 5 fps as a boat passes directly beneath him at 10 fps. If the bridge is 10 feet above the boat, how fast are the man and the boat separating 1 second later?

A. 8 fps
B. 8.25 fps
C. 8.33 fps
D. 8.67 fps

Problem 618:

An LRT train 6 m above the ground crosses a street at 9 m/s at the instant that a car approaching at a speed of 4 m/s is 12 m up the street. Find the rate of the LRT train and the car separating one second later.

A. 3.64 m/s
B. 3.94 m/s
C. 4.24 m/s
D. 4.46 m/s

Problem 619: EE Board October 1993

Water is flowing into a conical cistern at the rate of 8 m^3/min. If the height of the inverted cone is 12 m and the radius of its circular opening is 6 m. How fast is the water level rising when the water is 4 m deep?

A. 0.64 m/min
B. 0.56 m/min

C. 0.75 m/min
D. 0.45 m/min

Problem 620: CE Board November 1998

Water is pouring into a conical vessel 15 cm deep and having a radius of 3.75 cm across the top. If the rate at which the water rises is 2 cm/sec, how fast is the water flowing into the conical vessel when the water is 4 cm deep?

A. 2.37 m^3/sec
B. 5.73 m^3/sec
C. 6.28 m^3/sec
D. 4.57 m^3/sec

Problem 621: ME Board October 1996

Water is pouring into a swimming pool. After t hours, there are $t + \sqrt{t}$ gallons in the pool. At what rate is the water pouring into the pool when t = 9 hours?

A. 7/6 gph
B. 8/7 gph
C. 6/5 gph
D. 5/4 gph

Problem 622:

A helicopter is rising vertically from the ground at a constant rate of 4.5 meters per second. When it is 75 m off the ground, a jeep passed beneath the helicopter traveling in a straight line at a constant rate of 80 kph. Determine how fast the distance between them changing after 1 second.

A. 12.34 m/s
B. 11.10 m/s
C. 10.32 m/s
D. 9.85 m/s

Problem 623: ECE Board November 1991

A balloon is released from the ground 100 meters from an observer. The balloon rises directly upward at the rate of 4 meters per second. How fast is the balloon receding from the observer 10 seconds later?

A. 1.68 m/sec
B. 1.36 m/sec
C. 1.55 m/sec
D. 1.49 m/sec

Problem 624: ECE Board April 1998

A balloon is rising vertically over a point A on the ground at the rate of 15 ft./sec. A point B on the ground level with and 30 ft from A. When the balloon is 40 ft. from A, at what rate is its distance from B changing?

A. 13 ft /s

B. 15 ft /s
C. 12 ft /s
D. 10 ft /s

Problem 625: CE Board May 1997

Car A moves due East at 30 kph at the same instant car B is moving S 30° E, with a speed of 60 kph. The distance from A to B is 30 km. Find how fast is the distance between them separating after one hour.

A. 36 kph
B. 38 kph
C. 40 kph
D. 45 kph

Problem 626: CE Board November 1996

A car starting at 12:00 noon travels west at a speed of 30 kph. Another car starting from the same point at 2:00 P.M. travels north at 45 kph. Find how (in kph) fast the two are separating at 4:00 P.M.?

A. 49
B. 51
C. 53
D. 55

Problem 627: CE Board May 1996

Two railroad tracks are perpendicular to each other. At 12:00 P.M. there is a train at each track approaching the crossing at 50 kph, one being 100 km and the other 150 km away from the crossing. How fast in kph is the distance between the two trains changing at 4:00 P.M.?

A. 67.08
B. 68.08
C. 69.08
D. 70.08

Problem 628: CE Board May 1995

Water is running into a hemispherical bowl having a radius of 10 cm at a constant rate of 3 cm^3/min. When the water is x cm. deep, the water level is rising at the rate of 0.0149 cm/min. What is the value of x?

A. 3
B. 2
C. 4
D. 5

Problem 629: ECE Board November 1998

What is the allowable error in measuring the edge of the cube that is intended to hold 8 cu. m., if the error of the computed volume is not to exceed 0.03 cu. m?

A. 0.002
B. 0.003
C. 0.0025
D. 0.001

Problem 630: EE Board October 1993

A standard cell has an emf "E" of 1.2 volts. If the resistance "R" of the circuit is increasing at the rate of 0.03 ohm/sec, at what rate is the current "I" changing at the instant when the resistance is 6 ohms? Assume Ohm's law E = IR.

A. –0.002 amp/sec
B. 0.004 amp/sec
C. –0.001 amp/sec
D. 0.003 amp/sec

ANSWER KEY

596. B	606. C	616. C	626. B
597. C	607. B	617. C	627. A
598. A	608. B	618. A	628. C
599. A	609. B	619. A	629. C
600. A	610. B	620. C	630. C
601. B	611. D	621. A	
602. C	612. A	622. C	
603. A	613. C	623. D	
604. C	614. B	624. C	
605. C	615. D	625. D	

RATING

☐ 30-35 Topnotcher
☐ 21-29 Passer
☐ 18-20 Conditional
☐ 0-17 Failed

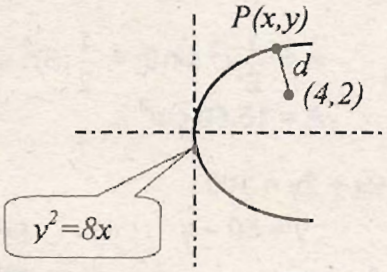
If FAILED, repeat the test.

SOLUTIONS TO TEST 14

596. $d = \sqrt{(x-4)^2 + (y-2)^2}$ ☞ ❶

$y^2 = 8x$

$x = \dfrac{y^2}{8}$ ☞ ❷



Substitute (2) in (1)

$d = \sqrt{(y^2/8 - 4)^2 + (y-2)^2}$

Note: $d\sqrt{u} = \dfrac{du}{2\sqrt{u}}$

Differentiate:

$$d' = \frac{2(y^2/8 - 4)(2y/8) + 2(y-2)(1)}{2\sqrt{(y^2/8-4)^2 + (y-2)^2}}$$

$$0 = 2(y^2/8 - 4)(2y/8) + 2(y-2)(1)$$

$$0 = \left(\frac{y^2}{4} - 8\right)\left(\frac{y}{4}\right) + 2y - 4 = \frac{y^3}{16} - 2y + 2y - 4$$

$y^3 = 64$

$y = 4$

Substitute y = 4 in (2):

$$x = \frac{(4)^2}{8} = 2$$

Substitute x = 2 and y = 4 in (1):

$d = \sqrt{(2-4)^2 + (4-2)^2} = \sqrt{8} = 2\sqrt{2}$ unit length

597. Let: x and y = two positive numbers

P = product of x and y

$x + y = 50$

$y = 50 - x$ ☞ ❶

$P = xy$ ☞ ❷

Substitute (1) in (2):

$P = x(50 - x) = 50x - x^2$

$\dfrac{dp}{dx} = 50 - 2x = 0$

$0 = 50 - 2x$

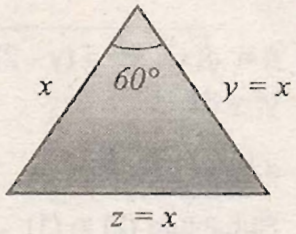
$x = 25$

Substitute x = 25 in (1):

$y = 50 - 25 = 25$

Thus, the numbers are 25 and 25.

598. Note: For maximum area, the triangle must be an equilateral triangle
Thus, x = y = z and θ = 60°



$$x + y + z = 18$$
$$3x = 18$$
$$x = 6$$

$$A = \frac{1}{2}x^2 \sin\theta = \frac{1}{2}(6)^2 \sin 60°$$
$$A = 15.59 \text{ cm}^2$$

599. $2x + 2y = 100$

$$y = 50 - x \quad ☞ ❶$$

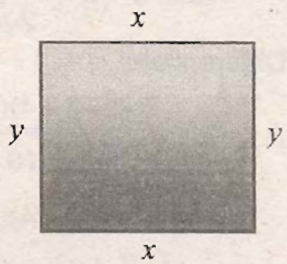
$$A = xy \quad ☞ ❷$$

Substitute (1) in (2):

$$A = x(50 - x) = 50x - x^2$$
$$\frac{dA}{dx} = 50 - 2x = 0$$
$$0 = 50 - 2x$$
$$x = 25$$

Substitute x = 25 in (1):

$$y = 50 - 25$$
$$y = 25$$

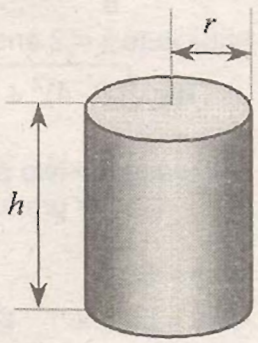


Thus the size of the field is 25 m x 25 m.

600.

$$V = \pi r^2 h$$
$$108 = \pi r^2 h$$
$$h = \frac{108}{\pi r^2} \quad ☞ ❶$$

$$A = 2\pi rh + \pi r^2(2) \quad ☞ ❷$$



Substitute (1) in (2):

$$A = 2\pi r\left[\frac{108}{\pi r^2}\right] + 2\pi r^2 = \frac{216}{r} + 2\pi r^2$$
$$\frac{dA}{dr} = \frac{-216}{r^2} + 4\pi r = 0$$
$$\frac{216}{r^2} = 4\pi r$$
$$r^3 = \frac{216}{4\pi}$$
$$r = 2.58 \text{ in.}$$

Substitute r in (1):

$$h = \frac{108}{\pi(2.58)^2} = 5.164 \text{ in.}$$

Substitute r and h in (2):

$A = 2\pi(2.58)(5.164) + 2\pi(2.58)^2$

$A = 125.\ 53\ in^2$

601.

$V = (20 - 2x)^2 x$ ☞ ❶

$V = 400x - 80x^2 + 4x^3$

$\frac{dV}{dx} = 400 - 160x + 12x^2 = 0$

$12x^2 - 160x + 400 = 0$

By factoring:

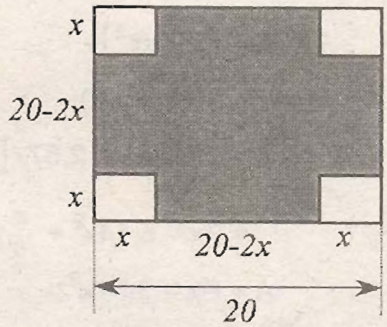
$(4x - 40)(3x - 10) = 0$

$4x = 40$

$x = 10$ in. (absurd)

$3x = 10$

$x = 3.33$ in.



Substitute x = 3.33 in (1):

$V = [20 - 2(3.33)]^2 (3.33) = 592.59$ cubic in.

602.

Let: A = total area

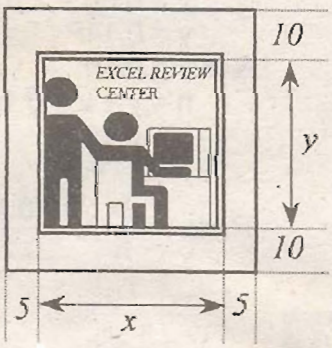
A_P = area with the picture

$A = (10 + x)(20 + y)$

$A = 200 + 10y + 20x + xy$ ☞ ❶

$A_P = xy = 300$

$y = \frac{300}{x}$ ☞ ❷



Substitute (2) in (1):

$$A = 200 + 10\left(\frac{300}{x}\right) + 20x + x\left(\frac{300}{x}\right)$$

$$A = 500 + \frac{3000}{x} + 20x$$

$$\frac{dA}{dx} = -\frac{3000}{x^2} + 20 = 0$$

$$0 = 20 - \frac{3000}{x^2}$$

$$x^2 = \frac{3000}{20}$$

$x = 12.24$ cm.

Substitute x = 12.24 in (2):

$$y = \frac{300}{12.24} = 24.5\ cm.$$

Thus, the dimension of the poster is = (10 + 12.24) x (20 + 24.5)

= 22.24 cm x 44.5 cm.

603. Let: P = perimeter

$$P = 2x + 2y + \frac{1}{2}(2\pi x)$$

$$P = 2x + 2y + \pi x$$

$$y = \frac{P - 2x - \pi x}{2} = 0.5P - 2.57x \quad \text{☞ ❶}$$

$$A = 2xy + \frac{1}{2}(\pi x^2) \quad \text{☞ ❷}$$

Substitute (1) in (2):

$$A = 2x[0.5P - 2.57x] + \frac{\pi}{2}x^2$$

$$= Px - 5.14x^2 + \frac{\pi}{2}x^2$$

$$A = Px - 3.57x^2$$

$$\frac{dA}{dx} = P - 7.14x = 0$$

$$0 = P - 7.14x$$

$$x = 0.14P$$

Substitute x in (1):

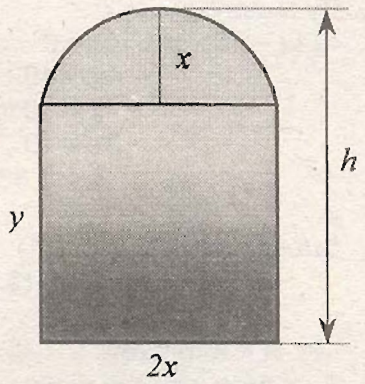
$$y = 0.5P - 2.57(0.14P)$$

$$y = 0.14P$$

Solving for h:

$$h = y + x = 0.14P + 0.14P$$

$$h = 0.28P$$



$$\text{Ratio} = \frac{2x}{h} = \frac{2(0.14P)}{0.28P} = 1$$

604.

$$V = \frac{\pi}{4}d^2h$$

$$11.3 = \frac{\pi d^2 h}{4}$$

$$h = \frac{45.2}{\pi d^2} \quad \text{☞ ❶}$$

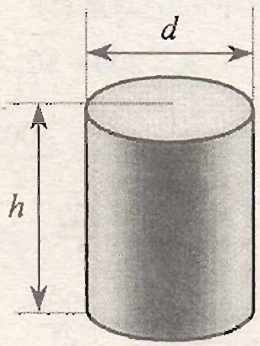
$$A = \pi dh + 2\left(\frac{\pi}{4}d^2\right) \quad \text{☞ ❷}$$

Substitute (1) in (2):

$$A = \pi d\left[\frac{45.2}{\pi d^2}\right] + \frac{\pi}{4}d^2 = \frac{45.2}{d} + \frac{\pi}{2}d^2$$

$$\frac{dA}{dd} = \frac{-45.2}{d^2} + \frac{\pi}{2}(2d) = 0$$

$$0 = \frac{-45.2}{d^2} + \pi d$$



$$\frac{45.2}{d^2} = \pi d$$

$$d^3 = \frac{45.2}{\pi}$$

$$d = 2.432 \text{ in.}$$

605. Let: C = total cost per hour
N = speed in miles per hour

C = fuel cost + others

$C = kN^2 + 100$ ☞ ❶

Fuel cost = kN^2

$$25 = k(25)^2$$

$$k = \frac{1}{25}$$

Substitute in (1):

$$C = \frac{1}{25}N^2 + 100$$

Let: x = total cost per mile

$$x = \frac{\text{total cost per hour}}{\text{speed in } \dfrac{\text{miles}}{\text{hour}}}$$

$$x = \frac{\dfrac{1}{25}N^2 + 100}{N}$$

$$\frac{dx}{dN} = \frac{N}{25} + \frac{100}{N} = 0$$

$$0 = \frac{1}{25} + \frac{(-100)}{N^2}$$

$$\frac{100}{N^2} = \frac{1}{25}$$

$$N^2 = 2500$$

$$N = 50 \text{ mph.}$$

606. $C = x^2 - 4000x + 50$

$$\frac{dC}{dx} = 2x - 4000 = 0$$

$$0 = 2x - 4000$$

$$2x = 4000$$

$$x = 2000 \text{ units}$$

607. Let: C = total cost

$C = x^2(15) + 4(xh)(6)$

$C = 15x^2 + 24xh$ ☞ ❶

$$V = x^2h$$
$$10 = x^2h$$
$$h = \frac{10}{x^2} \quad \text{☞ ❷}$$

Substitute (2) in (1):

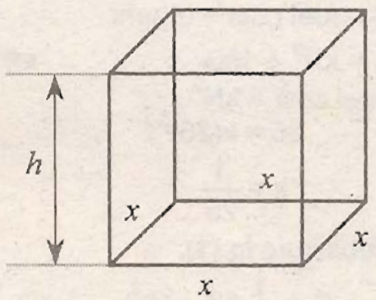
$$C = 15x^2 + 24x\left(\frac{10}{x^2}\right) = 15x^2 + \frac{240}{x}$$

$$\frac{dC}{dx} = 30x - \frac{240}{x^2} = 0$$

$$\frac{240}{x^2} = 30x$$

$$x^3 = \frac{240}{30}$$

$$x = 2 \text{ m}$$



Substitute x = 2 in (2):

$$h = \frac{10}{(2)^2} = 2.5 \text{ m}$$

Thus, the dimension of the tank is 2 m x 2 m x 2.5 m

608. $$P = 200{,}000 - x - \left(\frac{1.1}{x+1}\right)^8 \quad \text{☞ ❶}$$

$$\frac{dP}{dx} = -1 - 8\left(\frac{1.1}{x+1}\right)^7\left(\frac{-1.1}{(x+1)^2}\right) = 0$$

$$0 = -1 + \frac{8(1.1)^8}{(x+1)^9}$$

$$(x+1)^9 = 17.1487$$

$$x = 0.371$$

Substitute x in (1):

$$P = 200{,}000 - 0.371 - \left(\frac{1.1}{0.371+1}\right)^8 = 199{,}999.457$$

$$P = 200{,}000$$

609. $$d = \sqrt{(4.5)^2 + (9-x)^2}$$

Let: T = total time needed

T = time to row + time to walk

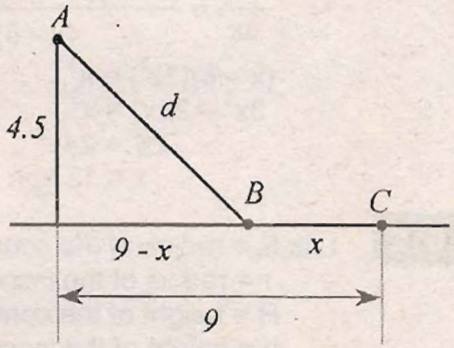
$$T = \frac{d}{6} + \frac{x}{7.5}$$

$$T = \frac{\sqrt{(4.5)^2 + (9-x)^2}}{6} + \frac{x}{7.5}$$

$$\frac{dT}{dx} = \frac{2(9-x)(-1)}{(6)\,2\sqrt{(45)^2 + (9-x)^2}} + \frac{1}{7.5} = 0$$

$$\frac{9-x}{\sqrt{(4.5)^2 + (9-x)^2}} = \frac{6}{7.5}$$

$$\sqrt{(4.5)^2 + (9-x)^2} = 1.25(9-x)$$



Square both sides:

$$(4.5)^2 + (9-x)^2 = (1.25)^2 (9-x)^2$$

$$20.25 = (9-x)^2 \left[(1.25)^2 - 1\right]$$

$$20.25 = 0.5625\,(9-x)^2$$

$$(9-x)^2 = \frac{20.25}{0.5625}$$

$$(9-x)^2 = 36$$

$$9 - x = 6$$

$$x = 3 \text{ km}$$

610. $x + y = 20$

$$y = 20 - x \quad \text{☞ ❶}$$

$A = xy$ ☞ ❷

Substitute (1) in (2):

$$A = x(20-x) = 20x - x^2$$

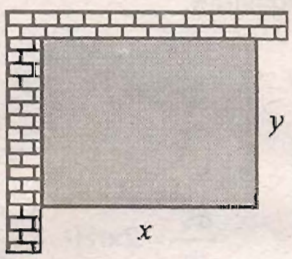
$$\frac{dA}{dx} = 20 - 2x = 0$$

$$0 = 20 - 2x$$

$$x = 10 \text{ ft.}$$

Substitute $x = 10$ in (1):

$$y = 20 - 10 = 10 \text{ ft.}$$



Substitute $x = 10$ and $y = 10$ in (2):

$$A = 10(10) = 100 \text{ ft}^2.$$

611. Let: C = cost per hour

x = speed of the motorboat

C_t = total cost

$C = k x^3$ ☞ ❶

where: k = proportionality constant

$$t = \frac{\text{distance}}{\text{speed}} = \frac{d}{x-8} \quad \text{☞ ❷}$$

$C_t = Ct$ ☞ ❸

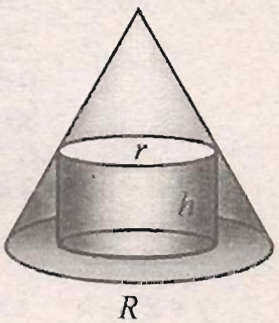
Substitute (1) and (2) in (3):

$$C_t = kx^3\left(\frac{d}{x-8}\right)$$

$$\frac{dC_t}{dx} = \frac{(x-8)(3kdx^2) - kdx^3(1)}{(x-8)^2} = 0$$

$$(x-8)(3x^2) = x^3$$
$$3x^3 - 24x^2 = x^3$$
$$2x^3 = 24x^2$$
$$x = 12 \text{ kph}$$

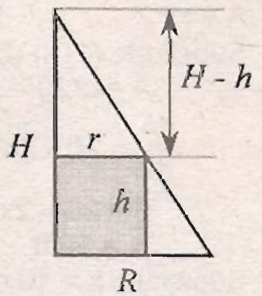
612. Let: R = radius of the cone
r = radius of the inscribed cylinder
H = height of the cone
h = height of the inscribed cylinder



$V = \pi r^2 h$ ☞ ❶

By ratio and proportion:

$$\frac{R}{H} = \frac{r}{H-h}$$
$$Hr = RH - Rh$$
$$h = \frac{RH - Hr}{R}$$ ☞ ❷



Substitute:

$$V = (\pi r^2)\frac{RH - Hr}{R}$$

$$V = \pi r^2 H - \frac{\pi r^3 H}{R}$$

$$\frac{dV}{dr} = 2\pi rH - \frac{3\pi r^2 H}{R} = 0$$

$$\frac{3\pi r^2 H}{R} = 2\pi rH$$

$$r = \frac{2}{3}R$$

Substitute r in (2):

$$h = \frac{RH - Hr}{R} = \frac{RH - H(2/3)R}{R} = \frac{1}{3}H$$

Substitute r and h in (1):

$V = \pi(2R/3)^2(H/3) = \pi(4/27)\,R^2H$

Let: V_C = volume of the cone

$V_C = (1/3)(\pi R^2)(H) = \pi/3\,R^2H$

$$\text{Ratio} = \frac{V}{V_C} = \frac{(4\pi/27)(R^2H)}{(\pi/3)(R^2H)} = 0.44 \text{ or } 44\%$$

613. Let: E = intensity of illumination
A & B = intensities of two light sources respectively.
k = proportionality constant

$$E = \frac{kA}{x^2} + \frac{kB}{(4-x)^2} = \frac{kA}{x^2} + \frac{k(8A)}{(4-x)^2}$$

$$\frac{dE}{dx} = \frac{kA(-2x)}{x^4} + \frac{(-8kA)(2)(4-x)(-1)}{(4-x)^4} = 0$$

$$\frac{2kA\,x}{x^4} = \frac{26kA(4-x)}{(4-x)^4}$$

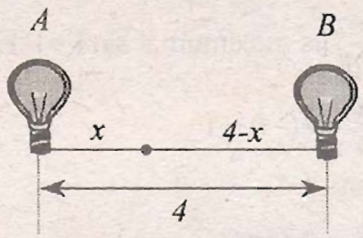
$$\frac{1}{x^3} = \frac{8}{(4-x)^3}$$

$$\left(\frac{4-x}{x}\right)^3 = 8$$

$$\frac{4-x}{x} = 2$$

$$4 - x = 2x$$

$$x = 1.5 \text{ m}$$



614. $\sin\theta = \frac{h}{x}$ $\qquad \cos\theta = \frac{2}{y}$

$$x = \frac{h}{\sin\theta} \qquad y = \frac{2}{\cos\theta}$$

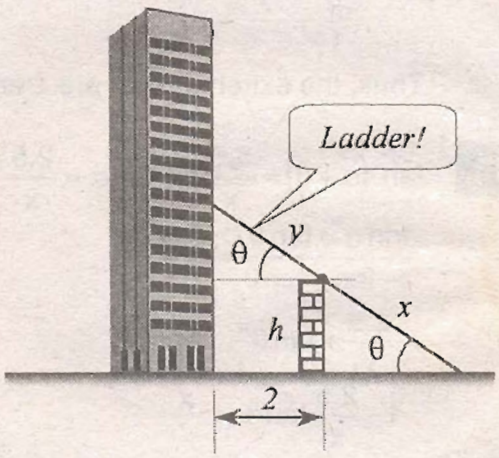
$$L = x + y = \frac{h}{\sin\theta} + \frac{2}{\cos\theta} \quad \text{☞ ❶}$$

$$\frac{dL}{d\theta} = \frac{-h\cos\theta}{\sin^2\theta} + \frac{-2(-\sin\theta)}{\cos^2\theta} = 0$$

$$\frac{h\cos\theta}{\sin^2\theta} = \frac{2\sin\theta}{\cos^2\theta}$$

$$h\cos^3\theta = 2\sin^3\theta$$

$$h = 2\tan^3\theta \quad \text{☞ ❷}$$



Substitute L = 6 and $h = 2\tan^3\theta$ in (1) and simplify:

$$6 = \frac{h\cos\theta + 2\sin\theta}{\sin\theta\cos\theta}$$

$$6\sin\theta\cos\theta = h\cos\theta + 2\sin\theta$$

$$= 2\tan^3\theta\cos\theta + 2\sin\theta$$

$$6 \sin\theta \cos\theta = 2\left(\frac{\sin\theta}{\cos\theta}\right)^3 \cos\theta + 2\sin\theta$$

$$6\cos\theta = 2\left(\frac{\sin\theta}{\cos\theta}\right)^2 + 2$$

$$6\cos^3\theta = 2\sin^2\theta + 2\cos^2\theta$$
$$= 2(\sin^2\theta + \cos^2\theta)$$
$$\cos^3\theta = 0.33333$$
$$\theta = 46.1°$$

Substitute θ in (2):

$$h = 2\tan^3 46.1° = 2.24 \text{ m.}$$

615. $x = \cos t - 1$

$$\frac{dx}{dt} = -\sin t$$

Note: $\frac{dx}{dt}$ is maximum if $\sin t = -1$

$$\frac{dx}{dt} = -(-1) = 1$$

$y = 2\sin t + 1$

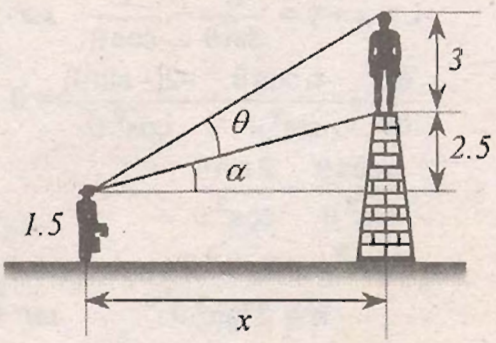
$$\frac{dy}{dt} = 2\cos t$$

Note: $\frac{dy}{dt}$ is maximum if $\cos t = 1$

$$\frac{dy}{dt} = 2(1) = 2$$

Thus, the extreme rates are 2 and 1.

616. $\tan(\alpha+\theta) = \frac{5.5}{x}$; $\tan\alpha = \frac{2.5}{x}$



$$\frac{\tan\alpha + \tan\theta}{1 - \tan\alpha\tan\theta} = \frac{5.5}{x}$$

$$\frac{\frac{2.5}{x} + \tan\theta}{1 - \frac{2.5}{x}\tan\theta} = \frac{5.5}{x}$$

$$2.5 + x\tan\theta = 5.5 - \frac{13.75}{x}\tan\theta$$

$$\tan\theta\left(x + \frac{13.75}{x}\right) = 3$$

$$\tan\theta = \frac{3}{x + \frac{13.75}{x}} = \frac{3x}{x^2 + 13.75}$$

$$\theta = \tan^{-1}\frac{3x}{x^2+13.75}$$

Note: $d\tan^{-1}u = \dfrac{du}{1+u^2}$

where:

$$u = \frac{3x}{x^2+13.75};\quad u^2 = \frac{9x^2}{(x^2+13.75)^2};\quad du = \frac{(x^2+13.75)3-3x(2x)}{(x^2+13.75)^2}$$

Thus,

$$\frac{d\theta}{dx} = \frac{du}{1+u^2} = 0$$

$$0 = du$$

$$0 = \frac{(x^2+13.75)3-3x(2x)}{(x^2+13.75)^2}$$

$$0 = 3x^2 + 41.25 - 6x^2$$

$$41.25 = 3x^2$$

$$x^2 = 13.75$$

$$x = 3.71 \text{ m.}$$

617.

$$S^2 = (10t)^2 + (5t)^2 + 10^2$$

$$S^2 = 125t^2 + 100$$

$$2S\frac{dS}{dt} = 250t$$

$$\frac{dS}{dt} = \frac{250t}{2S}$$ ☞ ❶

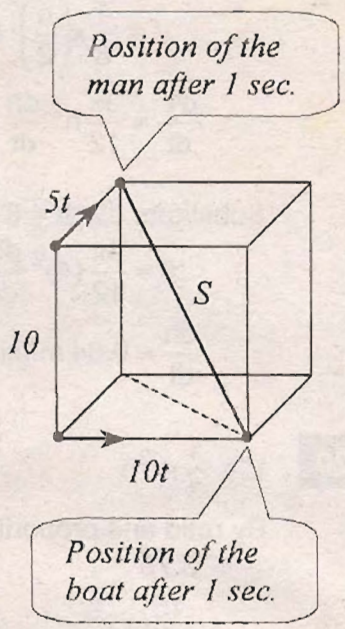
when t = 1 second

$$S^2 = 125(1)^2 + 100 = 225$$

$$S = 15 \text{ feet}$$

Substitute S = 15 in (1):

$$\frac{dS}{dt} = \frac{250(1)}{2(15)} = 8.33 \text{ fps.}$$



618.

$$S^2 = (12-4t)^2 + (9t)^2 + 6^2$$

$$S^2 = (12-4t)^2 + 81t^2 + 36$$

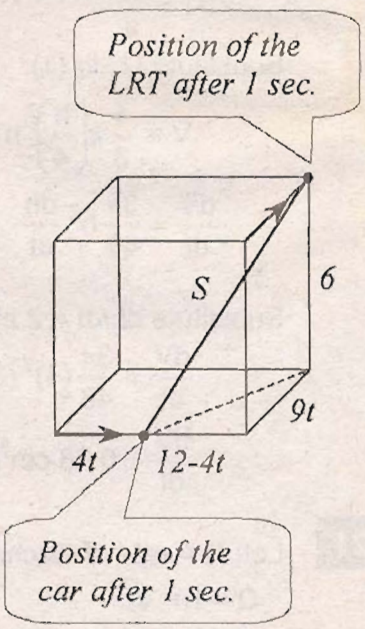
$$2S\frac{ds}{dt} = 2(12-4t)(-4) + 162t$$

$$\frac{dS}{dt} = \frac{162t - 8(12-4t)}{2S}$$

when t = 1 second,

$$S^2 = [12 - 4(1)]^2 + 81(1)^2 + 36 = 181$$

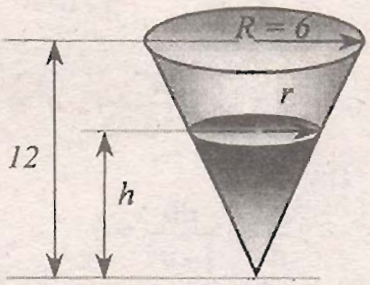
$$S = 13.45 \text{ m}$$



Substitute S = 13.45 in (1):

$$\frac{dS}{dt} = \frac{162(1) - 8[12 - 4(1)]}{2(13.45)} = 3.64 \text{ m/sec.}$$

619. $V = \frac{1}{3}\pi r^2 h$



By ratio and proportion:

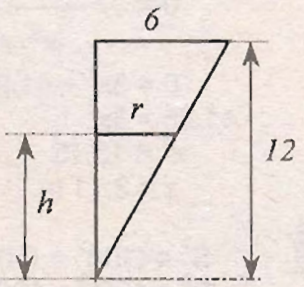
$$\frac{6}{12} = \frac{r}{h}$$

$$r = \frac{h}{2}$$

Substitute (2) in (1):

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3$$

$$\frac{dV}{dt} = \frac{3\pi}{12}h^2\frac{dh}{dt}$$

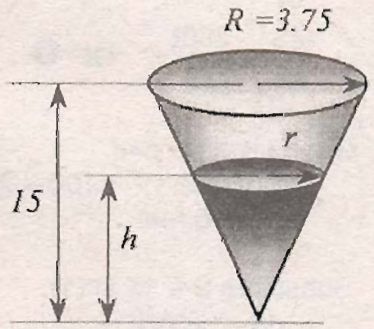


Substitute dV/dt = 8 and h = 4:

$$8 = \frac{3\pi}{12}(4)^2\frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.64 \text{ m/min}$$

620. $V = \frac{1}{3}\pi r^2 h$



By ratio and proportion:

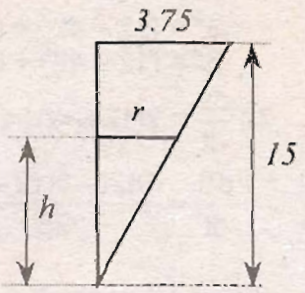
$$\frac{3.75}{15} = \frac{r}{h}$$

$$r = \frac{h}{4}$$

Substitute (2) in (1):

$$V = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h = \frac{\pi}{48}h^3$$

$$\frac{dV}{dt} = \frac{3\pi}{48}h^2\frac{dh}{dt}$$



Substitute dh/dt = 2 and h = 4:

$$\frac{dV}{dt} = \frac{3\pi}{48}(4)^2(2)$$

$$\frac{dV}{dt} = 6.28 \text{ cm}^3/\text{sec}$$

621. Let: Q = rate of discharge in gph

$$Q = t + \sqrt{t}$$

$$\frac{dQ}{dt} = 1 + \frac{1}{2\sqrt{t}}$$

when t = 9 hours

$$\frac{dQ}{dt} = 1 + \frac{1}{2\sqrt{9}} = 1 + \frac{1}{6}$$

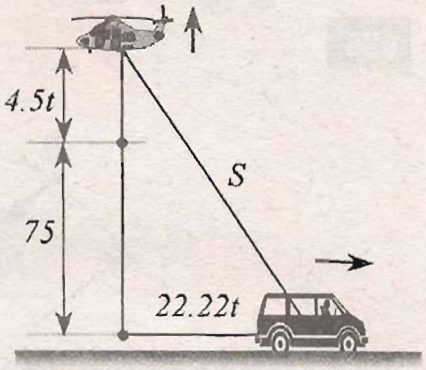
$$\frac{dQ}{dt} = \frac{7}{6}\text{ gph}$$

622. $\frac{80\text{ km}}{\text{hr}} \times \frac{1\text{ hr}}{3600\text{ sec}} \times \frac{1000\text{ m}}{1\text{ km}} = 22.22\text{ m/sec}$

$$S^2 = (22.22t)^2 + (75 + 4.5t)^2$$

$$2S\frac{ds}{dt} = 2(22.22t)(22.22) + 2(75 + 4.5t)(4.5)$$

$$\frac{dS}{dt} = \frac{(22.22)^2 t + 4.5(75 + 4.5t)}{S} \quad \text{☞ ❶}$$



when t = 1 second

$$S = \sqrt{(22.22t)^2 + (75 + 4.5t)^2}$$

$$= \sqrt{[(22.22(1)]^2 + [(75 + 4.5t)]^2}$$

S = 82.54 m.

Substitute S = 82.54, and t = 1 in (1):

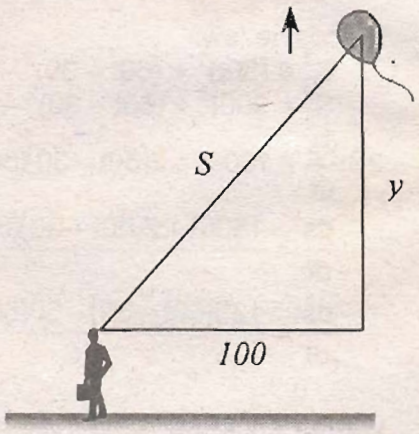
$$\frac{dS}{dt} = \frac{(22.22)^2(1) + 4.5[75 + 4.5(1)]}{82.54} = 10.32\text{ m/sec}$$

623.

$$S^2 = 100^2 + y^2 \quad \text{☞ ❶}$$

$$2S\frac{dS}{dt} = 2y\frac{dy}{dt}$$

$$\frac{dS}{dt} = \frac{y}{S}\left(\frac{dy}{dt}\right) \quad \text{☞ ❷}$$



when t = 10 seconds,

y = (4 m/sec)(10 sec) = 40 m.

Substitute y = 40 in (1):

$S^2 = 100^2 + 40^2 = 11600$

S = 107.7 m

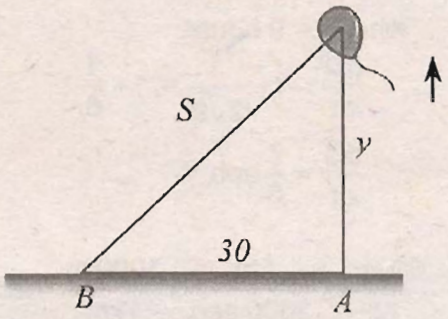
Substitute S = 107.7, y = 40 and dy/dt = 4 in (2):

$$\frac{dS}{dt} = \frac{40(4)}{107.7} = 1.49\text{ m/sec}$$

624. $S^2 = 30^2 + y^2$

$$2S\frac{ds}{dt} = 0 + 2y\frac{dy}{dt}$$

$$\frac{dS}{dt} = \frac{y\left(\frac{dy}{dt}\right)}{S} \quad \text{☞ 1}$$



when y = 40,

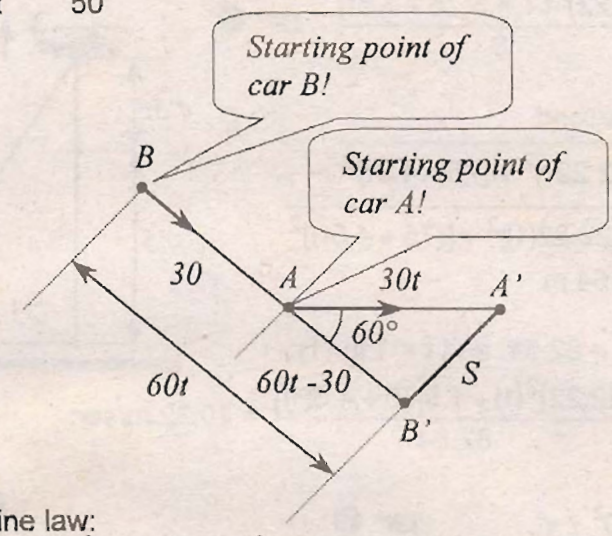
$S^2 = 30^2 + 40^2 = 2500$

S = 50 ft.

Substitute S = 50, y = 40 and dy/dt = 15 in (1):

$$\frac{dS}{dt} = \frac{40(15)}{50} = 12 \text{ fps}$$

625.



By cosine law:

$S^2 = (30t)^2 + (60t - 30)^2 - 2(30t)(60t - 30)\cos 60°$

$S^2 = 900t^2 + (60t - 30)^2 - 30t(60t - 30)$

$$2S\frac{ds}{dt} = 1800t + 2(60t - 30)(60) - [30t(60) + (60t - 30)(30)]$$

$$\frac{ds}{dt} = \frac{1800t + 2(60t - 30)(60) - 1800t - 30(60t - 30)}{2S}$$

$$\frac{ds}{dt} = \frac{120(60t - 30) - 30(60t - 30)}{2S} \quad \text{☞ 1}$$

when t = 1 hour,

$S^2 = [30(1)]^2 + [60(1) - 30]^2 - (30)(1)\,[60(1) - 30]$

$S^2 = 900$

S = 30 km.

Substitute S = 30 and t = 1 in (1):

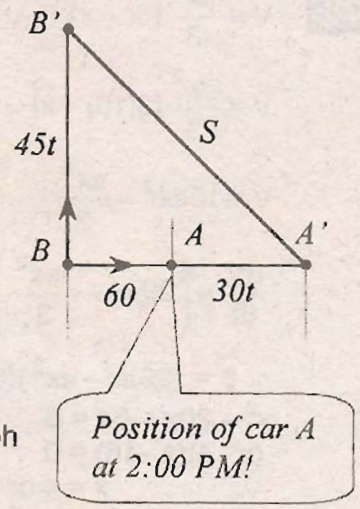
$$\frac{ds}{dt} = \frac{120[(60(1) - 30)] - 30[(609(1) - 30)]}{2(30)} = 45 \text{ kph}$$

626.

$$S^2 = (60t - 30t)^2 + (45t)^2$$

$$2S\frac{dS}{dt} = 2(60 - 30t)(30) + 2(45t)(45)$$

$$\frac{dS}{dt} = \frac{(60 - 30t)(30) + 45(45t)}{S} \quad \text{(1)}$$



when t = 2 hours,

$$S^2 = [60 + 30(2)]^2 + [45(2)]^2 = 22{,}500$$

$$S = 150 \text{ km.}$$

Substitute S = 150 and t = 2 in (1):

$$\frac{dS}{dt} = \frac{[60 + 30(2)]30 + 45[(45)(2)]}{150} = 51 \text{ kph}$$

627.

$$S^2 = (50t - 100)^2 + (50t - 150)^2$$

$$2S\frac{dS}{dt} = 2(50t - 100)(50) + 2(50t - 150)(50)$$

$$\frac{dS}{dt} = \frac{(50t - 100)(50) + (50t - 150)(50)}{S} \quad \text{(1)}$$

when t = 4 hours,

$$S^2 = [50(4) + 100]^2 + [50(4) + 150]^2 = 12500$$

$$S = 111.8 \text{ km.}$$

Substitute S = 111.8 and t = 4 in (1):

$$\frac{dS}{dt} = \frac{[50(4) - 100](50) + [50(4) - 150](50)}{111.8} = 67.08 \text{ kph}$$

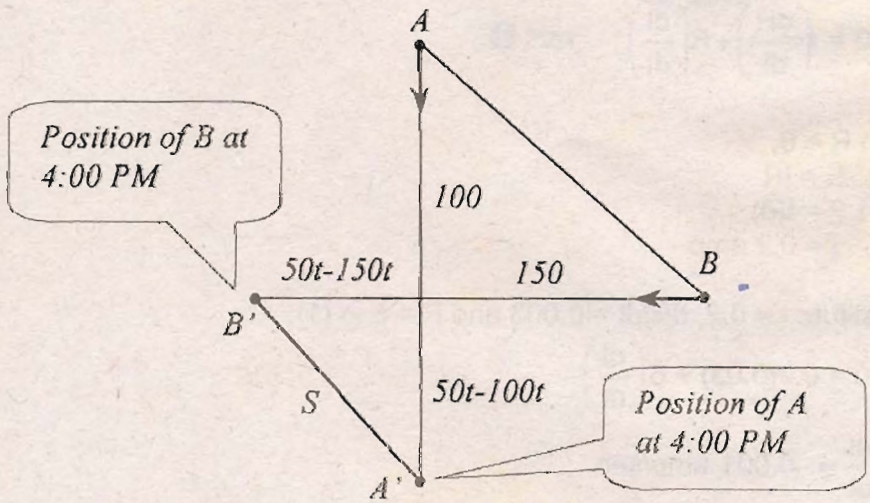


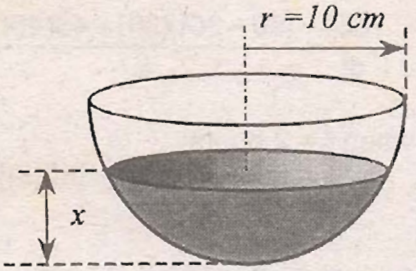
Figure for Prob. #627

628. $V = \frac{\pi x^2}{3}(3r - x)$

$V = \frac{\pi x^2}{3}[3(10) - x]$

$V = 10\pi x^2 - \frac{\pi x^3}{3}$

$\frac{dV}{dt} = \left(20\pi x - \frac{3\pi x^2}{3}\right)\frac{dx}{dt}$



$3 = (20\pi x - \pi x^2)(0.0149)$

$x^2 - 20x + 64 = 0$

$(x - 4)(x - 16) = 0$

$x = 4$ or 16 cm.

629. $V = x^3$

Differentiate both sides:

$dV = 3x^2 dx$ ☞ ❶

when $V = 8$,

$8 = x^3$

$x = 2$

Substitute $dV = 0.03$ and $x = 2$ in (1):

$0.03 = 3(2)^2 dx$

$dx = 0.0025$ m.

630. $E = IR$

Differentiate both sides:

$0 = I\left(\frac{dR}{dt}\right) + R\left(\frac{dI}{dt}\right)$ ☞ ❶

when $R = 6$,

$E = IR$

$1.2 = I(6)$

$I = 0.2$ amp

Substitute $I = 0.2$, $dR/dt = 0.003$ and $R = 6$ in (1):

$0 = 0.2(0.03) + 6\left(\frac{dI}{dt}\right)$

$\frac{dI}{dt} = -0.001$ amp/sec.

DAY 15

INTEGRAL CALCULUS

INTEGRALS

A. Basic Integrals:

1. $\int du = u + C$
2. $\int a\,du = au + C$
3. $\int u^n du = \frac{u^{n+1}}{n+1} + C \qquad (n \neq -1)$
4. $\int \frac{du}{u} = \ln u + C$

B. Exponential & Logarithmic Functions:

5. $\int e^u du = e^u + C$
6. $\int a^u du = \frac{a^u}{\ln a} + C$
7. $\int ue^u du = e^u(u-1) + C$
8. $\int \ln u\,du = u\ln u - u + C$
9. $\int \frac{du}{u\ln u} = \ln|\ln u| + C$

C. Trigonometric Functions:

10. $\int \sin u\,du = -\cos u + C$
11. $\int \cos u\,du = \sin u + C$

12. $\int \tan u\,du = \ln|\sec u| + C$

13. $\int \cot u\,du = \ln|\sin u| + C$

14. $\int \sec u\,du = \ln|\sec u + \tan u| + C$

15. $\int \csc u\,du = \ln|\csc u - \cot u| + C$

16. $\int \sec^2 u\,du = \tan u + C$

17. $\int \csc^2 u\,du = -\cot u + C$

18. $\int \sec u \tan u\,du = \sec u + C$

19. $\int \csc u \cot u\,du = -\csc u + C$

20. $\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$

21. $\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$

22. $\int \tan^2 u\,du = \tan u - u + C$

23. $\int \cot^2 u\,du = -\cot u - u + C$

D. Inverse Trigonometric Functions:

24. $\int \sin^{-1} u\,du = u\sin^{-1} u + \sqrt{1-u^2} + C$

25. $\int \cos^{-1} u\,du = u\cos^{-1} u - \sqrt{1-u^2} + C$

26. $\int \tan^{-1} u\,du = u\tan^{-1} u - \ln\sqrt{1-u^2} + C$

27. $\int \cot^{-1} u\,du = u\cot^{-1} u + \ln\sqrt{1+u^2} + C$

28. $\int \sec^{-1} u\,du = u\sec^{-1} u - \ln\left|u + \sqrt{u^2-1}\right| + C$

29. $\int \csc^{-1} u\,du = u\csc^{-1} u + \ln\left|u + \sqrt{u^2-1}\right| + C$

E. Hyperbolic Functions:

30. $\int \sinh u\,du = \cosh u + C$

31. $\int \cosh u\,du = \sinh u + C$

32. $\int \tanh u\,du = \ln|\cosh u| + C$

33. $\int \coth u\,du = \ln|\sinh u| + C$

34. $\int \operatorname{sech} u\,du = \tan^{-1}(\sinh u) + C$

35. $\int \operatorname{csch} u\,du = \ln\left|\tanh\frac{u}{2}\right| + C$

36. $\int \operatorname{sech}^2 u\,du = \tanh u + C$

37. $\int \operatorname{csch}^2 u\,du = -\coth u + C$

38. $\int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + C$

39. $\int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$

40. $\int \sinh^2 u\,du = \frac{1}{4}\sinh 2u - \frac{1}{2}u + C$

41. $\int \cosh^2 u\,du = \frac{1}{4}\sinh 2u + \frac{1}{2}u + C$

42. $\int \tanh^2 u\,du = u - \tanh u + C$

43. $\int \coth^2 u\,du = u - \coth u + C$

F. Trigonometric Substitution:

44. $\int \sqrt{a^2 - u^2}\,du$ let : $u = a \sin\theta$

45. $\int \sqrt{a^2 + u^2}\,du$ let : $u = a \tan\theta$

46. $\int \sqrt{u^2 - a^2}\,du$ let : $u = a \sec\theta$

G. Integration By Parts:

47. $\int udv = uv - \int vdu$

H. Wallis Formula: Formulated by John Wallis.

48. $\int_0^{\frac{\pi}{2}} \sin^m\theta \cos^n\theta d\theta = \dfrac{[(m-1)(m-3)\cdots 1 \text{ or } 2][(n-1)(n-3)\cdots 1 \text{ or } 2]}{(m+n)(m+n-2)(m+n-4)\cdots 1 \text{ or } 2}\ \alpha$

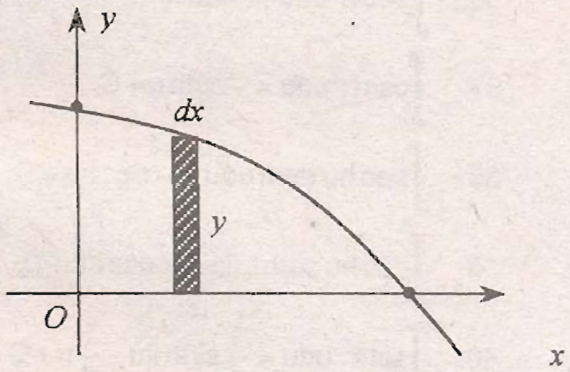
where: $\alpha = \dfrac{\pi}{2}$ if both m and n are even

$\alpha = 1$ if otherwise

PLANE AREAS

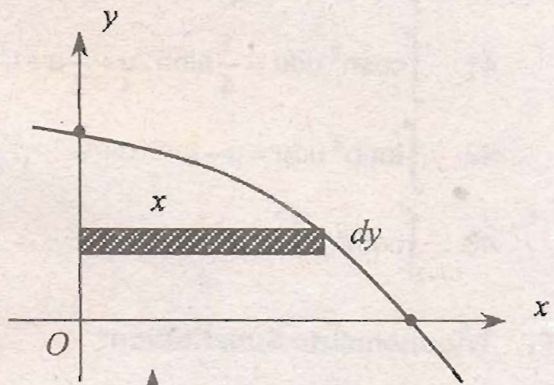
By rectangular coordinates:
Using a vertical differential strip:

$$A = \int ydx$$



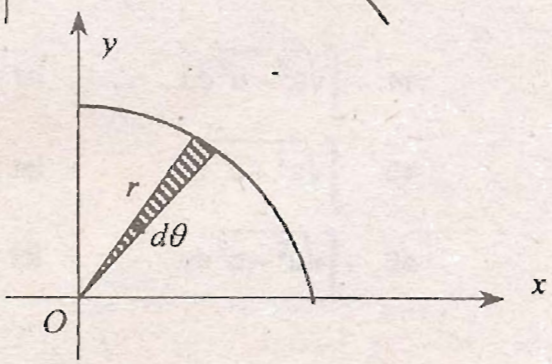
Using a horizontal differential strip:

$$A = \int xdy$$



By polar coordinates:

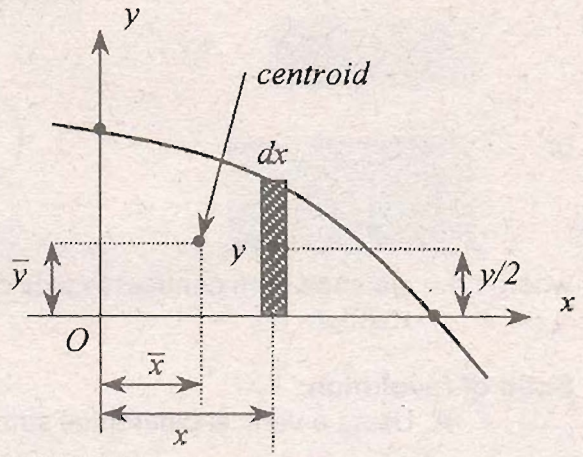
$$A = \int \frac{r^2}{2} d\theta$$



CENTROID

$$\bar{x} = \frac{\int dA \cdot x}{A}$$

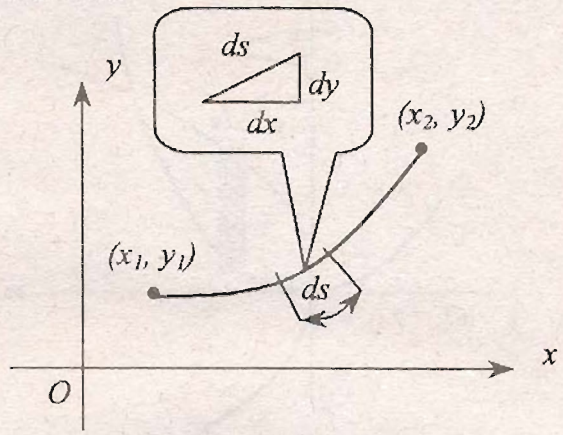
$$\bar{y} = \frac{\int dA \cdot \frac{y}{2}}{A}$$



LENGTH OF ARC

$$S = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

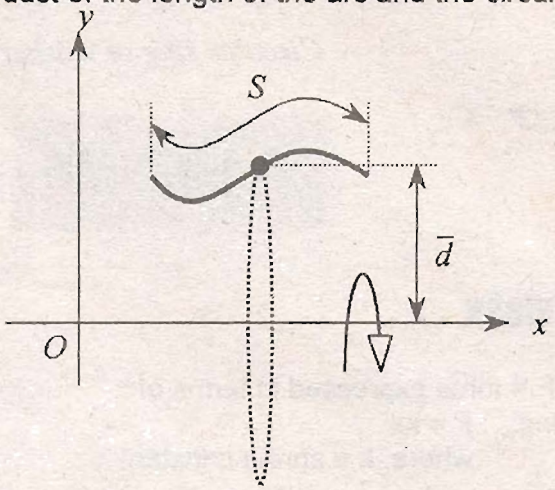


PROPOSITIONS OF PAPPUS

The following are the propositions or theorems of **Pappus** (A.D. c. 400) of Alexandria.

First Proposition of Pappus: (Surface Area)

"If an arc is rotated about an axis, it will generate a surface area equal to the product of the length of the arc and the circumference described the its centroid."



$$A = S \cdot 2\pi\bar{d}$$

where: S = length of arc
$\bar{d}$ = distance from centroid to the axis of rotation

or

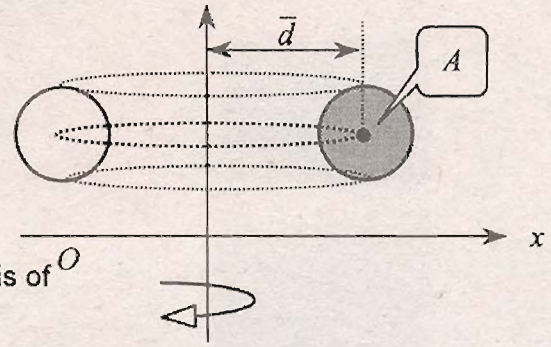
$$A = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \cdot 2\pi\bar{d}$$

Second Proposition of Pappus: (Volume)

$$V = A \cdot 2\pi\bar{d}$$

or

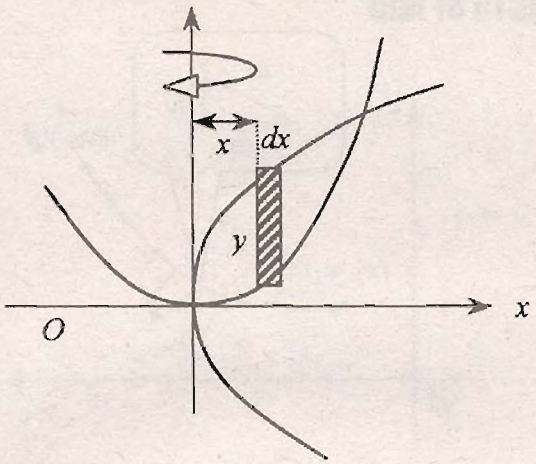
$$V = \int dA \cdot 2\pi\bar{d}$$

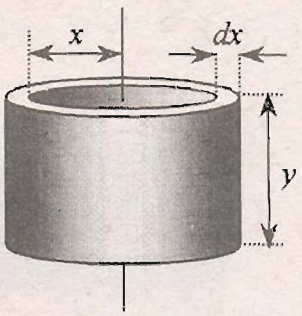


where : $\bar{d}$ = distance from centroid to axis of rotation

Solid of revolution:

A. Using a vertical differential strip:

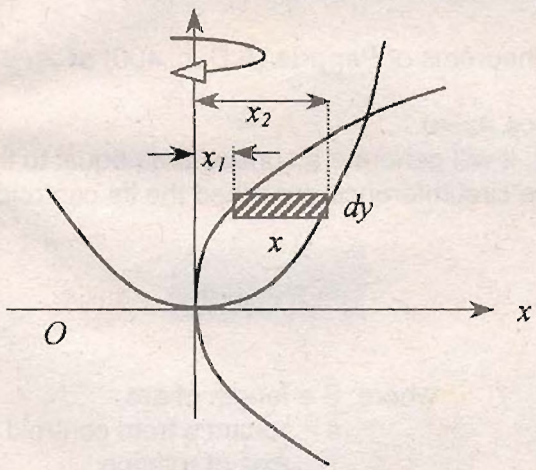


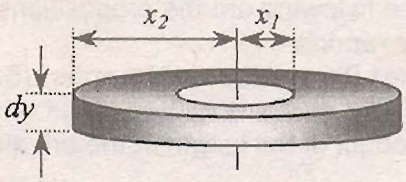


Hollow cylindrical shell

$$V = \int 2\pi yx\,dx$$

B. Using a horizontal differential strip:





Circular ring or washer

$$V = \int \pi\left(x_2^2 - x_1^2\right)dy$$

WORK

$$\text{Work} = \int F \cdot dx$$

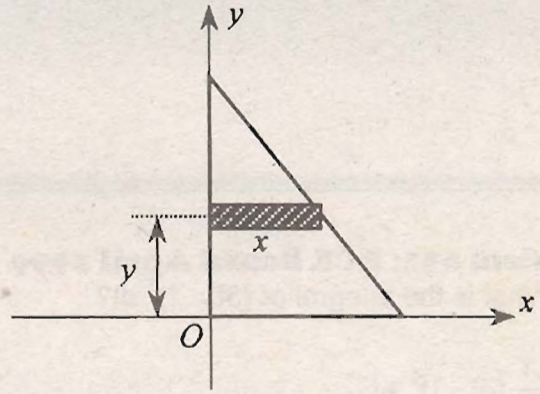
where: F = force expressed in terms of x
For spring: F = kx
where: k = spring constant

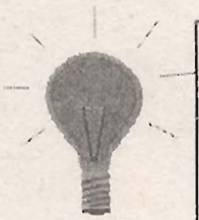
MOMENT OF INERTIA

(Second Moment of Area)

$$I_x = \int dA \cdot y^2$$

$$I_y = \int dA \cdot \left(\frac{x}{2}\right)^2$$





Tip: Pressure

The total hydrostatic pressure may be calculated directly by a formula rather than by integration.

$$P = \gamma \bar{h} A$$

where : γ = density of the liquid

$\bar{h}$ = distance from the centroid of the area to the liquid surface.

A = area subjected to pressure

Density of water = 1000 kg/m^3
= 9.81 kN/m^3
= 9810 N/m^3
= 62.4 lbs/ft^3
= 1 gram/cc

Density of other liquids = (density of H_2O)(sp. gr. of liquid)

Did you know that... the propositions of Pappus was inspired by two fruits, namely apple and lemon! Pappus imagined a circle cut by a line not passing through its center and rotate the area of the major part about the said line and produced a solid which looks like an apple while the remaining minor area when rotated about the same line forms what is known as a lemon.

Proceed to the next page for your 15th test. GOODLUCK ! ☞

Test 15

Time element: 4.0 hours

Problem 631: ECE Board April 1999
What is the integral of $(3t - 1)^3$ dt?

A. $\frac{1}{12}(3t-1)^4 + C$

B. $\frac{1}{12}(3t-4)^4 + C$

C. $\frac{1}{4}(3t-1)^4 + C$

D. $\frac{1}{4}(3t-1)^3 + C$

Problem 632: ECE Board November 1998
Evaluate the integral of dx / (x +2) from –6 to –10.

A. $2^{1/2}$
B. 1/2
C. ln 3
D. ln 2

Problem 633: ECE Board November 1998, ME Board April 1998
Integrate x cos ($2x^2$ + 7) dx.

A. $\frac{1}{4}\sin\left(2x^2+7\right)+C$

B. $\frac{1}{4}\cos\left(2x^2+7\right)+C$

C. $\frac{(\sin\theta)}{4\left(x^2+7\right)}+C$

D. $\sin\left(2x^2+7\right)+C$

Problem 634: ME Board April 1995, ME Board April 1997
Integrate: $(7x^3 + 4x^2)$ dx.

A. $\frac{7x^3}{3}+\frac{4x^2}{2}+C$

B. $\frac{7x^4}{4}+\frac{4x^2}{5}+C$

C. $\frac{7x^4}{4}+\frac{4x^3}{3}+C$

D. $7x^4-\frac{4x}{2}+C$

Problem 635: CE Board November 1995

What is the integral of $\sin^5 x \cos^3 x\, dx$ if the lower limit is zero and the upper limit is $\pi/2$?

A. 0.0203
B. 0.0307
C. 0.0417
D. 0.0543

Problem 636: CE Board November 1994

What is the integral of $\sin^5 x\, dx$ if the lower limit is 0 and the upper limit is $\pi/2$?

A. 0.233
B. 0.333
C. 0.433
D. 0.533

Problem 637: CE Board May 1996

Find the integral of $12 \sin^5 x \cos^5 x\, dx$ if lower limit = 0 and upper limit = $\pi/2$.

A. 0.2
B. 0.3
C. 0.4
D. 0.5

Problem 638: ECE Board April 1997

Evaluate the integral of $\sin^6 x\, dx$ from 0 to $\pi/2$.

A. $\frac{\pi}{32}$

B. $\frac{2\pi}{17}$

C. $\frac{3\pi}{32}$

D. $\frac{5\pi}{32}$

Problem 639: CE Board May 1997

Evaluate $\int_5^6 x(x-5)^{12}\,dx$.

A. 0.456
B. 0.556
C. 0.656
D. 0.756

Problem 640: CE Board November 1996

Evaluate $\int_0^1 \frac{x\,dx}{(x+1)^8}$.

A. 0.011
B. 0.022
C. 0.033
D. 0.044

Problem 641: ECE Board April 1998

Evaluate $\int_0^{\pi/6} (\cos 3A)^8\,dA$. .

A. $\frac{27\pi}{363}$
B. $\frac{35\pi}{768}$
C. $\frac{23\pi}{765}$
D. $\frac{12\pi}{81}$

Problem 642: EE Board March 1998

Integrate $\frac{1}{3x+4}$ with respect to x and evaluate the result from x = 0 and x = 2.

A. 0.278
B. 0.336
C. 0.252
D. 0.305

Problem 643: ECE Board November 1991

Evaluate the integral $\int \cos^2 y dy$

A. $\frac{y}{2} + \frac{\sin 2y}{4} + C$

B. $y + 2 \cos y + C$

C. $\frac{y}{4} + \frac{\sin 2y}{4} + C$

D. $y + \sin 2y + C$

Problem 644: ECE Board November 1998

Integrate the square root of (1 - cosx) dx.

A. $-2\sqrt{2} \cos \frac{1}{2} x + C$

B. $-2\sqrt{2} \cos x + C$

C. $2\sqrt{2} \cos \frac{1}{2} x + C$

D. $-2\sqrt{2} \cos x + C$

Problem 645: ME Board October 1997

Evaluate the integral of cos x dx limits from $\frac{\pi}{4}$ to $\frac{\pi}{2}$

A. 0.423
B. 0.293
C. 0.923
D. 0.329

Problem 646: EE Board April 1997

Evaluate the integral of ln x dx, the limits are 1 and e.

A. 0
B. 1
C. 2
D. 3

Problem 647: EE Board October 1997

Evaluate $\int_1^{10} \frac{2 \log_{10} e dx}{x}$

A. 2.0
B. 49.7
C. 3.0
D. 5.12

Problem 648: CE Board May 1995

What is the integral of cos 2x $e^{\sin 2x}$ dx?

A. $\frac{e^{\sin 2x}}{2} + C$

B. $-\frac{e^{\sin 2x}}{2} + C$

C. $-e^{\sin 2x} + C$

D. $e^{\sin 2x} + C$

Problem 649: ME Board April 1995, ME Board October 1997

The integral of cos x with respect to x is

A. sin x + C
B. sec x + C
C. – sin x + C
D. csc x + C

Problem 650: EE Board April 1997

Find the integral of [(e exp x – 1] divided by [e exp x + 1] dx

A. ln (e exp x – 1) square + x + C
B. ln (e exp x + 1) - x + C
C. ln (e exp x – 1) + x + C
D. ln (e exp x + 1) square - x + C

Problem 651: EE Board April 1997

Evaluate the double integral of r sin u dr du, the limits of r is 0 and cos u and the limits of u are 0 and pi.

A. 1
B. 1/2
C. 0
D. 1/3

Problem 652: CE Board November 1996

Evaluate the integral of $(3x^2 + 9y^2)$ dx dy if the interior limits has an upper limit of y and a lower limit of 0, and whose outer limit has an upper limit of 2 and lower limit of 0.

A. 10
B. 20
C. 30
D. 40

Problem 653: EE Board April 1996

Evaluate $\int_0^{\pi/2}\int_0^1\int_0^2 z\,dz\,r^2\,dr\,\sin u\,du$.

A. 2/3
B. 4/3
C. 1/3
D. 5/3

Problem 654: EE Board April 1993

Find the area of the region bounded by $y^2 = 8x$ and $y = 2x$.

A. 1.22 sq. units
B. 1.33 sq. units
C. 1.44 sq. units
D. 1.55 sq. units

Problem 655: CE Board November 1994

What is the area bounded by the curve $x^2 = -9y$ and the line $y + 1 = 0$?

A. 3 sq. units
B. 4 sq. units
C. 5 sq. units
D. 6 sq. units

Problem 656: CE Board May 1995

What is the area (in square units) bounded by the curve $y^2 = x$ and the line $x - 4 = 0$?

A. 30/3
B. 31/3
C. 32/3
D. 29/3

Problem 657: EE Board October 1997

Find the area bounded by the curve $y = x^2 + 2$, and the lines $x = 0$ and $y = 0$ and $x = 4$.

A. 88/3
B. 64/3
C. 54/3
D. 64/5

Problem 658: EE Board April 1997

Find the area bounded by the parabolas $y = 6x - x^2$ and $y = x^2 - 2x$. Note: The parabolas intersect at points (0,0) and (4,8).

A. 44/3 square units

B. 64/3 square units
C. 74/3 square units
D. 54/3 square units

Problem 659: ME Board April 1999

Find the area bounded by the parabola $x^2 = 4y$ and $y = 4$.

A. 21.33
B. 33.21
C. 31.32
D. 13.23

Problem 660: EE Board October 1997

Find the area bounded by the line $x - 2y + 10 = 0$, the x-axis, the y-axis and $x = 10$.

A. 75
B. 50
C. 100
D. 25

Problem 661: CE Board May 1996

What is the area (in square units) bounded by the curve $y^2 = 4x$ and $x^2 = 4y$?

A. 5.33
B. 6.67
C. 7.33
D. 8.67

Problem 662: CE Board May 1997

Find the area enclosed by the curve $x^2 + 8y + 16 = 0$, the x-axis, the y-axis and the line $x - 4 = 0$.

A. 7.67 sq. units
B. 8.67 sq. units
C. 9.67 sq. units
D. 10.67 sq. units

Problem 663: ME Board October 1997

What is the area bounded by the curve $y = x^3$, the x-axis and the line $x = -2$ and $x = 1$?

A. 4.25
B. 2.45
C. 5.24
D. 5.42

Problem 664: ME Board April 1999

Find the area in the first quadrant bounded by the parabola $y^2 = 4x$, $x = 1$ & $x = 3$

A. 9.555
B. 9.955
C. 5.955
D. 5.595

Problem 665: ECE Board April 1998

Find the area (in sq. units) bounded by the parabolas $x^2 - 2y = 0$ and $x^2 + 2y - 8 = 0$.

A. 11.7
B. 4.7
C. 9.7
D. 10.7

Problem 666: ME Board April 1998

What is the area between $y = 0$, $y = 3x^2$, $x = 0$ and $x = 2$?

A. 8
B. 24
C. 12
D. 6

Problem 667: CE Board May 1995

What is the area bounded by the curve $y^2 = x$ and the line $x - 4 = 0$?

A. 11
B. 31/3
C. 10
D. 32/3

Problem 668: CE Board November 1996, CE Board November 1998

Find the area of the curve $r^2 = a^2 \cos 2\theta$.

A. a
B. 2a
C. a^2
D. a^3

Problem 669:

Locate the centroid of the plane area bounded by $y = x^2$ and $y = x$.

A. 0.4 from the x-axis and 0.5 from the y-axis
B. 0.5 from the x-axis and 0.4 from the y-axis
C. 0.5 from the x-axis and 0.5 from the y-axis
D. 0.4 from the x-axis and 0.4 from the y-axis

Problem 670:

Find the coordinates of the centroid of the plane area bounded by the parabola $y = 4 - x^2$ and the x-axis.

A. (0,1)
B. (0,1.6)
C. (0,2)
D. (1,0)

Problem 671:

Locate the centroid of the plane area bounded by the equation $y^2 = 4x$, $x = 1$ and the x-axis on the first quadrant.

A. (3/4, 3/5)
B. (3/5, 3/4)
C. (3/5, 3/5)
D. (3/5, 2/3)

Problem 672:

Find the length of arc of the parabola $x^2 = 4y$ from $x = -2$ to $x = 2$.

A. 4.2 units
B. 4.6 units
C. 4.9 units
D. 5.2 units

Problem 673:

Find the surface area (in square units) generated by rotating the parabola arc $y = x^2$ about the x-axis from $x = 0$ to $x = 1$.

A. 5.33
B. 4.98
C. 5.73
D. 4.73

Problem 674: CE Board May 1997

The area enclosed by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is revolved about the line $x = 3$. What is the volume generated?

A. 355.3
B. 360.1
C. 370.3
D. 365.1

Problem 675: CE Board May 1996

The area in the second quadrant of the circle $x^2 + y^2 = 36$ is revolved about the line $y + 10 = 0$. What is the volume generated?

A. 2218.33
B. 2228.83
C. 2233.43
D. 2208.53

Problem 676: CE Board November 1995

The area bounded by the curve $y^2 = 12x$ and the line $x = 3$ is revolved about the line $x = 3$. What is the volume generated?

A. 179
B. 181
C. 183
D. 185

Problem 677: CE Board November 1994

Given the area in the first quadrant bounded by $x^2 = 8y$, the line $y - 2 = 0$ and the y-axis. What is the volume generated when the area is revolved about the line $y - 2 = 0$?

A. 28.41
B. 27.32
C. 25.83
D. 26.81

Problem 678:

Find the volume (in cubic units) generated by rotating a circle $x^2 + y^2 + 6x + 4y + 12 = 0$ about the y-axis.

A. 39.48
B. 47.23
C. 59.22
D. 62.11

Problem 679: CE Board May 1995

Given the area in the first quadrant by $x^2 = 8y$, the line $x = 4$and the x-axis. What is the volume generated by revolving this area about the y-axis.

A. 53.26
B. 52.26
C. 51.26
D. 50.26

Problem 680: CE Board November 1995

Find the moment of inertia, with respect to x-axis of the area bounded by the parabola $y^2 = 4x$ and the line $x = 1$.

A. 2.03
B. 2.13
C. 2.33
D. 2.53

ANSWER KEY

631. A	644. A	657. A	670. B
632. D	645. B	658. B	671. B
633. A	646. B	659. A	672. B
634. C	647. A	660. A	673. A
635. C	648. A	661. A	674. A
636. D	649. A	662. D	675. B
637. A	650. D	663. A	676. B
638. D	651. D	664. D	677. D
639. A	652. D	665. D	678. C
640. B	653. A	666. A	679. D
641. B	654. B	667. D	680. B
642. D	655. B	668. C	
643. A	656. C	669. A	

RATING

☐ 43-50 Topnotcher
☐ 30-42 Passer
☐ 25-29 Conditional
☐ 0-24 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 15

631. $\int(3t-1)^3\,dt = \frac{1}{3}\int(3t-1)^3\,3dt$

$$= \frac{1}{3}\left(\frac{(3t-1)^4}{4}\right) + C = \frac{1}{12}(3t-1)^4 + C$$

632. $\int_{-6}^{-10} \frac{dx}{x+2}$

Note: $\int\frac{du}{u} = \ln u$

$$\int_{-6}^{-10} \frac{dx}{x+2} = \ln(x+2)\Big|_{-6}^{-10}$$

$$= \ln(-10+2) - \ln(-6+2)$$
$$= \ln(-8) - \ln(-4)$$
$$= \ln\frac{-8}{-4} = \ln 2$$

633. $\int x\cos(2x^2+7)dx = \frac{1}{4}\int\cos(2x^2+7)4x\,dx$

$$= \frac{1}{4}\sin(2x^2+7) + C$$

634. $\int(7x^3+4x^2)dx = \frac{7x^4}{4} + \frac{4x^3}{3} + C$

635. $\int_0^{\pi/2}\sin^5 x\cos^3 x\,dx$

Using Wallis formula:

$$\int_0^{\pi/2}\sin^m x\cos^n x\,dx = \frac{(m-1)(m-3)(m-5)..{}_1^2\,(n-1)(n-3)(n-5)..{}_1^2}{(m+n)(m+n-2)(m+n-4)..{}_1^2}\cdot\alpha$$

In this problem; m = 5; n = 3; α = 1, since both m & n are odd numbers

$$\int_0^{\pi/2}\sin^5 x\cos^3 x\,dx = \frac{(5-1)(5-3)(3-1)}{8(6)(4)(2)}\cdot 1 = 0.0417$$

636. $\int_0^{\pi/2}\sin^5 x\,dx$

Using Wallis formula: m = 5; n = 0 and α = 1 since m is an odd number

$$\int_0^{\pi/2} \sin^5 x\,dx = \frac{4(2)}{5(3)(1)} \cdot 1 = 0.533$$

637. $12\int_0^{\pi/2} \sin^5 x \cos^5 x\,dx$

Using Wallis formula: m = 5; n = 5 and $\alpha = 1$

$$12\int_0^{\pi/2} \sin^5 x \cos^5 x\,dx = 12\left(\frac{(5-1)(5-3)(5-1)(5-3)}{10(8)(6)(4)(2)}\right)\cdot 1 = 0.2$$

638. $\int_0^{\pi/2} \sin^6 x\,dx$

Using Wallis formula: m = 6; n = 0 and $\alpha = \pi/2$, since m is an even number

$$\int_0^{\pi/2} \sin^6 x\,dx = \frac{5(3)(1)}{6(4)(2)} \cdot \frac{\pi}{2} = \frac{5\pi}{32}$$

639. $\int_5^6 x(x-5)^{12}\,dx$

Note: $\int udv = uv - \int vdu$

where: u = x; du = dx; dv = $(x-5)^{12}\,dx$; v = $\dfrac{(x-5)^{13}}{13}$

$$\int_5^6 x(x-5)^{12}\,dx = \frac{x(x-5)^{13}}{13} - \frac{1}{13}\int (x-5)^{13}\,dx$$

$$= \frac{x}{13}(x-5)^{13} - \frac{1}{13}\left[\frac{(x-5)^{14}}{14}\right]\Bigg|_5^6$$

$$= \frac{6}{13}(6-5)^{13} - \frac{5}{13}(5-5)^{13} - \frac{1}{182}\left[(6-5)^{14} - (5-5)^{14}\right]$$

$$= \frac{6}{13} - \frac{1}{182} = 0.456$$

640. $\int_0^1 \dfrac{xdx}{(x+1)^8} = (x+1)^{-8}xdx$

Let: u = x; du = dx; dv = $(x+1)^{-8}\,dx$; v = $\dfrac{(x+1)^{-7}}{-7}$

$$\int_0^1 (x+1)^{-8}\,x\,dx = uv - \int v\,du$$

$$= x\left[\frac{(x+1)^{-7}}{-7}\right] - \int \frac{(x+1)^{-7}}{-7}dx$$

$$= \frac{-x}{7(x+1)^7} + \frac{1}{7}\int (x+1)^{-7}\,dx$$

$$= \frac{-x}{7(x+1)^7} + \frac{1}{7}\left[\frac{(x+1)^{-6}}{-6}\right]$$

$$= \frac{-x}{7(x+1)^7} - \frac{1}{42}\left[(x+1)^{-6}\right]\Big|_0^1$$

$$= \frac{-1}{7(1+1)^7} - \frac{0}{7(0+1)^7} - \frac{1}{42}\left[(1+1)^{-6} - (0+1)^{-6}\right]$$

$$= 0.022$$

641. $\int_0^{\pi/6} \cos^8 3A\,dA$

Let: u = 3A; du = 3dA thus, dA = du/3

Change limits:

at A = 0, u = 0; at A = π/6, u = π/2

Substitute:

$$\int_0^{\pi/2} \cos^8 u \frac{du}{3} = \frac{1}{3}\int_0^{\pi/2} \cos^8 u\,du$$

Using Wallis formula: n = 8 and $\alpha = \pi/2$, since n is an even number

$$= \frac{1}{3}\left(\frac{7(5)(3)(1)}{8(6)(4)(2)}\right)\left(\frac{\pi}{2}\right) = \frac{105\pi}{2304} = \frac{35\pi}{768}$$

642. $\int_0^2 \frac{dx}{3x+4} = \frac{1}{3}\int_0^2 \frac{dx}{3x+4}$

$$= \frac{1}{3}\ln(3x+4)\Big|_0^2$$

$$= \frac{1}{3}\ln[(3)(2)+4] - \frac{1}{3}\ln[(3)(0)+4]$$

$$= 0.305$$

643. $\int \cos^2 y\,dy = \int \frac{1+\cos 2y}{2}\,dy = \frac{1}{2}\int (1+\cos 2y)\,dy$

$$= \frac{1}{2}\int dy + \frac{1}{4}\int \cos 2y(2dy))$$

$$= \frac{y}{2} + \frac{\sin 2y}{4} + C$$

644. $\int \sqrt{1-\cos x}\,dx$

Note: $\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}}$

Thus, $\sqrt{2}\sin\frac{x}{2} = \sqrt{1-\cos x}$

$$\sqrt{2}\int \sin\frac{x}{2} = 2\sqrt{2}\int \sin\frac{x}{2}\frac{dx}{2}$$

$$= 2\sqrt{2}\left(-\cos\frac{x}{2}\right) + C$$

$$= -2\sqrt{2}\cos\frac{x}{2} + C$$

645. $\int_{\pi/4}^{\pi/2} \cos x\,dx = \sin x\Big|_{\pi/4}^{\pi/2} = \sin\frac{\pi}{2} - \left(\sin\frac{\pi}{4}\right) = 0.293$

Note: $\pi/2 = 90°$ and $\pi/4 = 45°$

646. $\int_1^e \ln x\,dx$

Let: $u = \ln x$; $du = \frac{dx}{x}$; $dv = dx$; $v = x$

$$\int_1^e \ln x\,dx = uv - \int v\,du$$

$$= (\ln x)(x) - \int x\frac{dx}{x} = x\ln x - x\Big|_0^1$$

$$= e\ln e - e - (\ln 1 - 1) = 1$$

647. $\int_1^{10} \frac{2\log_{10} e\,dx}{x} = 2\log_{10} e\int_1^{10}\frac{dx}{x} = (2\log_{10} e)\ln x\Big|_1^{10}$

$$= (2\log_{10} e)[\ln 10 - \ln 1] = 2$$

Note: $e = 2.718$

648. $\int \cos 2x\ e^{\sin 2x} dx$

Let: u = sin 2x; du = cos 2x(2)

$$\int \cos 2x\ e^{\sin 2x} dx = \frac{1}{2}\int e^{\sin 2x}(2\cos 2x)dx$$

$$= \frac{1}{2}e^{\sin 2x} + C$$

649. $\int \cos x dx = \sin x + C$

650. $$\int \frac{e^x - 1}{e^x + 1} dx = \int \frac{e^x dx}{e^x + 1} - \int \frac{dx}{e^x + 1}$$

$$= \ln(e^x + 1) - \int \frac{dx}{e^x + 1}$$

Let: $u = e^x + 1$, thus $e^x = u - 1$; $du = e^x dx$

$$\int \frac{dx}{e^x + 1} = \int \frac{du}{e^x(e^x + 1)} = \int \frac{du}{(u-1)(u-1+1)} = \int \frac{du}{u(u-1)}$$

$$\frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1}$$

$$1 = A(u-1) + Bu = Au - A + Bu$$

Equate constant:

$1 = -A$, thus $A = -1$

Equate coefficients of u:

$0 = A + B$

$0 = -1 + B$, thus $B = 1$

$$\int \frac{du}{u(u-1)} = \int \frac{-du}{u} + \int \frac{du}{u-1}$$

$$= -\ln u + \ln(u-1) = -\ln(e^x + 1) + \ln(e^x)$$

Thus, $$\int \frac{e^x - 1}{e^x + 1} dx = \ln(e^x + 1) - \left[-\ln(e^x + 1) + \ln(e^x)\right] + C$$

$$= 2\ln(e^x + 1) - x + C$$

$$= \ln(e^x + 1)^2 - x + C$$

651. $$\int_0^{\pi}\int_0^{\cos u} r \sin u\, dr\, du = \int_0^{\pi} \frac{r^2}{2}\Big|_0^{\cos u} \sin u\, du = \frac{1}{2}\int_0^{\pi} \cos^2 u \sin u\, du$$

$$= -\frac{1}{2}\int_0^{\pi} \cos^2 u(-\sin u)du = -\frac{1}{2}\frac{\cos^3 u}{3} = \frac{-\cos^3 u}{6}\Big|_0^{\pi}$$

$$= -\frac{1}{6}(\cos^3 \pi - \cos^3 0) = -\frac{1}{6}(-1-1) = \frac{1}{3}$$

652. $$\int_0^2\int_0^y (3x^2+9y^2)dxdy = \int_0^2 \frac{3x^3}{3}+9y^2x\Big|_0^y dy = \int_0^2 (y^3+9y^3)dy$$

$$= \int_0^2 10y^3dy = \frac{10y^4}{4}\Big|_0^2 = \frac{10(2)^4}{4} = 40$$

653. $$\int_0^{\pi/2}\int_0^1\int_0^2 zdz\ r^2dr\ \sin u\,du = \int_0^{\pi/2}\int_0^1 \frac{z^2}{2}\Big|_0^2 r^2dr\sin u\,du$$

$$= \int_0^{\pi/2}\int_0^1 2r^2dr\sin u\,du = 2\int_0^{\pi/2}\frac{r^3}{3}\Big|_0^1 \sin u\,du$$

$$= \frac{2}{3}\int_0^{\pi/2}\sin u\,du = \frac{2}{3}(-\cos u)\Big|_0^{\pi/2}$$

$$= \frac{2}{3}\left(-\cos\frac{\pi}{2}+\cos 0°\right) = \frac{2}{3}$$

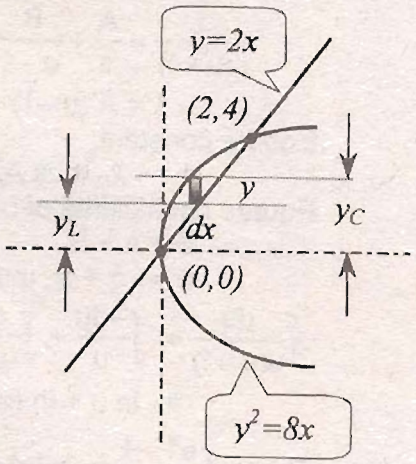
654.

$y^2 = 8x$ $\qquad$ $y = 2x$

$(2x)^2 = 8x$ $\qquad$ $= 2(2)$

$4x^2 = 8x$ $\qquad$ $y = 4$

$x = 2$



$$A = \int_0^2 ydx = \int_0^2 (y_P - y_L)dx$$

$$= \int_0^2 \left(\sqrt{8x}-2x\right)dx$$

$$= \frac{\sqrt{8}x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{2x^2}{2} = \frac{2\sqrt{8}}{3}x^{\frac{3}{2}} - x^2\Big|_0^2 = \frac{2\sqrt{8}}{3}(2)^{\frac{3}{2}} - (2)^2$$

A = 1.33 square units

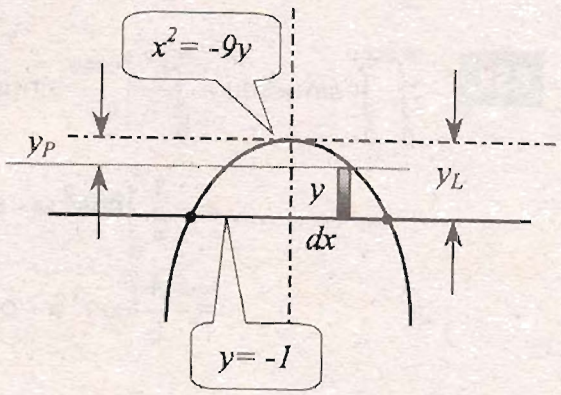
655.

$x^2 = -9y$

$x^2 = -9(-1)$

$x^2 = 9$

$x = \pm 3$



$$A = 2\int_0^3 ydx = 2\int_0^3 -(y_L - y_P)dx$$

$$= -2\int_0^3 \left[-1-\left(\frac{x^2}{9}\right)\right]dx$$

$$= -2\int_0^3 \left(\frac{x^2}{9}-1\right)dx = -2\left[\frac{(x)^3}{27}-x\right]\Bigg|_0^3$$

$$A = -2\left[\frac{(3)^3}{27}-3\right] = -2(-2) = 4 \text{ square units}$$

656. $y^2 = x$

$y^2 = (4)$

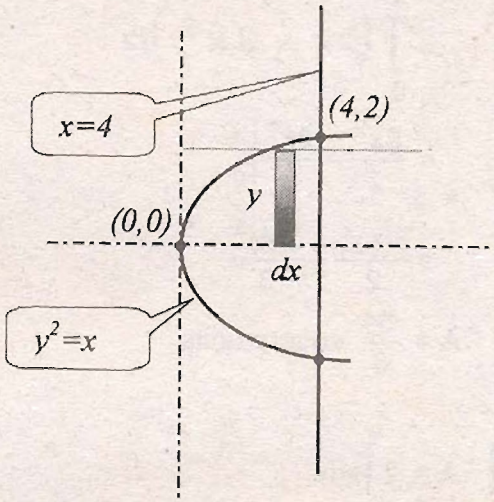
$y = \pm 2$

$$A = 2\int_0^4 y\,dx$$

$$= 2\int_0^4 \sqrt{x}\,dx$$

$$= \frac{2(x)^{\frac{3}{2}}}{\frac{3}{2}} = \frac{4}{3}(x)^{\frac{3}{2}}\Bigg|_0^4$$

$$A = \frac{4}{3}(4)^{\frac{3}{2}} = \frac{32}{3} \text{ square units}$$



657.

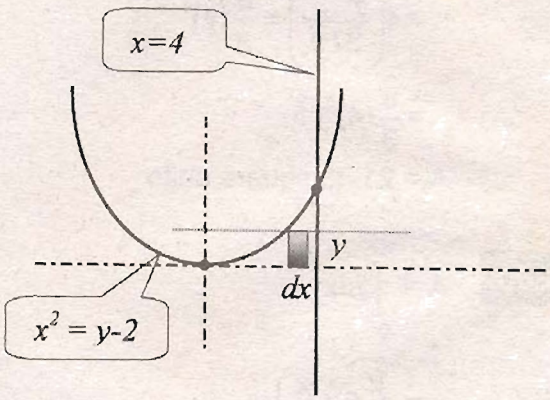
$$A = \int_0^4 y\,dx$$

$$= \int_0^4 (x^2+2)dx$$

$$= \frac{x^3}{3}+2x\Big|_0^4$$

$$= \frac{(4)^3}{3}+2(4)$$

$$A = \frac{88}{3} \text{ square units}$$



658.

$y = 6x - x^2$

$x^2 - 6x = -y$

$(x-3)^2 = -y + 9$

$(x-3)^2 = -(y-9)$

Thus the vertex is at (3,9)

$y = x^2 - 2x$

$x^2 - 2x = y$

$(x-1)^2 = y + 1$

Thus, the vertex is at (1,-1)

$$A = \int_0^4 y\,dx = \int_0^4 (y_{P1} - y_{P2})\,dx$$

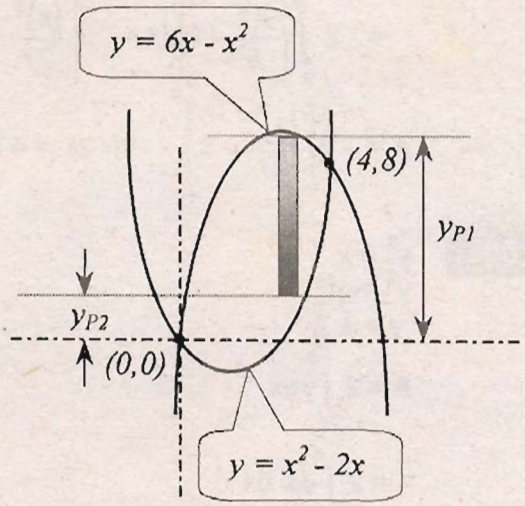
$$= \int_0^4 \left[\left(6x - x^2\right) - \left(x^2 - 2x\right)\right] dx$$

$$= \int_0^4 \left(8x - 2x^2\right) dx$$

$$= \frac{8x^2}{2} - \frac{2x^3}{3}\Bigg|_0^4$$

$$= \frac{8(4)^2}{2} - \frac{2(4)^3}{3}$$

$$A = \frac{64}{3} \text{ square units}$$



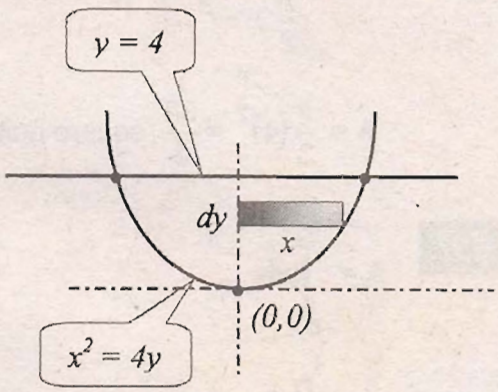
659. $$A = 2\int_0^4 x\,dy$$

$$= 2\int_0^4 (\sqrt{4y}\,dy = 4\int_0^4 y^{1/2}\,dy$$

$$= 4\left(\frac{y^{3/2}}{3/2}\right) = \frac{8}{3}(y)^{3/2}\Bigg|_0^4$$

$$= \frac{8}{3}(4)^{3/2}$$

$$A = 21.33 \text{ square units}$$



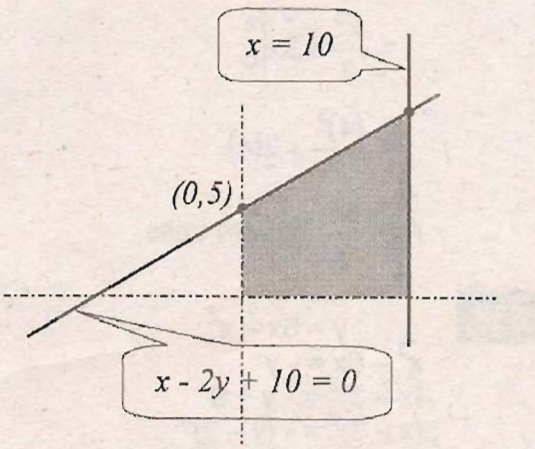
660. $$A = \int_0^{10} y\,dx$$

$$= \int_0^{10} \left(5 + \frac{x}{2}\right) dx$$

$$= 5x + \frac{x^2}{2(2)}\Bigg|_0^{10}$$

$$= 5(10) + \frac{(10)^2}{4}$$

$$A = 75 \text{ square units}$$



Another solution, use the formula of a trapezoid:

$$A = \frac{1}{2}(a+b)h = \frac{1}{2}(10+5)(10) = 75 \text{ square units}$$

661. $y^2 = 4x$

$$x = \frac{y^2}{4} \quad ☞ ❶$$

$x^2 = 4y$ ☞ ❷

Substitute (1) in (2):

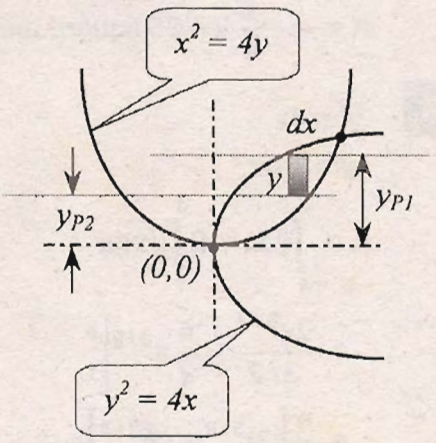
$$\left(\frac{y^2}{4}\right)^2 = 4y$$

$$\frac{y^4}{16} = 4y$$

$$y^3 = 64$$

$$y = 4$$

$$x = \frac{y^2}{4} = \frac{(4)^2}{4} = 4$$



Thus the parabolas will intersect at (0,0) and (4,4)

$$A = \int_0^4 y\,dx = \int_0^4 (y_{P1} - y_{P2})\,dx = \int_0^4 \left(\sqrt{4x} - \frac{x^2}{4}\right) dx$$

$$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = \frac{2x^{3/2}}{3/2} - \frac{x^3}{4(3)}\Bigg|_0^4 = \frac{4}{3}(4)^{3/2} - \frac{(4)^3}{12}$$

A = 5.33 square units

662. $x^2 = -8y - 16$

$x^2 = -8(y+2)$

Thus, vertex is at (0,-2)

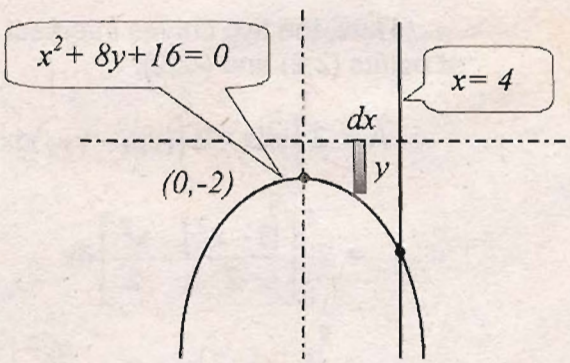
$$A = \int_0^4 -y\,dx$$

Note:

$x^2 + 8y + 16 = 0$

$-8y = x^2 + 16$

$$-y = \frac{x^2}{8} + 2$$



$$A = \int_0^4 \left(\frac{x^2}{8} + 2\right) dx = \frac{x^3}{3(8)} + 2x\Bigg|_0^4$$

$$A = \frac{(4)^3}{24} + 2(4) = 10.67 \text{ square units}$$

663. $A = \int_{-2}^{0} -x^3 dy + \int_{0}^{1} x^3 dy$

$$= \frac{-x^4}{4}\Big|_{-2}^{0} \quad \frac{x^4}{4}\Big|_{0}^{1}$$

$$= -\frac{(0)^4}{4} + \frac{(-2)^4}{4} + \frac{(1)^4}{4} - \frac{(0)^4}{4}$$

$A = 4 + \frac{1}{4} = 4.25$ square units

664. $y^2 = 4x$

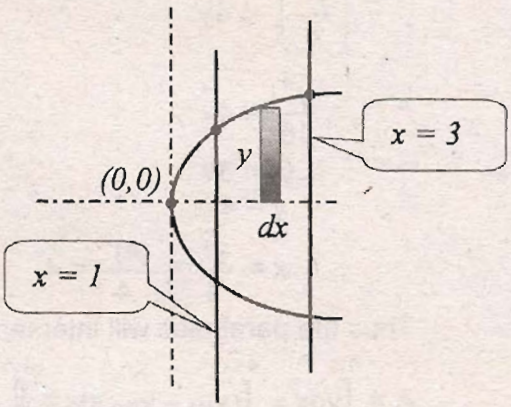
$y = 2\sqrt{x}$

$$A = \int_{1}^{3} y dx = 2\int_{1}^{3} \sqrt{x} dx$$

$$= \frac{2x^{3/2}}{3/2} = \frac{4}{3}x^{3/2}\Big|_{1}^{3}$$

$$= \frac{4}{3}\left[(3)^{3/2} - (1)^{3/2}\right]$$

$A = 5.595$ square units



665. $x^2 + 2y - 8 = 0$

$x^2 = -2y + 8$ ☞ ❶

$x^2 = -2(y - 4)$, thus vertex is at (2,4)

$x^2 = 2y$ ☞ ❷

Equate (1) to (2):

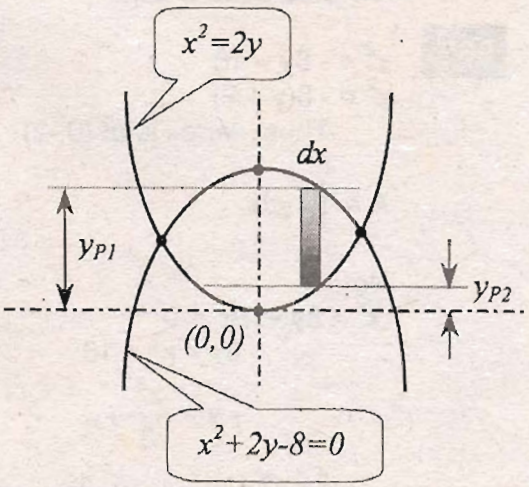
$-2y + 8 = 2y$

$y = 2$

$x^2 = 2(2) = 4$

$x = \pm 2$

Thus, the two curves intersect at points (2,2) and (-2,2).



$$A = 2\int_{0}^{2} y dx = 2\int_{0}^{2} (y_{P1} - y_{P2}) dx$$

$$= 2\int_{0}^{2} \left[\frac{\left(8 - x^2\right)}{2} - \frac{x^2}{2}\right] dx$$

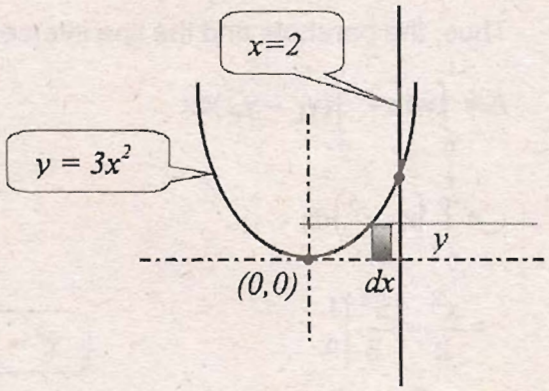
$$= \int_{0}^{2} \left(8 - 2x^2\right) dx = 8x - \frac{2x^3}{3}\Big|_{0}^{2}$$

$$A = 8(2) - \frac{2(2)^3}{3} = 10.7 \text{ square units}$$

666. $A = \int_0^2 y\,dx = 3\int_0^2 x^2 dx$

$= 3\frac{x^3}{3} = x^3\Big|_0^2$

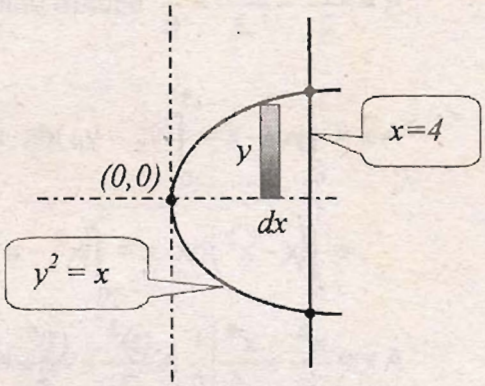
$A = (2)^3 = 8$ square units



667. $A = 2\int_0^4 y\,dx = 2\int_0^4 \sqrt{x}\,dx$

$= 2\frac{x^{3/2}}{3/2} = \frac{4}{3}x^{3/2}\Big|_0^4$

$A = \frac{4}{3}(4)^{3/2} = \frac{32}{3}$ square units



668. $r^2 = a^2 \cos 2\theta$

θ	r
0°	±a
30°	±0.707 a
45°	0
60°	i
90°	i
120°	i
150°	±0.707a
180°	±a

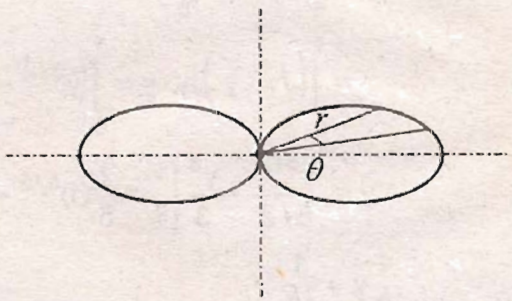
Note: Differential area of a sector

$dA = \frac{1}{2}r^2 d\theta$

$A = 4\int_0^{45°} \frac{1}{2}r^2 d\theta = 2\int_0^{45°} r^2 d\theta$

$= 2\left(\frac{1}{2}\right)\int_0^{45°} a^2 \cos 2\theta(2d\theta)$

$A = a^2 \sin 2\theta\Big|_0^{45°} = a^2 \sin[(2)(45°)] = a^2$ square units



669. $x^2 = y$ ☞ ❶

$y = x$ ☞ ❷

Equate (1) to (2):

$x^2 = x$

$x = 1;\ y = 1$

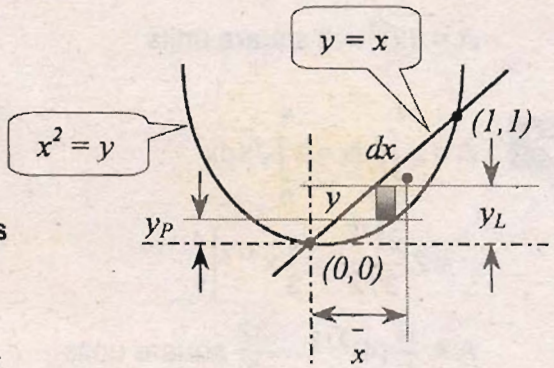
Thus, the parabola and the line intersect at points (0,0) and (1,1)

$$A = \int_0^1 y\,dx = \int_0^1 (y_L - y_P)\,dx$$

$$= \int_0^1 \left(x - x^2\right)dx$$

$$= \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1$$

$$A = \frac{(1)^2}{2} - \frac{(1)^3}{3} = \frac{1}{6} \text{ square units}$$



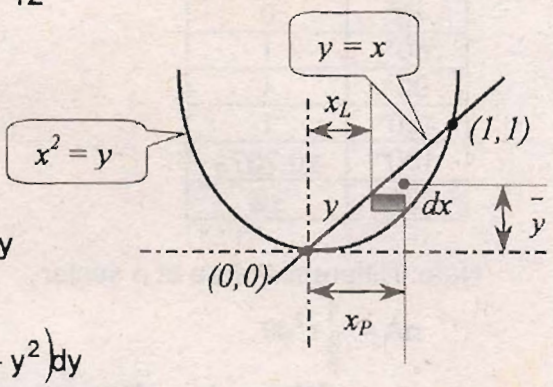
$$A\bar{x} = \int_0^1 y\,dx \cdot x = \int_0^1 (y_L - y_P)\,dx \cdot x$$

$$= \int_0^1 \left(x - x^2\right)dx \cdot x = \int_0^1 \left(x^2 - x^3\right)dx$$

$$A\bar{x} = \frac{x^3}{3} - \frac{x^4}{4}\Big|_0^1 = \frac{(1)^3}{3} - \frac{(1)^4}{4} = \frac{1}{12}$$

$$\frac{1}{6}\bar{x} = \frac{1}{12}$$

$$\bar{x} = 0.5$$



$$A\bar{y} = \int_0^1 x\,dy \cdot y = \int_0^1 (x_P - x_L)\,dy \cdot y$$

$$= \int_0^1 \left(\sqrt{y} - y\right)dy \cdot y = \int_0^1 \left(y^{3/2} - y^2\right)dy$$

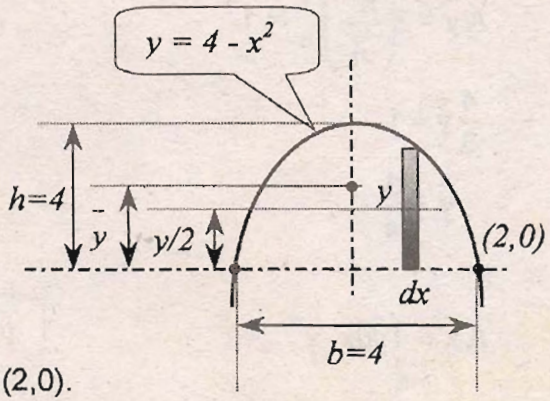
$$A\bar{y} = \frac{y^{5/2}}{5/2} - \frac{y^3}{3}\Big|_0^1 = \frac{2}{5}(1)^{5/2} - \frac{1}{3}(1)^3 = \frac{1}{15}$$

$$\frac{1}{6}\bar{y} = \frac{1}{15}$$

$$\bar{y} = 0.4$$

Thus, the coordinates of the center is at (0.5, 0.4)

670. $y = 4 - x^2$
$x^2 = -y + 4$
$x^2 = -(y - 4)$
Thus the vertex is at (0,4).



at y = 0
$y = 4 - x^2$
$0 = 4 - x^2$
$x = \pm 2$
Thus the parabola intersects the x – axis at points (-2,0) and (2,0).

Solving for the area of the parabola:

$$A = \frac{2}{3}bh$$

where: b = 4 and h = 4

$$A = \frac{2}{3}(4)(4) = 10.667 \text{ square units}$$

By inspection, $\bar{x} = 0$.

$$A\bar{y} = 2\int_0^2 ydx \cdot \frac{y}{2} = \int_0^2 y^2 dx = \int_0^2 (4 - x^2)^2 dx$$

$$= \int_0^2 (16 - 8x^2 + x^4)dx = 16x - \frac{8x^3}{3} + \frac{x^5}{5}\Big|_0^2$$

$$= 16(2) - \frac{8}{3}(2)^3 + \frac{1}{5}(2)^5 = 17.067$$

$(10.667)\ \bar{y} = 17.067$

$\bar{y} = 1.6$

Thus, the center is at point (0,1.6)

671. $y^2 = 4x$
at x = 1
$y^2 = 4(1)$
$y = \pm 2$
Thus, the parabola and the line intersect at points (1,2) and (1,-2).

Solving for area:

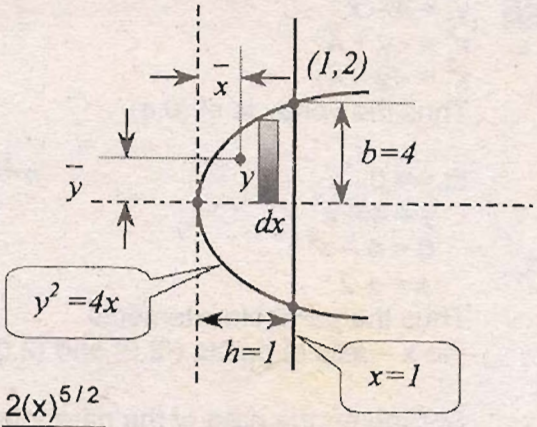
$$A = \left[\frac{2}{3}bh\right] = \left(\frac{2}{3}\right)(2)(1) = \frac{4}{3} \text{ square units}$$

$$A\bar{y} = \int_0^1 ydx \cdot \frac{y}{2} = \frac{1}{2}\int_0^1 y^2 dx = \frac{1}{2}\int_0^1 (4x)dx = 2\int_0^1 xdx$$

$$A\bar{y} = 2\left(\frac{x^2}{2}\right)\Big|_0^1 = 1$$

$$\frac{4}{3}\bar{y} = 1$$

$$\bar{y} = \frac{3}{4}$$



$$A\bar{x} = \int_0^1 y\,dx \cdot x$$

$$= \int_0^1 \left(\sqrt{4x}\ dx \cdot x\right) = 2\int_0^1 x^{\frac{3}{2}}dx = \frac{2(x)^{5/2}}{5/2}$$

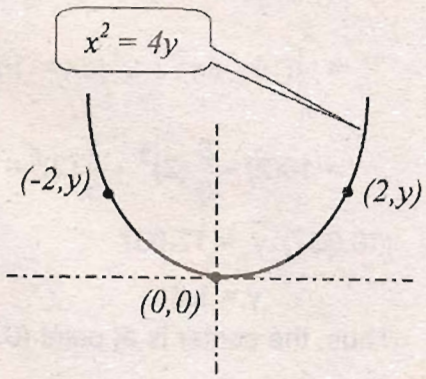
$$A\bar{x} = \frac{4}{5}x^{5/2}\Big|_0^1 = \frac{4}{5}(1)^{5/2} = \frac{4}{5}$$

$$\left(\frac{4}{5}\right)\bar{x} = \frac{4}{5}$$

$$\bar{x} = \frac{3}{5}$$

Thus, the center is at point (3/5, 3/4).

672. $$S = 2\int_0^2 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$



$$x^2 = 4y$$

$$2x\,dx = 4\,dy$$

$$\frac{dy}{dx} = \frac{x}{2}$$

$$S = 2\int_0^2 \sqrt{1+\frac{x^2}{4}}\,dx = 2\int_0^2 \sqrt{\frac{4+x^2}{4}}\,dx$$

$$S = \int_0^2 \sqrt{4+x^2}\,dx$$

Let: $x = 2\tan\theta$ Limits: $x = 0;\ \theta = 0$

$dx = 2\sec^2\theta\,d\theta$ $x = 2;\ \theta = 45°$

$$S = \int_0^{45°} \sqrt{4+4\tan^2\theta}\ (2\sec^2\theta\,d\theta) = \int_0^{45°} \sqrt{4(1+\tan^2\theta)}\ (2\sec^2\theta\,d\theta)$$

$$= \int_0^{45°} \sqrt{4(\sec^2\theta)}\ (2\sec^2\theta\,d\theta) = \int_0^{45°} 4\sec^3\theta\,d\theta$$

$$S = 4\left(\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln(\sec\theta + \tan\theta)\right)\Big|_0^{45°}$$

$$S = 4\left(\frac{1}{2}\sec 45°\tan 45° + \frac{1}{2}\ln(\sec 45° + \tan 45°)\right) = 4.6 \text{ unit length}$$

673.

$A = \int dS \cdot 2\pi d$

$A = \int\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \cdot 2\pi x$

$y = x^2$

$dy = 2x\,dx$

$\frac{dy}{dx} = 2x$

$A = 2\pi\int\sqrt{1+(2x)^2}\,xdx$

$A = 2\pi\int_0^1\left(1+4x^2\right)^{1/2}xdx$

$A = \frac{2\pi}{8}\int_0^1\left(1+4x^2\right)^{1/2}8xdx$

$A = \frac{\pi}{4}\frac{\left(1+4x^2\right)^{3/2}}{\frac{3}{2}}\Big|_0^1$

$A = \frac{\pi}{6}\left\{\left[1+4(1)^2\right]^{3/2} - \left[1+4(0)^2\right]^{3/2}\right\}$

$A = 5.33$

674. Note: Standard equation of an ellipse with center at (0,0) and with major-axis horizontal.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

By inspection:

$a = 3$ and $b = 2$

$A = \pi ab$

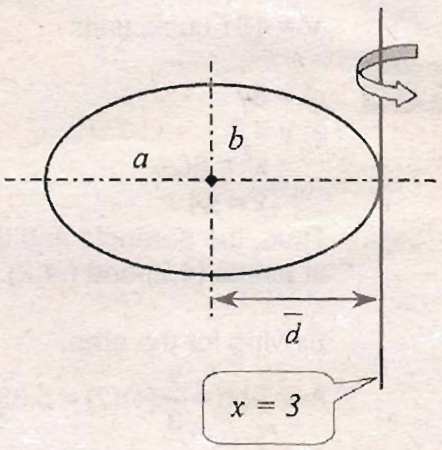
$A = \pi(3)(2) = 6\pi$

Using the second proposition of Pappus:

$V = A \cdot 2\pi\bar{d}$

$= 6\pi \cdot 2\pi(3)$

$V = 355.3$ cubic units



675. $x^2 + y^2 = 36 = (6)^2$

Standard equation of a circle with center at the origin: $x^2 + y^2 = r^2$

By inspection, $r = 6$.

Let: $\bar{y}$ = distance of the center of a gravity of the quarter circle from the x-axis

$\bar{y} = \dfrac{4r}{3\pi}$ ☞ *Formula!*

$\bar{y} = \dfrac{4(6)}{3\pi} = 2.546$

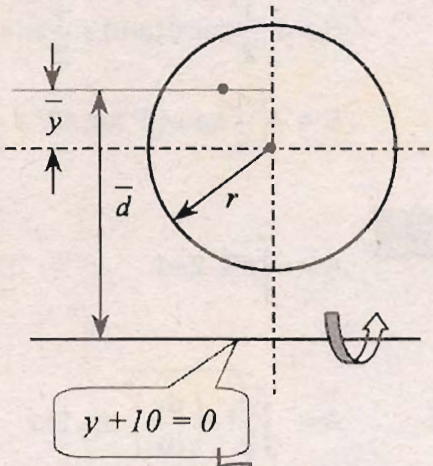
Using the second proposition of Pappus:

$V = A \cdot 2\pi\, \bar{d}$

$= \dfrac{1}{4}(\pi r^2) \cdot 2\pi(10 + \bar{y})$

$= \left(\dfrac{\pi(6)^2}{4}\right)(2\pi)(10 + 2.546)$

$V = 2228.83$ cubic units



676. $y^2 = 12x$

at $x = 3$

$y^2 = 12(3)$

$y = \pm 6$

Thus, the parabola and the line intersect at points (3,6) and (3,-6).

Solving for the area:

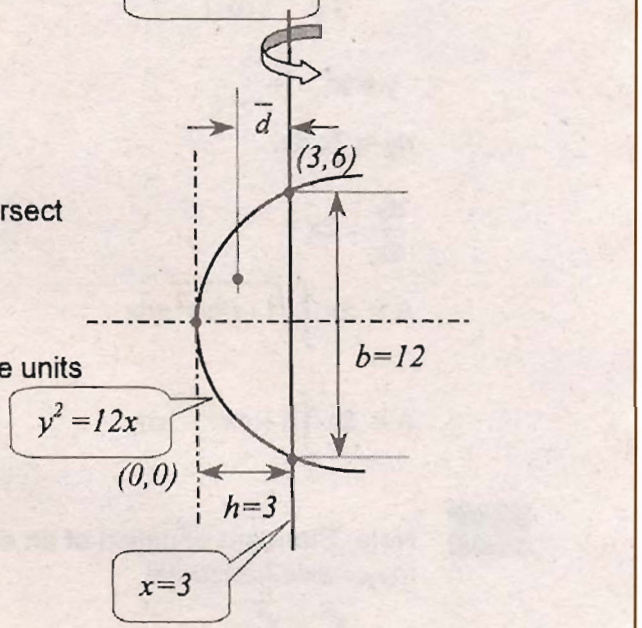
$A = \dfrac{2}{3}bh = \dfrac{2}{3}(12)(3) = 24$ square units

$\bar{d} = \dfrac{2}{5}h$ ☞ *Formula!*

$\bar{d} = \dfrac{2}{5}(3) = 1.2$

$V = A \cdot 2\pi\, \bar{d} = 24.2\pi(1.2)$

$V = 181$ cubic units



677. $x^2 = 8y$

at $y = 2$

$x^2 = 8(2)$

$x = \pm 4$

Thus, the parabola and the line intersect at points (4,2) and (-4,2).

Solving for the area:

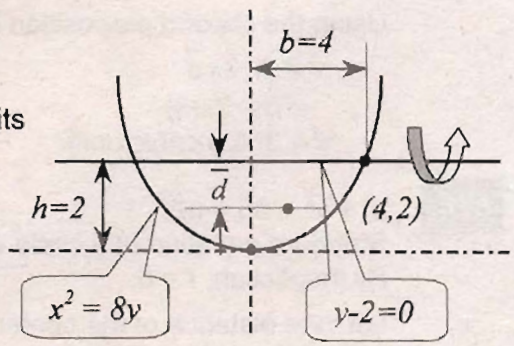
$A = \dfrac{2}{3}bh = \dfrac{2}{3}(4)(2) = 5.333$ square units

$\bar{d} = \dfrac{2}{5}h$ ☞ *Formula!*

$\bar{d} = \dfrac{2}{5}(2) = 0.8$

$V = A \cdot 2\pi\, \bar{d} = (5.333)(2\pi)(0.8)$

$V = 26.81$ cubic units



678. $x^2 + y^2 + 6x + 4y + 12 = 0$

$x^2 + 6x + y^2 + 4y = -12$

By completing square:

$(x + 3)^2 + (y + 2)^2 = -12 + 9 + 4$

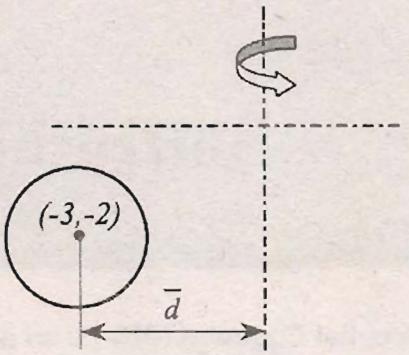
$(x + 3)^2 + (y + 2)^2 = 1$

Thus, the center of circle is at (-3,-2) and the radius is 1.

By inspection, $\bar{d} = 3$

$V = A \cdot 2\pi \bar{d} = \pi r^2 \cdot 2\pi \bar{d} = \pi(1)^2(2\pi)(3)$

$V = 59.22$ cubic units



679. $x^2 = 8y$

at $x = 4$

$(4)^2 = 8y$

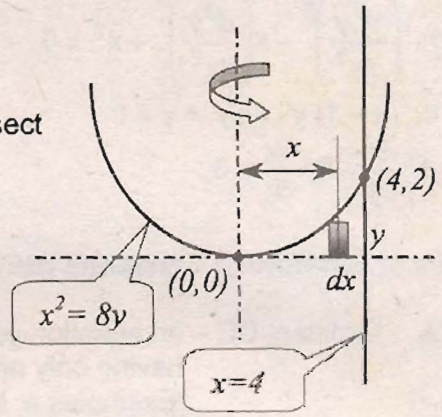
$y = 2$

Thus, the parabola and the line intersect at point (4,2).

$$V = \int_0^4 y\,dx \cdot 2\pi x = \int_0^4 \frac{x^2}{8} \cdot 2\pi x\,dx$$

$$= \frac{\pi}{4}\int_0^4 x^3 dx = \frac{\pi}{4}\frac{x^4}{4} = \frac{\pi}{16}x^4\Big|_0^4$$

$$V = \frac{\pi(4)^4}{16} = 50.265 \text{ cubic units}$$



680. $y^2 = 4x$

at $x = 1$

$y^2 = 4(1)$

$y = \pm 2$

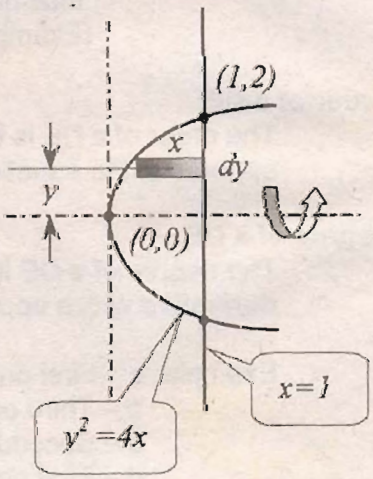
Thus, the parabola and the line intersect at points (1,2) and (1,-2).

$$I_X = 2\int_0^2 y^2(x\,dy) = 2\int_0^2 y^2(y_L - y_P)dy$$

$$= 2\int_0^2 y^2\left(1 - \frac{y^2}{4}\right)dy = 2\int_0^2\left(y^2 - \frac{y^4}{4}\right)dy$$

$$= 2\left(\frac{y^3}{3} - \frac{y^5}{4(5)}\right)\Bigg|_0^2$$

$$I_X = 2\left(\frac{(2)^3}{3} - \frac{(2)^5}{4(20)}\right) = 2.13$$



DAY 16

DIFFERENTIAL EQUATIONS

Differential Equation (DE)– is an equation containing at least one derivative.

Examples:

a. $(2xy + x^2)\,dx + 2y\,dy = 0$

b. $\left(\frac{d^3y}{dx^3}\right)^2 - x\left(\frac{d^2y}{dx^2}\right)^3 + x^3 = 0$

c. $(x - 1)\,y'' + xy' + y = 0$

d. $\frac{\partial x}{\partial y} + 2y\frac{\partial x}{\partial z} = 3$

Types of Differential Equations (DE):

a. Ordinary DE – an equation containing only one independent variable, thus having only **ordinary derivatives** in the equation. (Examples a, b & c)

b. Partial DE – an equation containing two or more independent variables, thus having **partial derivatives** in the equation. (Example d)

Order of a DE:

The order of a DE is the **order of the highest ordered derivative** which appears in the equation.

Degree of a DE:

The degree of a DE is the **degree (exponent) of the highest ordered derivative** which appears in the equation.

Example: a – First order, First degree
b – Third order, Second degree
c – Second order, First degree
d – First order, First degree

Types of solutions of a DE:

a. General solution – the solution has **at least one** arbitrary constant.

Example: $x^3 - 3y = C$, where: C = arbitrary constant

b. Particular solution – the solution has **no** arbitrary constant.

Example: $2xy + y^2 - 2 = 0$

Solutions to first order DE:

a. Variable separable type

Standard form: ☞ $P(x)dx + Q(y)dy = 0$

General solution: ☞ $\int P(x)dx + \int Q(y)dy = C$

b. Homogeneous type

Standard form: ☞ $M(x,y)dx + N(x,y)dy = 0$

General solution: ☞ Substitute $y = vx$ or $x = vy$ and the resulting DE becomes a variable separable type

Test for homogeneity of degree n:
Let: $f(x,y) = M(x,y)\,dx + N(x,y)\,dy$
$f(rx, ry) = r^n f(x,y)$

c. Exact type

Standard form: ☞ $M(x,y)dx + N(x,y)dy = 0$

General solution: ☞ $F(x,y) = C$

where: $F(x,y) = \int M(x,y)\partial x + k_1$

$F(x,y) = \int N(x,y)\partial y + k_2$

Test for exactness: ☞ $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

d. Linear type

Standard form: ☞ $$\frac{dy}{dx} + y\,P(x) = Q(x)$$

General solution: ☞ $$y(\text{i.f.}) = \int Q(x)(\text{i.f.})\,dx + C$$

where: i.f. = integrating factor = $e^{\int P(x)dx}$

e. Bernoulli's equation type

Standard form: ☞ $$\frac{dy}{dx} + y\,P(x) = y^n\,Q(x)$$

General solution: ☞ $$y^{1-n} = \frac{1}{e^{\int (1-n)\,P(x)\,dx}} \int (1-n)\,Q(x)\,e^{\int (1-n)P(x)dx}\,dx$$

Applications of first order DE

a. Population growth problems

$$\frac{dP}{dt} = kP$$

where:

$\frac{dP}{dt}$ = rate of change of the population

P = number of inhabitants at any time t

k = constant of proportionality

b. Decay problems

$$\frac{dQ}{dt} = kQ$$

where:

$\frac{dQ}{dt}$ = rate of change of the substance

Q = amount of the substance present at any time t

k = constant of proportionality

c. Continuous compound interest problems

$$\frac{dP}{dt} = rP$$

where:

$\frac{dP}{dt}$ = rate of change of the account

P = money present in the account at any time t

r = nominal rate of interest per year

d. Flow problems

$$\frac{dQ}{dt} = \text{Rate of gain} - \text{Rate of loss}$$

where: Q = concentration of the mixture at any time t

$\frac{dQ}{dt}$ = rate of change of the concentration of the mixture

e. Cooling and heating problems

$$\frac{dT}{dt} = k(T - t_s)$$

☞ *Cooling process*

$$\frac{dT}{dt} = k(t_s - T)$$

☞ *Heating process*

where:

$\frac{dT}{dt}$ = rate of change of the body temp.

T = temperature of the body at any time t

t_s = surrounding temperature

f. Newton's second law of motion

$$F = m\frac{dV}{dt}$$

where:

F = force

m = mass of the body

$\frac{dV}{dt}$ = rate of change of velocity

g. Geometrical problems (Orthogonal trajectory)

A curve which intersects all curves of a given family at the same angles is referred to as a **trajectory**; if the intersection is at right angle, the curve is called **orthogonal trajectory**.

$$\left(\frac{dy}{dx}\right)_t = -\left(\frac{dx}{dy}\right)_g$$

where:

$\left(\frac{dy}{dx}\right)_t$ = slope of the orthogonal trajectory

$\left(\frac{dy}{dx}\right)_g$ = slope of the given family of curves

$$\left(\frac{dx}{dy}\right)_g = \frac{1}{\left(\frac{dy}{dx}\right)_g}$$



Did you know that... The binary number system using 2 as the base, which was investigated and set into serious numerical system by eminent German mathematician, Gottfried Wilhelm von Leibniz, was used by Chinese mathematicians as early as 5th century A.D.!

Proceed to the next page for your 16th test. GOODLUCK ! ☞

Test 16

Time element: 2.0 hours & 30 minutes

Problem 681:
Determine the order and degree of the differential equation

$$2x\frac{d^4y}{dy^4}+5x^2\left(\frac{dy}{dx}\right)^3-xy=0\,.$$

A. Fourth order, first degree
B. Third order, first degree
C. First order, fourth degree
D. First order, third degree

Problem 682:
Which of the following equations is an exact DE?

A. $(x^2 + 1)\,dx - xy\,dy = 0$
B. $x\,dy + (3x - 2y)\,dx = 0$
C. $2xy\,dx + (2 + x^2)\,dy = 0$
D. $x^2y\,dy - y\,dx = 0$

Problem 683:
Which of the following equations is a variable separable DE?

A. $(x + x^2y)\,dy = (2x + xy^2)\,dx$
B. $(x + y)\,dx - 2y\,dy = 0$
C. $2y\,dx = (x^2 + 1)\,dy$
D. $y^2\,dx + (2x - 3y)\,dy = 0$

Problem 684: ECE Board April 1998
The equation $y^2 = cx$ is the general solution of:

A. $y' = \frac{2y}{x}$
B. $y' = \frac{2x}{y}$
C. $y' = \frac{y}{2x}$
D. $y' = \frac{x}{2y}$

Problem 685: EE Board March 1998

Solve the differential equation: x (y – 1) dx + (x + 1) dy = 0. If y = 2 when x = 1, determine y when x = 2.

A. 1.80
B. 1.48
C. 1.55
D. 1.63

Problem 686: EE Board October 1997

If $dy = x^2dx$; what is the equation of y in terms of x if the curve passes through (1,1)?

A. $x^2 - 3y + 3 = 0$
B. $x^3 - 3y + 2 = 0$
C. $x^3 + 3y^2 + 2 = 0$
D. $2y + x^3 + 2 = 0$

Problem 687: ECE Board November 1998

Find the equation of the curve at every point of which the tangent line has a slope of 2x.

A. $x = -y^2 + C$
B. $y = -x^2 + C$
C. $y = x^2 + C$
D. $x = y^2 + C$

Problem 688: ECE Board April 1995

Solve (cos x cos y – cot x) dx – sin x sin y dy = 0

A. sin x cos y = ln (c cos x)
B. sin x cos y = ln (c sin x)
C. sin x cos y = - ln (c sin x)
D. sin x cos y = - ln (c cos x)

Problem 689: EE Board October 1997

Solve the differential equation dy –xdx = 0, if the curve passes through (1,0)?

A. $3x^2 + 2y - 3 = 0$
B. $2y + x^2 - 1 = 0$
C. $x^2 - 2y - 1 = 0$
D. $2x^2 + 2y - 2 = 0$

Problem 690: ME Board April 1996

What is the solution of the first order differential equation y(k+1) = y(k) + 5.

A. $y(k) = 4 - \frac{5}{k}$

B. $y(k) = 20 + 5k$

C. $y(k) = C - k$, where C is constant

D. The solution is non-existent for real values of y

Problem 691: EE Board April 1995

Solve $\left(y - \sqrt{x^2 + y^2}\right) dx - xdy = 0$

A. $\sqrt{x^2 + y^2} + y = C$

B. $\sqrt{x^2 + y^2 + y} = C$

C. $\sqrt{x + y} + y = C$

D. $\sqrt{x^2 - y} + y = C$

Problem 692: ECE Board November 1994

Find the differential equation whose general solution is $y = C_1x + C_2e^x$.

A. $(x - 1)\, y'' - xy' + y = 0$

B. $(x + 1)\, y'' - xy' + y = 0$

C. $(x - 1)\, y'' + xy' + y = 0$

D. $(x + 1)\, y'' + xy' + y = 0$

Problem 693: EE Board October 1995

Find the general solution of $y' = y \sec x$

A. $y = C (\sec x + \tan x)$

B. $y = C (\sec x - \tan x)$

C. $y = C \sec x \tan x$

D. $y = C (\sec^2 x \tan x)$

Problem 694: EE Board April 1996

Solve $xy' (2y - 1) = y (1 - x)$

A. $\ln (xy) = 2 (x - y) + C$

B. $\ln (xy) = x - 2y + C$

C. $\ln (xy) = 2y - x + C$

D. $\ln (xy) = x + 2y + C$

Problem 695: EE Board April 1996

Solve $(x + y)\,dy = (x - y)\,dx$.

A. $x^2 + y^2 = C$
B. $x^2 + 2xy + y^2 = C$
C. $x^2 - 2xy - y^2 = C$
D. $x^2 - 2xy + y^2 = C$

Problem 696:

Solve the linear equation: $\frac{dy}{dx} + \frac{y}{x} = x^2$

A. $xy^2 = \frac{x^3}{4} + C$

B. $xy = \frac{x^4}{4} + C$

C. $x^2y = \frac{x^4}{4} + C$

D. $y = \frac{x^3}{4} + C$

Problem 697: CE Board May 1997

Find the differential equations of the family of lines passing through the origin.

A. $ydx - xdy = 0$
B. $xdy - ydx = 0$
C. $xdx + ydy = 0$
D. $ydx + xdy = 0$

Problem 698: CE Board May 1996

What is the differential equation of the family of parabolas having their vertices at the origin and their foci on the x-axis.

A. $2xdy - ydx = 0$
B. $xdy + ydx = 0$
C. $2ydx - xdy = 0$
D. $\frac{dy}{dx} - x = 0$

Problem 699: CE Board November 1995

Determine the differential equation of the family of lines passing through (h, k).

A. $(y-k)dx-(x-h)dy=0$

B. $(y-h)+(y-k)=\dfrac{dy}{dx}$

C. $(x-h)dx-(y-k)dy=0$

D. $(x+h)dx-(y-k)dy=0$

Problem 700:

Determine the differential equation of the family of circles with center on the y-axis.

A. $(y")^3 - xy" + y' = 0$
B. $y" - xyy' = 0$
C. $xy" - (y')^3 - y' = 0$
D. $(y')^3 + (y")^2 + xy = 0$

Problem 701: EE Board April 1997

Radium decomposes at a rate proportional to the amount at any instant. In 100 years, 100 mg of radium decomposes to 96 mg. How many mg will be left after 100 years?

A. 88.60
B. 95.32
C. 92.16
D. 90.72

Problem 702:

The population of a country doubles in 50 years. How many years will it be five times as much? Assume that the rate of increase is proportional to the number of inhabitants.

A. 100 years
B. 116 years
C. 120 years
D. 98 years

Problem 703:

Radium decomposes at a rate proportional to the amount present. If half of the original amount disappears after 1000 years, what is the percentage lost in 100 years?

A. 6.70%
B. 4.50 %
C. 5.36 %
D. 4.30%

Problem 704: ECE Board November 1998

Find the equation of the family of orthogonal trajectories of the system of parabolas $y^2 = 2x + C$.

A. $y = Ce^{-x}$

B. $y = Ce^{2x}$

C. $y = Ce^{x}$

D. $y = Ce^{-2x}$

Problem 705:

According to Newton's law of cooling, the rate at which a substance cools in air is directly proportional to the difference between the temperature of the substance and that of air. If the temperature of the air is 30° and the substance cools from 100° to 70° in 15 minutes, how long will it take to cool 100° to 50°?

A. 33.59 min.
B. 43.50 min
C. 35.39 min
D. 45.30 min

Problem 706:

An object falls from rest in a medium offering a resistance. The velocity of the object before the object reaches the ground is given by the differential equation dV/dt + V/10 = 32, ft/sec. What is the velocity of the object one second after it falls?

A. 40.54
B. 38.65
C. 30.45
D. 34.12

Problem 707:

In a tank are 100 liters of brine containing 50 kg. total of dissolved salt. Pure water is allowed to run into the tank at the rate of 3 liters a minute. Brine runs out of the tank at the rate of 2 liters a minute. The instantaneous concentration in the tank is kept uniform by stirring. How much salt is in the tank at the end of one hour?

A. 15.45 kg.
B. 19.53 kg.
C. 12.62 kg.
D. 20.62 kg.

Problem 708

A tank initially holds 100 gallons of salt solution in which 50 lbs of salt has been dissolved. A pipe fills the tank with brine at the rate of 3 gpm, containing 2 lbs of dissolved salt per gallon. Assuming that the mixture is kept uniform by stirring, a drain pipe draws out of the tank the mixture at 2 gpm. Find the amount of salt in the tank at the end of 30 minutes.

A. 171.24 lbs.
B. 124.11 lbs.
C. 143.25 lbs.
D. 105.12 lbs.

Problem 709: ME Board April 1998

If the nominal interest rate is 3%, how much is P5,000 worth in 10 years in a continuously compounded account?

A. P5,750
B. P6,750
C. P7,500
D. P6,350

Problem 710: ME Board October 1997

A nominal interest of 3% compounded continuously is given on the account. What is the accumulated amount of P10,000 after 10 years?

A. P13,620.10
B. P13,500.10
C. P13,650.20
D. P13,498.60

ANSWER KEY

681. A	691. A	701. C
682. C	692. A	702. B
683. C	693. A	703. A
684. C	694. D	704. A
685. C	695. C	705. A
686. B	696. B	706. C
687. C	697. B	707. B
688. B	698. A	708. A
689. C	699. A	709. B
690. B	700. C	710. D

RATING

- ☐ 25-30 Topnotcher
- ☐ 18-24 Passer
- ☐ 15-17 Conditional
- ☐ 0-14 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 16

681. Fourth order, since the highest derivative in the equation is 4th derivative.
First degree, since the exponent or power of the 4th derivative is 1.

682. A DE whose equation is M(x,y)dx + N(x,y)dy = 0 is exact if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Note: From the choices, choice (c) is an exact DE.

$$2xydx + (2 + x^2)dy = 0$$

$M = 2xy$ $\qquad$ $N = 2 + x^2$

$$\frac{\partial(2xy)}{\partial y} = 2x \qquad \frac{\partial(2 + x^2)}{\partial x} = 2x$$

683. A DE is a variable separable if the coefficient of dx shall be functions of x only and the coefficient of dy shall be functions of y only.

Note: From the choices, choice (c) is a variable separable DE

$$2y\,dx = (x^2 + 1)\,dy$$

$$\frac{2ydx}{y(x^2+1)} = \frac{(x^2+1)dy}{y(x^2+1)}$$

$$\left(\frac{2}{x^2+1}\right)dx = \left(\frac{1}{y}\right)dy$$

684. $y^2 = cx$

$$c = \frac{y^2}{x}$$

Differentiate:

$$0 = \frac{x(2yy') - y^2}{x^2}$$

$$y^2 = 2xyy'$$

$$y' = \frac{y^2}{2xy} = \frac{y}{2x}$$

685. $x(y - 1)\,dx + (x + 1)\,dy = 0$

$$\frac{xdx}{x+1} + \frac{dy}{y-1} = 0$$

$$\left(1 - \frac{1}{x+1}\right)dx + \frac{dy}{y-1} = 0$$

Integrate:

$$\int dx - \int\frac{dx}{x+1} + \int\frac{dy}{y-1} = \int 0$$

$$x - \ln(x+1) + \ln(y-1) = C$$

at $x = 1; y = 2$

$$1 - \ln(1+1) + \ln(2-1) = C$$

$$C = 0.307$$

at $x = 2; y = ?$

$$2 - \ln(2+1) + \ln(y-1) = 0.307$$

$$\ln(y-1) = -0.594$$

$$y - 1 = e^{-0.594}$$

$$y = 1.55$$

686. $dy = x^2\,dx$

$$\int dy = \int x^2 dx$$

$$y = \frac{x^3}{3} + C$$

at $x = 1; y = 1$

$$1 = \frac{(1)^3}{3} + C\,;\; C = \frac{2}{3}$$

Thus the solution is,

$$y = \frac{x^3}{3} + \frac{2}{3}$$

$$3y = x^3 + 2$$

$$x^3 - 3y + 2 = 0$$

687. $\text{slope} = \dfrac{dy}{dx} = 2x$

Note: Since tangent, the slope of the unknown curve is equal to the slope of the line

$$\frac{dy}{dx} = 2x\,;\; dy = 2x\,dx$$

$$\int dy = 2\int x dx$$

$$y = \frac{2x^2}{2} + C$$

$$y = x^2 + C$$

688. $(\cos x \cos y - \cot x)\,dx - \sin x \sin y\,dy = 0$

$M = \cos x \cos y - \cot x$ $\qquad$ $N = -\sin x \sin y$

$$\frac{\partial(\cos x \cos y - \cot x)}{\partial y} = -\cos x \sin y \qquad \frac{\partial(-\sin x \sin y)}{\partial x} = -\cos x \sin y$$

Note: $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$, then the given DE is an exact DE.

$$\int(\cos x \cos y - \cot x)\partial x + k_1 = C$$

$$\sin x \cos y - \ln \sin x + k_1 = C \quad \text{☞ ❶}$$

$$\int(-\sin x \sin y)\partial y + k_2 = C$$

$$-\sin x(-\cos y) + k_2 = C$$

$$\sin x \cos y + k_2 = C \quad \text{☞ ❷}$$

By comparing (1) and (2), $k_1 = 0$ and $k_2 = -\ln \sin x$

Thus, the solution is,

$$\sin x \cos y - \ln \sin x = C$$
$$\sin x \cos y = \ln \sin x + C$$
$$= \ln \sin x + \ln c \text{, where: } C = \ln c$$
$$= \ln (c \sin x)$$
$$\sin x \cos y = \ln (c \sin x)$$

689. $dy - xdx = 0$

$$dy = xdx$$

$$\int dy = \int xdx$$

$$y = \frac{x^2}{2} + C$$

at $x = 1; y = 0$

$$0 = \frac{(1)^2}{2} + C;\ C = -\frac{1}{2}$$

Thus the solution is,

$$y = \frac{x^2}{2} - \frac{1}{2}$$
$$2y = x^2 - 1$$
$$x^2 - 2y - 1 = 0$$

690. Assume the solution has the form: $y(k) = 20 + 5k$

Solve $y(k + 1)$ using the assumed solution:

$$y(k + 1) = 20 + 5(k + 1)$$
$$= 20 + 5k + 5$$
$$= (20 + 5k) + 5$$
$$y(k + 1) = y(k) + 5$$

Since, the resulting equation is the same to the given DE, then the assumed equation is the solution to the DE.

691. $\left(y-\sqrt{x^2+y^2}\right)dx - xdy = 0$

By inspection, this equation is a homogeneous DE, since the coefficient of the dx and dy are of the same degree (degree 1).

Let: $y = vx$; $dy = vdx + xdv$

Substitute:

$$vx\,dx - \sqrt{x^2+v^2x^2}\,dx - x(vdx + xdv) = 0$$

$$vx\,dx - \sqrt{1+v^2}\,xdx - vxdx - x^2dv = 0$$

$$-\sqrt{1+v^2}\,dx - xdv = 0$$

$$\int\frac{dx}{x} + \int\frac{dv}{\sqrt{1+v^2}} = 0$$

Let: $v = \tan\theta$; $dv = \sec^2\theta\,d\theta$

$$\int\frac{dx}{x} + \int\frac{\sec^2\theta\,d\theta}{\sqrt{1+\tan^2\theta}} = 0$$

$$\int\frac{dx}{x} + \int\sec\theta\,d\theta = 0$$

$$\ln x + \ln(\sec\theta + \tan\theta) = c$$

$$\ln[x(\sec\theta + \tan\theta)] = c$$

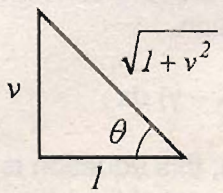
$$x(\sec\theta + \tan\theta)] = C, \text{ where: } e^C = C$$

Refer to the triangle, substitute values of $\sec\theta$ & $\tan\theta$ to the solution:

$$x\sqrt{1+v^2} + v = C$$

$$x\sqrt{1+\left(\frac{y}{x}\right)^2} + \frac{y}{x} = C$$

$$\sqrt{x^2+y^2} + y = C$$



692.

$y = C_1x + C_2e^x$ ☞ ❶

$y' = C_1 + C_2e^x$ ☞ ❷

$y'' = C_2e^x$ ☞ ❸

Substitute (3) in (2):

$y' = C_1 + y''$

$C_1 = y' - y''$ ☞ ❹

Substitute (3) in (1):

$y = C_1x + y''$ ☞ ❺

Substitute (4) in (5):

$y = (y' - y'')x + y'' = y'x - y''x + y''$

$(x-1)y'' - xy' + y = 0$

693. $y' = y \sec x$

$$\frac{dy}{dx} = y \sec x$$

$$\int \frac{dy}{y} = \int \sec x\,dx$$

$$\ln y = \ln(\sec x + \tan x) + c$$

$$\ln \frac{y}{\sec x + \tan x} = c$$

$$\frac{y}{\sec x + \tan x} = C \text{, where: } e^c = C$$

$$y = C(\sec x + \tan x)$$

694. $xy'(2y - 1) = y(1 - x)$

$$x\frac{dy}{dx}(2y-1) = y(1-x)$$

$$x(2y-1)dy = y(1-x)dx$$

$$\frac{2y-1}{y}dy - \frac{1-x}{x}dx = 0$$

$$2\int dy - \int \frac{dy}{y} - \int \frac{dx}{x} + \int dx = C$$

$$2y - \ln y - \ln x + x = C$$

$$2y + x + C = \ln x + \ln y$$

$$\ln(xy) = 2y + x + C$$

Note: C is an arbirary constant which can be placed in either side of the equal sign.

695. $(x + y)\,dy = (x - y)\,dx$

By inspection, this equation is a homogeneous DE, since the coefficient of the dx and dy are of the same degree (degree 1).

Let: $y = vx$; $dy = vdx + xdv$

Substitute:

$$x(vdx + xdv) + vx(vdx + xdv) = xdx - vxdx$$

$$vxdx + x^2dv + v^2xdx + vx^2dv = xdx - vxdx$$

$$xdx(2v + v^2 - 1) + x^2\,dv(1 + v) = 0$$

$$\frac{dx}{x} + \frac{(1+v)dv}{v^2 + 2v - 1} = 0$$

$$\int \frac{dx}{x} + \frac{1}{2}\int \frac{(2+2v)dv}{v^2 + 2v - 1} = 0$$

$$\ln x + \frac{1}{2}\ln(v^2 + 2v - 1) = c$$

Multiply 2 on both sides:

$$2\ln x + \ln(v^2 + 2v - 1) = k, \text{ where: } k = 2c$$

$$\ln [x^2 (v^2 + 2v - 1)] = k$$
$$x^2 (v^2 + 2v - 1) = C, \text{ where } C = e^k$$

Substitute $v = \frac{y}{x}$:

$$x^2 \left(\frac{y^2}{x^2} + \frac{2y}{x} - 1 \right) = C$$
$$y^2 + 2xy - x^2 = C$$
$$x^2 - 2xy - y^2 = C$$

Note: C is an arbirary constant which can be placed in either side of the equal sign.

696. $\frac{dy}{dx} + \left(\frac{1}{x}\right) y = x^2$

Note: This equation is a linear DE:

$y\,(\text{i.f.}) = \int Q(x)(\text{i.f.})\,dx + C$ ☞ *General solution*

where: $P(x) = \frac{1}{x}$; $Q(x) = x^2$

$$\text{i.f.} = e^{\int P(x)dx} = e^{\int \frac{dx}{x}} = e^{\ln x} = x$$

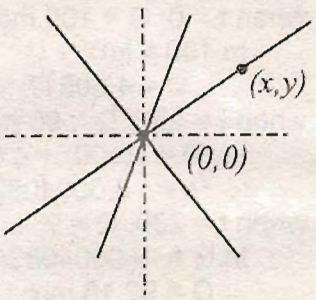
Substitute:

$$y(x) = \int x^2(x)dx + C$$
$$xy = \frac{x^4}{4} + C$$

697. $m = \frac{y}{x}$; where: m = slope (constant)

Differentiate:

$$0 = \frac{xdy - ydx}{x^2}$$
$$xdy - ydx = 0$$

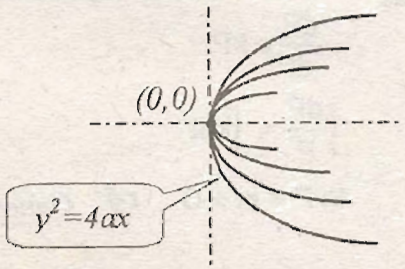


698. $y^2 = 4ax$

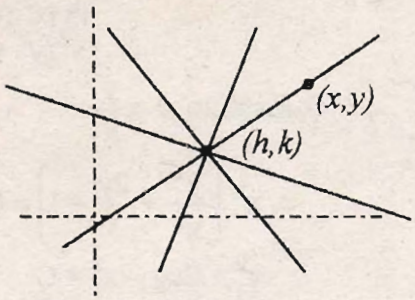
$$4a = \frac{y^2}{x}$$

Differentiate:

$$0 = \frac{x(2ydy) - y^2 dx}{x^2}$$
$$2xydy - y^2\,dx = 0$$
$$2xdy - ydx = 0$$



699. $m = \dfrac{y-k}{x-h}$; where: m = slope (constant)



Differentiate:

$$0 = \frac{(x-h)dy - (y-k)dx}{(x-h)^2}$$

$$(x-h)dy - (y-k)dx = 0$$

$$(y-k)dx - (x-h)dy = 0$$

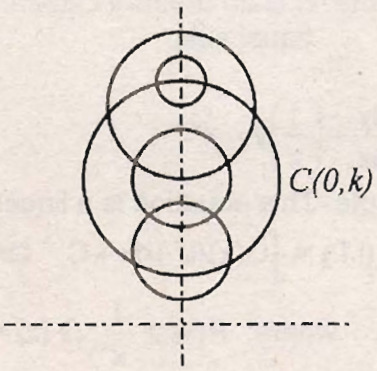
700. $x^2 + (y-k)^2 = r^2$

Differentiate:

$$2x + 2(y-k)y' = 0$$

$$x + yy' - ky' = 0$$

$$k = \frac{x}{y'} + y$$



Differentiate:

$$0 = \frac{y'(1) - xy''}{(y')^2} + y'$$

$$y' - xy'' + (y')^3 = 0$$

$$xy'' - (y')^3 - y' = 0$$

701. Let: Q = amount of substance present

$$\frac{dQ}{dt} = kQ$$

$$\int \frac{dQ}{Q} = \int k\,dt$$

$\ln Q = kt + C$ ☞ *General solution*

when t = 0; Q = 100 mg

$\ln 100 = k(0) + C$

$C = 4.60517$

when t = 100; Q = 96 mg

$\ln 96 = k(100) + 4.60517$

$k = -0.0004082$

when t = 200; Q = ?

$\ln Q = -0.0004082(200) + 4.60517$

$Q = 92.16$ mg

702. Let: P = number of population

$$\frac{dP}{dt} = kP$$

$$\int \frac{dP}{P} = \int k\,dt$$

$\ln P = kt + C$ ☞ *General solution*

when $t = 0$; $P = P_0$

$\ln P_0 = k(0) + C$

$C = \ln P_0$

when $t = 50$ years; $P = 2P_0$

$\ln 2P_0 = k(50) + \ln P_0$

$k = 0.0138629$

when $t = ?$ $Q = 5P_0$

$\ln 5P_0 = 0.0138629(t) + \ln P_0$

$t = 116$ years

703. Let: Q = amount of radium present

$$\frac{dQ}{dt} = kQ$$

$$\int \frac{dQ}{Q} = \int k\,dt$$

$\ln Q = kt + C$ ☞ *General solution*

when $t = 0$; $Q = Q_0$

$\ln Q_0 = k(0) + C$

$C = \ln Q_0$

when $t = 1000$ years; $Q = 0.5\,Q_0$

$\ln 0.5Q_0 = k(1000) + \ln Q_0$

$k = -0.000693$

when $t = 100$ years; $Q = ?$

$\ln Q = -0.000693(100) + \ln Q_0$

$e^{\ln Q} = e^{-0.0693 + \ln Q_0}$

$Q = Q_0 e^{-0.0693}$

$Q = 0.933\,Q_0$

$$\% \text{ lost} = \frac{Q_0 - 0.933Q_0}{Q_0} \times 100\% = 6.7\%$$

704. $y^2 = 2x + C$

Differentiate:

$$2y\frac{dy}{dx} = 2$$

$$\left(\frac{dy}{dx}\right)_c = \frac{1}{y}$$

Note: The slope of the orthogonal trajectories is given by:

$$\frac{dy}{dx} = -\frac{1}{\left(\frac{dy}{dx}\right)_c} = -\left(\frac{dx}{dy}\right)_c$$

Substitute:

$$\frac{dy}{dx} = -y$$

$$\int \frac{dy}{y} = -\int dx$$

$$\ln y = -x + c$$

$$e^{\ln y} = e^{-x+c} = e^{-x}e^{c}$$

$$y = Ce^{-x}, \text{ where: } C = e^{c}$$

705. Let: T = temperature of the body

$$\frac{dT}{dt} = k(T - 30°)$$

$$\int \frac{dt}{T - 30°} = \int k dt$$

$\ln (T - 30°) = kt + C$ ☞ *General solution*

when t = 0; T = 100°

$\ln (100° - 30°) = k(0) + C$

$C = 4.24849$

when t = 15 min; T = 70°

$\ln (70° - 30°) = k(15) + 4.24849$

$k = -0.0373$

when t = ?; T = 50°

$\ln (50° - 30°) = -0.0373(t) + 4.24849$

$t = 33.59$ min.

706.

$$\frac{dV}{dt} + \frac{V}{10} = 32$$

$$10\frac{dV}{dt} + V = 320$$

$$10\int \frac{-dV}{320 - V} = \int dt$$

$-10 \ln (320 - V) = t + C$ ☞ *General solution*

when t = 0; V = 0 (at rest)

$-10 \ln (320 - 0) = 0 + C$

$C = -57.6832$

when t = 1 sec; V = ?

$-10 \ln (320 - V) = 1 - 57.6832$

$\ln (320 - V) = 5.66832$

$320 - V = e^{5.66832}$

$V = 30.45$ ft/sec

707. Let: Q = amount of salt in the mixture

$100 + (3 - 2)t = 100 + t$ = volume of mixture at any time t

$\frac{Q}{100+t}$ = concentration of salt

$\frac{dQ}{dt}$ = rate of gain – rate of loss

$$\frac{dQ}{dt} = 0 - 2\left(\frac{Q}{100+t}\right)$$

$$\int\frac{dQ}{Q} = -\int\frac{dt}{100+t} + C$$

$$\ln Q = -2\ln(100+t) + C$$

$$\ln Q = -\ln(100+t)^2 + C$$ ☞ *General solution*

when t = 0; Q = 50 kg.

$\ln 50 = -\ln(100+1)^2 + C$

$C = 13.12236$

when t = 60 min; Q = ?

$\ln Q = -\ln(100+60)^2 + 13.12236$

$\ln Q = 2.972$

$Q = 19.53$ kg.

708. Let: Q = amount of salt in the mixture

$100 + (3 - 2)t = 100 + t$ = volume of mixture at any time t

$\frac{Q}{100+t}$ = concentration of salt

$\frac{dQ}{dt}$ = rate of gain – rate of loss

$$\frac{dQ}{dt} = 3(2) - 2\left(\frac{Q}{100+t}\right)$$

$$\frac{dQ}{dt} + Q\left(\frac{2}{100+t}\right) = 6$$

Note: This is a linear DE

$Q(\text{i.f.}) = \int A(t)(\text{i.f.})dt + C$ ☞ *Formula for the general solution of a linear DE*

$\text{i.f.} = e^{\int P(t)dt}$

where: $P(t) = \frac{2}{100+t}$; $A(t) = 6$

$$\text{i.f.} = e^{\int P(t)dt} = e^{\int\frac{2}{100+t}} = e^{2\ln(100+t)} = e^{\ln(100+t)^2} = (100+t)^2$$

Substitute:

$$Q(100 + t)^2 = \int 6(100 + t)^2 dt + C$$

$$Q(100 + t)^2 = 6\frac{(100 + t)^3}{3} + C$$

$$Q(100 + t)^2 = 2(100 + t)^3 + C$$

when t = 0; Q = 50

$$50(100)^2 = 2(100)^3 + C$$

$$C = -1.5 \times 10^6$$

when t = 30 min; Q = ?

$$Q(100 + 30)^2 = 2(100 + 30)^3 - 1.5 \times 10^6$$

$$Q = 171.24 \text{ lbs.}$$

709. $\frac{dP}{dt} = 0.03P$

$$\int \frac{dP}{P} = 0.03 \int dt$$

$$\ln P = 0.03t + C$$

when t = 0; P = 5,000

$$\ln 5000 = 0.03(0) + C$$

$$C = 8.51719$$

when t = 10 years; P = ?

$$\ln P = 0.03(10) + 8.51719$$

$$P = 6{,}749.30$$

710. $\frac{dP}{dt} = 0.03P$

$$\int \frac{dP}{P} = 0.03 \int dt$$

$$\ln P = 0.03t + C$$

when t = 0; P = 10,000

$$\ln 10000 = 0.03(0) + C$$

$$C = 9.21034$$

when t = 10 years; P = ?

$$\ln P = 0.03(10) + 9.21034$$

$$P = 13{,}498.60$$

DAY 17

ADVANCED ENGINEERING MATH

COMPLEX NUMBERS

By definition, a complex number is any number expressible in the standard form a + bi or a + jb

where: i or $j = \sqrt{-1}$

i^2 or $j^2 = -1$

Example: Solve the solution of $x^2 + 1 = 0$.

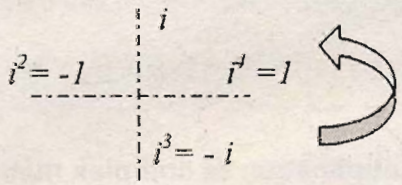
Simplifying we get $x = \pm\sqrt{-1}$

Thus the solutions are i and – i.

Successive integral powers of i or j

a. $j^2 = -1$
b. $j^3 = j(j^2) = -j$
c. $j^4 = j^2(j^2) = 1$
d. $j^5 = j^3(j^2) = j$
e. $j^6 = j^4(j^2) = -1$
f. $j^7 = j^5(j^2) = -j$
g. $j^8 = j^4(j^4) = 1$

...etc



Different forms of complex numbers

a. Rectangular form

$z = a + jb$

where:
a = real part
b = imaginary part

b. Trigonometric form

$z = r\cos\theta + j\, r\sin\theta$

$z = r\,\text{cis}\,\theta$

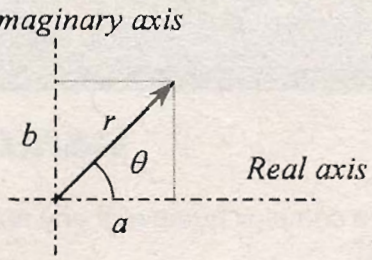
c. Polar form

$$z = r\angle\theta$$

where:
r = modulus or absolute value
θ = argument or amplitude in degrees

$$r = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\frac{b}{a}$$



d. Exponential form

$$z = r\,e^{j\theta}$$

where:
θ = argument in radians

Mathematical operations:

a. **Addition and Subtraction of complex numbers**

Rule: Use the rectangular form

$$(a_1 + jb_1) + (a_2 + jb_2) = (a_1 + a_2) + j\,(b_1 + b_2)$$

$$(a_1 + jb_1) - (a_2 + jb_2) = (a_1 - a_2) + j\,(b_1 - b_2)$$

b. **Multiplication of complex numbers**

Rule: Use the polar form

$$(r_1\angle\theta_1)(r_2\angle\theta_2) = r_1 r_2\angle(\theta_1 + \theta_2)$$

c. **Division of complex numbers**

Rule: Use the polar form

$$\frac{r_1\angle\theta_1}{r_2\angle\theta_2} = \frac{r_1}{r_2}\angle(\theta_1 - \theta_2)$$

d. **Complex number raised to exponent "n"**

Rule: Use the polar form

$$(r\angle\theta)^n = r^n\angle(n\theta)$$

e. n^{th} root of a complex number

Rule: Use the polar form

$$(r\angle\theta)^{\frac{1}{n}} = r^{\frac{1}{n}}\angle\frac{\theta + k(360°)}{n}$$

where: k = 0, first root or principal root
k = 1, second root
k = 2, third root
k = n -1, n^{th} root

f. Logarithm of a complex number

Rule: Use the trigonometric form

$$\ln z = r\,e^{j\theta} = \ln r + \ln e^{j\theta}$$
$$\ln z = \ln r + j\,\theta$$

g. Exponential & trigonometric functions of a complex number

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{j2}$$
$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

Note: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\csc\theta = \dfrac{1}{\sin\theta}$

h. Hyperbolic functions

$$\sin h\theta = \frac{e^{\theta} - e^{-\theta}}{2}$$
$$\cos h\theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

Note: $\tan h\theta = \dfrac{\sinh\theta}{\cosh\theta}$

$\cot h\theta = \dfrac{\cosh\theta}{\sinh\theta}$

$\sec h\theta = \dfrac{1}{\cosh\theta}$

$\csc h\theta = \dfrac{1}{\sinh\theta}$

i. Inverse trigonometric functions of complex numbers

$$\text{Arc sin } z = -j \ln\left(jz \pm \sqrt{1-z^2}\right)$$

$$\text{Arc cos } z = -j \ln\left(z \pm \sqrt{z^2-1}\right)$$

$$\text{Arc tan } z = \frac{-j}{2}\ln\left(\frac{1+jz}{1-jz}\right)$$

j. Inverse hyperbolic functions of complex numbers

$$\text{Arc sinh } z = \ln\left(z \pm \sqrt{z^2+1}\right)$$

$$\text{Arc cosh } z = \ln\left(z \pm \sqrt{z^2-1}\right)$$

$$\text{Arc tanh } z = \frac{1}{2}\ln\left(\frac{1+z}{1-z}\right)$$

MATRICES

Matrix is a rectangular array of real numbers arranged in m rows and n columns. The term "matrix" was introduced by the English mathematician **James Joseph Sylvester** (1814 – 1897) in 1850. The size of a matrix is determined by the number of rows and columns. The expression "m x n" is the **dimension** or **order** of the matrix. If the matrix has only one column, it is called a **column matrix** and if it has only one row, it is called a **row matrix**. The following is a 3 x 3 matrix or **square matrix** (i.e. 3 rows and 3 columns).

$$A = \begin{bmatrix} 6 & 5 & 4 \\ 2 & 1 & 4 \\ 3 & -2 & -2 \end{bmatrix}$$

The first non-zero entry in a row of a matrix is known as the **leading entry** or the **leading element**. In the matrix above, 6 is the leading entry. The diagonal from the upper left to the lower right is called the **principal diagonal** or **main diagonal** and all entries in the said diagonal are called as **diagonal entries**.

If all entries in a matrix above the main diagonal are zero, then it is said to be a **lower triangular matrix**, and if all the entries below the main diagonal are zero, the matrix is referred to as the **upper diagonal matrix**.

$$A = \begin{bmatrix} 6 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -2 & -2 \end{bmatrix}$$

Lower Triangular Matrix

$$B = \begin{bmatrix} 6 & 5 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & -2 \end{bmatrix}$$

Upper Triangular Matrix

$$C = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

Diagonal Matrix

Two matrices are said to be equal if and only if there orders are equal and their corresponding elements are equal.

Example: Matrices B and C are equal.

$$B = \begin{bmatrix} 3 & 2 \\ 2 & -1 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 2 \\ 2 & -1 \end{bmatrix}$$

Sum of two matrices:
If A and B are two matrices of the same order, the sum of A and B, denoted by A + B, is the matrix for which each of its elements is the sum of the corresponding elements of A and B.

Example: Find A + B.

$$A = \begin{bmatrix} 5 & -2 & 6 \\ -1 & 8 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 7 & 1 \\ 7 & 0 & 4 \end{bmatrix}$$

Solution:

$$A + B = \begin{bmatrix} 5+4 & -2+7 & 6+1 \\ -1+3 & 8+0 & -3-4 \end{bmatrix} = \begin{bmatrix} 9 & 5 & 7 \\ 2 & 8 & -7 \end{bmatrix}$$

Difference of two matrices:
If A and B are matrices having the same order, then the difference of A and B, denoted by A – B, is defined as

A – B = A + (-B)

Example: Find A – B.

$$A = \begin{bmatrix} 4 & -3 \\ -5 & -1 \\ 3 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 3 \\ 7 & -5 \\ 8 & 2 \end{bmatrix}$$

Solution:

$$A - B = \begin{bmatrix} 4 & -3 \\ -5 & -1 \\ 3 & 8 \end{bmatrix} + \begin{bmatrix} -4 & -3 \\ -7 & 5 \\ -8 & -2 \end{bmatrix} = \begin{bmatrix} 0 & -6 \\ -12 & 4 \\ -5 & 6 \end{bmatrix}$$

Product of two matrices:
Supposed that A is a matrix of order m x p and B is a matrix of order p x n, then the product of A and B, denoted by AB, is the n x n matrix for which the element in the ith row and the jth column is the sum of the products formed by multiplying each element in the ith row of A by the corresponding element in the jth column of B.

Example: Find DC.

$$D=\begin{bmatrix}2 & -3\\ 4 & -1\\ 1 & 5\end{bmatrix} \qquad C=\begin{bmatrix}-4 & 1 & 3\\ -1 & 2 & 5\end{bmatrix}$$

D is a 3 x 2 matrix and C is a 2 x 3 matrix. The product DC can be obtained because the number of columns of D (2 columns) is equal to the number of rows of C (2 rows). The product DC will be a 3 x 3 matrix.

$$DC=\begin{bmatrix}2(-4)+(-3)(-1) & 2(1)+(-3)(2) & 2(3)+(-3)(5)\\ 4(-4)+(-1)(-1) & 4(1)+(-1)(2) & 4(3)+(-1)(5)\\ 1(-4)+5(-1) & 1(1)+5(2) & 1(3)+5(5)\end{bmatrix}=\begin{bmatrix}-5 & -4 & -9\\ -15 & 2 & 7\\ -9 & 11 & 28\end{bmatrix}$$

Division of matrices

Division of matrices are done by multiplying the numerator by the inverse matrix of the denominator.

Let: A and B are matrices

$\frac{A}{B} = AB^{-1}$, where B^{-1} is called the inverse matrix of matrix B

Transpose Matrix

If matrix A is reflected in its main diagonal, so that all rows become columns and all columns become rows without changing their relative order of entries in the rows and columns, the result is a **transpose matrix, A^T**.

Example:

$$A=\begin{bmatrix}-4 & 1 & 3\\ -1 & 2 & 5\end{bmatrix} \qquad A^T=\begin{bmatrix}-4 & -1\\ 1 & 2\\ 3 & 5\end{bmatrix}$$

Cofactor of an entry of a matrix

A cofactor of an entry of a matrix is the same as the cofactor of the same entry in the determinant of the matrix and thus, is defined only for **square matrices**.

Sign conventions:

$$\begin{bmatrix}+ & -\\ - & +\end{bmatrix} \qquad \begin{bmatrix}+ & - & +\\ - & + & -\\ + & - & +\end{bmatrix}$$

Example: Find the co-factor of 6 in the following matrix.

$$A=\begin{bmatrix}1 & 2 & 0\\ 2 & 1 & 4\\ 4 & 2 & 6\end{bmatrix}$$

The equivalent matrix is:

$$A = +\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

The determinant of A is:

$$\det A = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = (1)(1) - (2)(2) = -3$$

Thus, the cofactor of 6 is – 3.

Cofactor Matrix

A cofactor matrix is formed by replacing each element in the given matrix by its cofactor.

Example: Find cofactor matrix of A

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \qquad \text{Cofactor matrix of } A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

Inverse Matrix

Steps required to find for the Inverse Matrix (say. A):

a. Form the cofactor matrix of matrix A
b. Form the transpose matrix of the cofactor matrix A
c. Evaluate the determinant of matrix A
d. Divide each element in the $(\text{matrix cofactor})^T$

Example: Find the inverse matrix of A.

$$A = \begin{vmatrix} 1 & 2 & 0 \\ 2 & 1 & 4 \\ 4 & 2 & 6 \end{vmatrix}$$

$$\text{Cofactor matrix } A = \begin{vmatrix} -2 & 4 & 0 \\ -12 & 6 & 6 \\ 8 & -4 & -3 \end{vmatrix}$$

$$(\text{Cofactor matrix } A)^T = \begin{vmatrix} -2 & -12 & 0 \\ 4 & 6 & 6 \\ 0 & 6 & -3 \end{vmatrix}$$

$$\det A = \begin{vmatrix} 1 & 2 & 0 \\ 2 & 1 & 4 \\ 4 & 2 & 6 \end{vmatrix} = 6$$

$$\text{Thus, } A^{-1} = \frac{1}{6}\begin{vmatrix} -2 & -12 & 0 \\ 4 & 6 & 6 \\ 0 & 6 & -3 \end{vmatrix}$$

DETERMINANTS

Determinant is a square matrix (i.e. the number of rows = the number of column). Every determinant can be associated with a real number.

Properties of determinants:

❶ If the rows of one determinant are the same as the columns of another, and in the same order, the two determinants are equal.

❷ If two columns (or rows) of a determinant are interchanged, the value of the resulting determinant is equal to the negative of the value of the given determinant.

❸ If two columns (or rows) of a determinant are identical, the value of the determinant is zero.

❹ If the elements of a column (or row) of a determinant are multiplied by k, the value of the determinant is multiplied by k.

❺ If the elements of the jth column of a determinant D are the sum $a_{ij} + b_{ij}$, then D is the sum of the determinants D' and D" in which all the columns of D, D' and D: are the same except the jth; furthermore, the jth column of D' is a_{ij}, i = 1, 2, 3,..., n, and the jth column of D" is b_{ij}, I = 1, 2, 3,..., n. Similarly for rows.

❻ The value of the determinant is not changed if a column is replaced by the column plus a multiple of another column. Similarly for rows.

Determinant of a 2 x 2 matrix:

$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\det A = ad - bc$$

Determinant of a 3 x 3 matrix:

$$\det A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$\det A = \begin{vmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

Determinant of a 4 x 4 matrix:

a. Using Pivotal Element method:

Example: Find det A.

$$A = \begin{vmatrix} 2 & -4 & 3 & -1 \\ -1 & 1 & -2 & 2 \\ 3 & 2 & -4 & -1 \\ -2 & 5 & 1 & 4 \end{vmatrix}$$

2 for 2nd row and 2 for 2nd column

$$\det A = \begin{vmatrix} 2-(-4)(-1) & 3-(-4)(-2) & -1-(-4)(2) \\ 3-(2)(-1) & -4-(2)(-2) & -1-(2)(2) \\ -2-(5)(-1) & 1-(5)(-2) & 4-(5)(2) \end{vmatrix} (-1)^{2+2}$$

$$\det A = \begin{vmatrix} -2 & -5 & 7 \\ 5 & 0 & -5 \\ 3 & 11 & -6 \end{vmatrix} = \begin{vmatrix} -2 & -5 & 7 & -2 & -5 \\ 5 & 0 & -5 & 5 & 0 \\ 3 & 11 & -6 & 3 & 11 \end{vmatrix}$$

$\det A = 2(0)(-6) + (-5)(-5)(3) + 7(5)(11) - [3(0)(7) + (11)(-5)(-2) + (-6)(5)(-5)]$
$\det A = 200$

b. Using Modification method:

$$A = \begin{vmatrix} 2 & -4 & 3 & -1 \\ (-1) & 1 & (-2) & (2) \\ 3 & 2 & -4 & -1 \\ -2 & 5 & 1 & 4 \end{vmatrix}$$

Set the encircled numbers to zero by:

Multiplying column 2 by 1 and add it to column 1
Multiplying column 2 by 2 and add it to column 3
Multiplying column 2 by –2 and add it to column 4

The new matrix becomes,

$$A = \begin{vmatrix} -2 & -4 & -5 & 7 \\ 0 & 1 & 0 & 0 \\ 5 & 2 & 0 & -5 \\ 3 & 5 & 11 & -6 \end{vmatrix}$$

$$A = (1)\begin{vmatrix} -2 & -5 & 7 \\ 5 & 0 & -5 \\ 3 & 11 & -6 \end{vmatrix}(-1)^{2+2}$$

Set the encircled number to zero by multiplying column 1 by 1 and add it to column 3

$$A = \begin{vmatrix} -2 & -5 & 5 \\ 5 & 0 & 0 \\ 3 & 11 & -3 \end{vmatrix}$$

2 for 2nd row
1 for 1st column

$$\det A = (5)\begin{vmatrix} -5 & 5 \\ 11 & -3 \end{vmatrix}(-1)^{2+1}$$

$$= 5\,[(-5)(-3) - (11)(5)]\,(-1)$$

$$\det A = 200$$

LAPLACE TRANSFORMS

The Laplace transform of a function f(t) denoted by $\mathcal{L}\,[\,f(t)\,]$ is defined as a function of a variable "s" by the integral:

$$F(s) = \mathcal{L}\,[\,f(t)\,] = \int_0^\infty f(t)\,e^{-st}dt$$

where: $t > 0$ and s is any number (real or complex)

Laplace transform of some elementary functions:

f(t)	F(s)
1. 1	$\frac{1}{s}$
2. t	$\frac{1}{s^2}$
3. t^n	$\frac{n!}{s^{n+1}}$
4. $e^{\pm at}$	$\frac{1}{s \mp a}$
5. $t^n\ e^{\pm at}$	$\frac{n!}{(s \mp a)^{n+1}}$
6. $e^{\pm at} \sin kt$	$\frac{k}{(s \mp a)^2 + k^2}$
7. $e^{\pm at} \cos kt$	$\frac{s \mp a}{(s \mp a)^2 + k^2}$
8. sin at	$\frac{a}{s^2 + a^2}$
9. cos at	$\frac{s}{s^2 + a^2}$
10. sinh at	$\frac{a}{s^2 - a^2}$
11. cosh at	$\frac{s}{s^2 - a^2}$
12. t sin at	$\frac{2as}{(s^2 + a^2)^2}$
13. t cos at	$\frac{s^2 - a^2}{(s^2 + a^2)^2}$
14. sin (wt + θ)	$\frac{s \sin\theta + k \cos\theta}{s^2 + k^2}$
15. cos (wt + θ)	$\frac{s \sin\theta - k \cos\theta}{s^2 + k^2}$



Did you know that... The theory of determinants dates back to the ancient Chinese who use bamboo rods in representing the coefficient of unknown quantities, and gain acceptance when introduced in Japan by the 17th century greatest Japanese mathematician Seki Kowa (1642 – 1708). Even if German Mathematician Gottfried Wilhelm von Leibniz (1646 – 1716) and Swiss mathematician Gabriel Cramer (1704 – 1752) gave their valuable contribution to the subject, it was Alexandre-Theophile Vandermonde (1735 – 1796) the one regarded as the formal founder of determinant theory!

Proceed to the next page for your 17th test. GOODLUCK ! ☞

Test 17

Time element: 4.0 hours

Problem 711: ECE Board April 1999

Simplify the expression $i^{1997} + i^{1999}$, where i is an imaginary number.

A. 0
B. - i
C. 1+ i
D. 1 - i

Problem 712: EE Board April 1997

Simplify: $i^{29} + i^{21} + i$

A. 3i
B. 1- i
C. 1 + i
D. 2i

Problem 713: EE Board April 1997

Write in the form a + bi the expression $i^{3217} - i^{427} + i^{18}$

A. 2i + 1
B. – i + 1
C. 2i – 1
D. 1 + i

Problem 714: CE Board May 1994

The expression 3 + 4i is a complex number. Compute its absolute value.

A. 4
B. 5
C. 6
D. 7

Problem 715: EE Board October 1993

Write the polar form of the vector 3 + j4.

A. $6\angle 53.1°$
B. $10\angle 53.1°$
C. $5\angle 53.1°$
D. $8\angle 53.1°$

Problem 716: ME Board April 1997

Evaluate the value of $\sqrt{-10} \times \sqrt{-7}$.

A. i
B. $-\sqrt{70}$
C. $\sqrt{70}$
D. $\sqrt{17}$

Problem 717: EE Board April 1996

Simplify $(3 - i)^2 - 7(3 - i) + 10$

A. $-(3 + i)$
B. $3 + i$
C. $3 - i$
D. $-(3 - i)$

Problem 718: EE Board April 1996

If $A = 40\, e^{j120°}$, $B = 20\angle -40°$, $C = 26.46 + j0$, solve for A + B + C.

A. $27.7\angle 45°$
B. $35.1\angle 45°$
C. $30.8\angle 45°$
D. $33.4\angle 45°$

Problem 719: EE Board October 1997

What is 4i cube times 2i square?

A. $-8i$
B. $8i$
C. -8
D. $-8i^2$

Problem 720: EE Board April 1997

What is the simplified complex expression of $(4.33 + j2.5)$ square?

A. $12.5 + j21.65$
B. $20 + j20$
C. $15 + j20$
D. $21.65 + j12.5$

Problem 721: ECE Board November 1998

Find the value of $(1 + i)^5$, where i is an imaginary number.

A. $1 - i$
B. $-4(1 + i)$
C. $1 + I$

D. $4(1 + i)$

Problem 722: EE Board October 1997

Find the principal 5th root of [$50(\cos 150° + j\sin 150°)$].

A. $1.9 + j1.1$
B. $3.26 - j2.1$
C. $2.87 + j2.1$
D. $2.25 - j1.2$

Problem 723: ECE Board April 1999

What is the quotient when $4 + 8i$ is divided by i^3 ?

A. $8 - 4i$
B. $8 + 4i$
C. $-8 + 4i$
D. $-8 - 4i$

Problem 724: EE Board October 1997

If $A = -2 - 3i$, and $B = 3 + 4i$, what is $\frac{A}{B}$?

A. $\frac{18 - i}{25}$

B. $\frac{-18 - i}{25}$

C. $\frac{-18 + i}{25}$

D. $\frac{18 + i}{25}$

Problem 725: EE Board October 1997

Rationalize $\frac{4 + 3i}{2 - i}$.

A. $1 + 2i$

B. $\frac{11 + 10i}{5}$

C. $\frac{5 + 2i}{5}$

D. $2 + 2i$

Problem 726: EE Board October 1997

Simplify $\frac{(2+3i)(5-i)}{(3-2i)^2}$.

A. $\frac{221-91i}{169}$

B. $\frac{21+52i}{13}$

C. $\frac{-7+17i}{13}$

D. $\frac{-90+220i}{169}$

Problem 727: EE Board April 1996

What is the simplified expression of the complex number $\frac{6+j2.5}{3+j4}$?

A. – 0.32 + j 0.66
B. 1.12 – j0.66
C. 0.32 – j0.66
D. – 1.75 + j 1.03

Problem 728: EE Board April 1997

Perform the operation: 4 (cos 60° + i sin 60°) divided by 2 (cos 30° + i sin 30°) in rectangular coordinates.

A. square root of 3 – 2i
B. square root of 3 – i
C. square root of 3 + i
D. square root of 3 + 2i

Problem 729: EE Board June 1990

Find the quotient of $\frac{50+j35}{8+j5}$.

A. $6.47\angle 3°$
B. $4.47\angle 3°$
C. $7.47\angle 30°$
D. $2.47\angle 53°$

Problem 730: EE Board March 1998

Three vectors A, B and C are related as follows: $\frac{A}{B} = 2$ at 180°, A + C = - 5 + j15, C = conjugate of B. Find A.

A. 5 – j5

B. $-10 + j10$
C. $10 - j10$
D. $15 + j15$

Problem 731: EE Board April 1999

Evaluate $\cosh\left(j\frac{\pi}{4}\right)$.

A. 0.707
B. 1.41 + j0.866
C. 0.5 + j0.707
D. j0.707

Problem 732: EE Board April 1999

Evaluate $\tanh\left(j\frac{\pi}{3}\right)$.

A. 0.5 + j1.732
B. j0.866
C. j1.732
D. 0.5 + j0.866

Problem 733: EE Board April 1999

Evaluate ln (2 + j3).

A. 1.34 + j0.32
B. 2.54 + j0.866
C. 2.23 + j0.21
D. 1.28 + j 0.98

Problem 734: EE Board October 1997

Evaluate the terms of a Fourier series $2\ e^{j10\pi t} + 2\ e^{-j10\pi t}$ at t = 1.

A. 2 + j
B. 2
C. 4
D. 2 + j2

Problem 735: EE Board March 1998

Given the following series:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\ldots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$$

What relation can you draw from these series?

A. $e^x = \cos x + \sin x$

B. $e^{ix} = \cos x + i \sin x$

C. $e^{ix} = i\cos x + \sin x$

D. $ie^x = i\cos x + i \sin x$

Problem 736: EE Board October 1997

One term of a Fourier series in cosine form is 10 cos 40πt. Write it in exponential form.

A. $5\ e^{j40\pi t}$

B. $5\ e^{j40\pi t} + 5e^{-j40\pi t}$

C. $10\ e^{-j40\pi t}\ 0$

D. $10\ e^{j40\pi t}$

Problem 737: EE Board April 1997

Evaluate the determinant:

$$\begin{vmatrix} 1 & 2 & 3 \\ -2 & -1 & -2 \\ 3 & 1 & 4 \end{vmatrix}$$

A. 4
B. 2
C. 5
D. 0

Problem 738: ECE Board November 1991

Evaluate the determinant:

$$\begin{vmatrix} 1 & 6 & 0 \\ 4 & 2 & 7 \\ 0 & 5 & 3 \end{vmatrix}$$

A. 110
B. --101
C. 101
D. –110

Problem 739: EE Board April 1997

Evaluate the determinant:

$$\begin{vmatrix} 2 & 14 & 3 & 1 \\ 1 & 5 & -1 & 3 \\ 1 & -2 & 2 & -3 \\ 3 & -4 & -3 & -4 \end{vmatrix}$$

A. 489
B. 389
C. 326
D. 452

Problem 740: CE Board November 1996

Compute the value of x by determinant.

$$x = \begin{vmatrix} 4 & -1 & 2 & 3 \\ 2 & 0 & 2 & 1 \\ 10 & 3 & 0 & 1 \\ 14 & 2 & 4 & 5 \end{vmatrix}$$

A. –32
B. –28
C. 16
D. 52

Problem 741: EE Board April 1997

Given the equations:

$$x + y + z = 2$$
$$3x - y - 2z = 4$$
$$5x - 2y + 3z = -7$$

Solve for y by determinants.

A. 1
B. –2
C. 3
D. 0

Problem 742: EE Board April 1997

Solve the equations by Cramer's Rule:

$$2x - y + 3z = -3$$
$$3x + 3y - z = 10$$
$$-x - y + z = -4$$

A. (2, 1, -1)
B. (2, -1, -1)
C. (1, 2, -1)
D. (-1, -2, 1)

Problem 743: EE Board October 1997

If $A = \begin{vmatrix} 2 & 3 & 1 \\ -1 & 2 & 4 \\ 0 & 5 & 7 \end{vmatrix}$, what is the cofactor of the second row, third column element?

A. $\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix}$

B. $-\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix}$

C. $-\begin{vmatrix} 1 & 7 \\ 2 & 0 \end{vmatrix}$

D. $\begin{vmatrix} 3 & 1 \\ 5 & 7 \end{vmatrix}$

Problem 744: EE Board October 1997

If $A = \begin{vmatrix} 3 & 1 & 2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{vmatrix}$, what is the cofactor with the first row, second column element?

A. $-\begin{vmatrix} 3 & 2 \\ 0 & -1 \end{vmatrix}$

B. $\begin{vmatrix} -2 & -1 \\ 0 & 2 \end{vmatrix}$

C. $\begin{vmatrix} 3 & 2 \\ 0 & -1 \end{vmatrix}$

D. $-\begin{vmatrix} -2 & 0 \\ 0 & -1 \end{vmatrix}$

Problem 745: EE Board October 1997

If a 3 x 3 matrix and its inverse are multiplied together, write the product.

A. $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

B. $\begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$

C. $\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$

D. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$

Problem 746: EE Board April 1996

If matrix $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 0 & -1 & 1 \end{bmatrix}$ is multiplied by $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is equal to zero, then matrix $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is

A. 3
B. 1
C. 0
D. –2

Problem 747: CE Board November 1997

Given the matrix equation, solve for x and y.

$$\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

A. – 4, 6
B. – 4, 2
C. – 4, - 2
D. – 4, - 6

Problem 748: EE Board April 1996

If matrix $\begin{bmatrix} 1 & 4 \\ -4 & 1 \end{bmatrix}$ is multiplied by $\begin{bmatrix} x \\ y \end{bmatrix}$ is equal to zero, then matrix $\begin{bmatrix} x \\ y \end{bmatrix}$ is

A. 8
B. 1
C. –4
D. 0

Problem 749: EE Board October 1997

If $A = \begin{vmatrix} 4 & 5 & 0 \\ 6 & 7 & 3 \\ 1 & 2 & 5 \end{vmatrix}$ and $B = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$, what is A times B equal to?

A. $\begin{vmatrix} 4 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 5 \end{vmatrix}$

B. $\begin{vmatrix} 0 & 0 & 0 \\ 0 & 7 & 0 \\ 1 & 0 & 0 \end{vmatrix}$

C. $\begin{vmatrix} 6 & 7 & 0 \\ 8 & 9 & 4 \\ 2 & 3 & 5 \end{vmatrix}$

D. $\begin{vmatrix} 4 & 5 & 0 \\ 6 & 7 & 3 \\ 1 & 2 & 5 \end{vmatrix}$

Problem 750: EE Board April 1997

Matrix $\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix}$ + 2 Matrix $\begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix}$ =

A. Matrix $\begin{vmatrix} -2 & 4 \\ 2 & 2 \end{vmatrix}$

B. Matrix $\begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix}$

C. Matrix $\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix}$

D. Matrix $\begin{vmatrix} 0 & 5 \\ 1 & 5 \end{vmatrix}$

Problem 751: CE Board May 1996

Elements of matrix B = $\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix}$

Elements of matrix C = $\begin{bmatrix} 3 & 6 \\ 4 & 1 \end{bmatrix}$

Find the elements of the product of the two matrices, matrix BC.

A. $\begin{bmatrix} 11 & 8 \\ -20 & -5 \end{bmatrix}$

B. $\begin{bmatrix} -11 & 8 \\ 19 & 5 \end{bmatrix}$

C. $\begin{bmatrix} -10 & 9 \\ -19 & 6 \end{bmatrix}$

D. $\begin{bmatrix} -11 & 9 \\ -20 & -4 \end{bmatrix}$

Problem 752: EE Board October 1997

Transpose the matrix $\begin{vmatrix} 3 & 1 & 2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{vmatrix}$.

A. $\begin{vmatrix} -1 & 2 & 0 \\ 0 & -1 & -2 \\ 2 & 1 & 3 \end{vmatrix}$

B. $\begin{vmatrix} 3 & -2 & 0 \\ 1 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix}$

C. $\begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & -1 \\ -2 & -1 & 0 \end{vmatrix}$

D. $\begin{vmatrix} 1 & 3 & 2 \\ -1 & -2 & 0 \\ 2 & 2 & -1 \end{vmatrix}$

Problem 753:

Determine the inverse matrix of $\begin{pmatrix} 1 & 2 \\ 5 & 9 \end{pmatrix}$.

A. $\begin{pmatrix} -9 & 5 \\ 2 & -1 \end{pmatrix}$

B. $\begin{pmatrix} 9 & 5 \\ 2 & 1 \end{pmatrix}$

C. $\begin{pmatrix} 2 & 5 \\ 9 & 1 \end{pmatrix}$

D. $\begin{pmatrix} -9 & -5 \\ 2 & 1 \end{pmatrix}$

Problem 754: EE Board April 1997

k divided by [(s square) + (k square)] is the inverse laplace transform of:

A. cos kt
B. sin kt
C. (e exponent kt)

D. 1.0

Problem 755: EE Board April 1996, EE Board April 1997

The laplace transform of cos wt is

A. s / [(s square) + (w square)]
B. w / [(s square) + (w square)]
C. w / (s + w)
D. s / (s + w)

Problem 756: EE Board April 1997

Find the laplace transform of [2/(s +1)] – [4/(s + 3)].

A. [2 e(exp –t) – 4 e(exp – 3t)]
B. [e(exp –2t) + e(exp – 3t)]
C. [e(exp –2t) - e(exp – 3t)]
D. [2 e(exp –t)][1-2 e(exp – 3t)]

Problem 757: EE Board March 1998

Determine the inverse laplace transform of $I(s) = \dfrac{200}{s^2 - 50s + 10625}$

A. $I(s) = 2e^{-25t} \sin 100t$
B. $I(s) = 2t\,e^{-25t} \sin 100t$
C. $I(s) = 2e^{-25t} \cos 100t$
D. $I(s) = 2t\,e^{-25t} \cos 100t$

Problem 758: EE Board April 1997

The inverse laplace transform of s / [(s square) + (w square)] is

A. sin wt
B. w
C. (e exponent wt)
D. cos wt

Problem 759:

Find the inverse laplace transform of $\dfrac{2s - 18}{s^2 + 9}$ as a function of x.

A. 2 cos x – sin 3x
B. 2 cos 3x – 6 sin 3x
C. 3 cos 2x – 2 sin 6x
D. 6 cos x – 3 sin 2x

Problem 760:

Determine the inverse laplace transform of $\frac{1}{4s^2 - 8s}$.

A. $\frac{1}{4}e^t \sinh t$

B. $\frac{1}{2}e^{2t} \sinh t$

C. $\frac{1}{4}e^t \cosh t$

D. $\frac{1}{2}e^{2t} \cosh t$

ANSWER KEY

711. A	724. B	737. C	750. D
712. A	725. A	738. B	751. A
713. C	726. C	739. C	752. B
714. B	727. B	740. B	753. A
715. C	728. C	741. C	754. B
716. B	729. A	742. C	755. A
717. D	730. B	743. B	756. A
718. C	731. A	744. D	757. A
719. B	732. C	745. A	758. D
720. A	733. D	746. C	759. B
721. B	734. C	747. A	760. A
722. A	735. B	748. D	
723. C	736. B	749. D	

RATING

☐ 43-50 Topnotcher

☐ 30-42 Passer

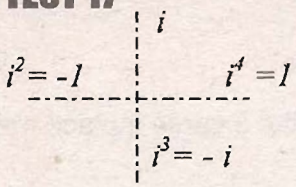
☐ 25-29 Conditional

☐ 0-24 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 17

711. Note: $i^2 = -1$
$i^3 = -i$
$i^4 = 1$



If the exponent of " i " is exactly divisible by 4, then the simplified equivalent of the imaginary number is equal to 1.

$i^{1996} = 1$, since 1996 is exactly divisible by 4
$i^{1997} = i$
$i^{1998} = -1$
$i^{1999} = -i$

Substitute:
$i^{1997} + i^{1999} = i + (-i) = 0$

712. Note: $i^{20} = 1$, since 20 is exactly divisible by 4
$i^{21} = i$
$i^{28} = 1$, since 28 is exactly divisible by 4
$i^{29} = i$

Substitute:
$i^{29} + i^{21} + i = i + i + i = 3i$

713. Note: $i^{3216} = 1$, since 3216 is exactly divisible by 4
$i^{3217} = i$

$i^{424} = 1$, since 424 is exactly divisible by 4
$i^{425} = i$
$i^{426} = -1$
$i^{427} = -i$

$i^{16} = 1$, since 16 is exactly divisible by 4
$i^{17} = i$
$i^{18} = -1$

Substitute:
$i^{3217} - i^{427} + i^{18} = i - (-i) + (-1) = 2i - 1$

714. Let: r = absolute value of the complex no. (a + bi)

$$r = \sqrt{a^2 + b^2}$$

Substitute:

$$r = \sqrt{(3)^2 + (4)^2} = 5$$

715. The polar form of the complex number, "a + jb" is given by: $z = r\angle\theta$

where: $r = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\frac{b}{a}$

Substitute: $r = \sqrt{3^2 + 4^2} = 5$

$$\theta = \tan^{-1}\frac{4}{3} = 53.1°$$

Thus, the complex number is $5\angle 53.1°$

716. $\sqrt{-10} = \sqrt{10}\sqrt{-1} = \sqrt{10}\,i$

$\sqrt{-7} = \sqrt{7}\sqrt{-1} = \sqrt{7}\,i$

Substitute:

$$\sqrt{-10} \times \sqrt{-7} = \left(\sqrt{10}\,i\right)\left(\sqrt{7}\,i\right) = \sqrt{70}\,i^2 = \sqrt{70}\,(-1) = -\sqrt{70}$$

717. $(3 - i)^2 - 7(3 - i) + 10 = 9 - 6i + i^2 - 21 + 7i + 10$

$= 9 - 6i + (-1) - 21 + 7i + 10 = -3 + i$

$= -(3 - i)$

718. Note: Convert all the complex number in rectangular form

$A = 40\,e^{j120°} = 40\angle 120° = -20 + j\,34.64$

$B = 20\angle -40° = 15.32 - j12.855$

$C = 26.46$

$A + B + C = -20 + j\,34.64 + 15.32 - j\,12.855 + 26.46$

$= 21.78 + j\,21.785$

$= 30.8\angle 45°$

719. Note: $i^2 = -1$

$i^3 = -i$

$(4i^3)(2i^2) = (4)(-i)(2)(-1) = 8i$

720. $(4.33 + j2.5)^2 = (4.33)^2 + 2(4.33)(j2.5) + (2.5)^2 j^2$

$= 18.749 + j21.65 + 6.25(-1)$

$= 12.5 + j\,21.65$

721. Note: $(r\angle\theta)^n = r^n\angle n\theta$

$1 + i = 1.4142\angle 45°$

$(1 + i)^5 = (1.4142\angle 45°)^5 = (1.4142)^5\angle 5(45°) = 5.656\angle 225°$

$= -4 - 4i = -4(1 + i)$

722. $50\,(\cos 150° + j\sin 150°) = 50\angle 150°$

$\sqrt[5]{50\angle 150°} = (50)^{1/5}\angle 150°(1/5) = 2.1867\angle 30°$

$= 1.893 + j\,1.093$

$= 1.9 + j1.1$

723. $\dfrac{4+8i}{i^3} = \dfrac{4+8i}{-i}$; since $i^3 = -i$

Rationalizing: Multiply the denominator with its conjugate.

$$= \frac{4+8i}{-i} x \left(\frac{i}{i}\right) = \frac{4i+8i^2}{-i^2} = \frac{4i+8(-1)}{-(-1)} = -8+4i$$

724. $\dfrac{-2-3i}{3+4i}$

Rationalizing:

$$= \frac{-2-3i}{3+4i} x \frac{3-4i}{3-4i} = \frac{-6+8i-9i+12i^2}{9-12i+12i-16i^2} = \frac{-6+8i-9i+12(-1)}{9-12i+12i-16(-1)}$$

$$= \frac{-18-i}{25}$$

725. $\dfrac{4+3i}{2-i}$

Rationalizing:

$$= \frac{4+3i}{2-i} x \frac{2+i}{2+i} = \frac{8+4i+6i+3i^2}{4+2i-2i-i^2} = \frac{8+4i+6i+3(-1)}{4+2i-2i-(-1)} = \frac{5+10i}{5}$$

$$= 1 + 2i$$

726. $(2 + 3i)(5 - i) = 10 - 2i + 15i - 3i^2 = 13 + 13i$

$(3 - 2i)^2 = 9 - 12i + 4i^2 = 5 - 12i$

$$\frac{(2+3i)(5-i)}{(3-2i)^2} = \frac{13+13i}{5-12i}$$

Rationalizing:

$$= \frac{13+13i}{5-12i} x \left(\frac{5+12i}{5+12i}\right) = \frac{65+156i+65i+156i^2}{25+60i-60i-144i^2}$$

$$= \frac{65+156i+65i+156(-1)}{25+60i-60i-144(-1)} = \frac{-91+221i}{169} \div \frac{13}{13} = \frac{-7+17i}{13}$$

727. $\dfrac{6+j2.5}{3+j4} = \dfrac{6.5\angle 22.619°}{5\angle 53.13°} = 1.3\angle -30.5° = 1.12 - j0.66$

728. $\dfrac{4(\cos 60° + i \sin 60°)}{2(\cos 30° + i \sin 30°)} = \dfrac{4\angle 60°}{2\angle 30°} = 2\angle 30°$

$$= 1.732 + i = \sqrt{3} + i$$

729. $\dfrac{50+j35}{8+j5} = \dfrac{61.03\angle 35°}{9.43\angle 32°} = 6.47\angle 3°$

730. $\frac{A}{B} = 2\angle 180° = -2$, thus $A = -2B$

Let: $B = a + jb$; $C = a - jb$

$$A + C = -5 + j15$$
$$-2B + C = -5 + j15$$
$$-2(a + jb) + (a - jb) = -5 + j15$$
$$-2a - j2b + a - jb = -5 + j15$$
$$-a - j3b = -5 + j15$$

By inspection:

$a = 5$

$-3b = 15$

$b = -5$

Thus, $A = -2(5 - j5) = -10 + j10$

731. $\cosh x = \frac{e^{x} + e^{-x}}{2}$; $\cosh\left(j\frac{\pi}{4}\right) = \frac{e^{j\frac{\pi}{4}} + e^{-j\frac{\pi}{4}}}{2}$

Note: $e^{j\theta} + e^{-j\theta} = 2\cos\theta$ ☞ *Euler's equation*

$$e^{j\frac{\pi}{4}} + e^{-j\frac{\pi}{4}} = 2\cos\left(\frac{\pi}{4} x \frac{180°}{\pi}\right) = 1.4142$$

Thus, $\cosh\left(j\frac{\pi}{4}\right) = \frac{1.4142}{2} = 0.707$

732. $\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$; $\tanh\left(j\frac{\pi}{3}\right) = \frac{e^{j\frac{\pi}{3}} - e^{-j\frac{\pi}{3}}}{e^{j\frac{\pi}{3}} + e^{-j\frac{\pi}{3}}}$

Note: $e^{j\theta} + e^{-j\theta} = 2\cos\theta$

$e^{j\theta} - e^{-j\theta} = j2\sin\theta$ } *Euler's equations*

$$e^{j\frac{\pi}{3}} + e^{-j\frac{\pi}{3}} = 2\cos\left[\frac{\pi}{3} x \frac{180°}{\pi}\right] = 2\cos 60° = 1$$

$$e^{j\frac{\pi}{3}} - e^{-j\frac{\pi}{3}} = j2\sin 60° = j1.732$$

Thus, $\tanh\left(j\frac{\pi}{3}\right) = \frac{j1.732}{1} = j1.732$

733. Convert $(2 + j3)$ to polar form, then to exponential form:

$$2 + j3 = 3.6\angle 56.3° = 3.6\, e^{j56.3° x \frac{\pi}{180°}} = 3.6\, e^{j0.98}$$

Let: $x = \ln(2 + j3)$

$$x = \ln\left(3.6\, e^{j0.98}\right) = \ln 3.6 + \ln e^{j0.98}$$

$$x = 1.28 + j\,0.98$$

734. Let : $x = 2\, e^{j10\pi t} + 2\, e^{-j10\pi t}$

Substitute: $t = 1$

$$x = 2\, e^{j10\pi(1)} + 2\, e^{-j10\pi(1)}$$

$$x = 2\left(e^{j10\pi} + e^{-j10\pi}\right)$$

Note: $e^{j\theta} + e^{-j\theta} = 2\cos\theta$ ☞ *Euler's equation*

$$x = 2\left(e^{j10\pi} + e^{-j10\pi}\right) = 2\left[2\cos\left(10\pi\, x\, \frac{180°}{\pi}\right)\right] = 4$$

735. Let: $x = ix$

$$e^{ix} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!} \cdots\cdots$$

$$= 1 + ix + \frac{-x^2}{2!} + \frac{-ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} + \frac{-x^6}{6!} \cdots\cdots$$

$$e^{ix} = \underbrace{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\cdots}_{\cos x} + \underbrace{i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\cdots\right)}_{i \sin x}$$

Thus, $e^{ix} = \cos x + i \sin x$

736. $e^{j\theta} + e^{-j\theta} = 2\cos\theta$ ☞ *Euler's equation*

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$10\cos(40\pi t) = 10\left[\frac{e^{j40\pi t} + e^{-j40\pi t}}{2}\right] = 5\, e^{j40\pi t} + 5\, e^{-j40\pi t}$$

737.
$$D = \begin{vmatrix} 1 & 2 & 3 \\ -2 & -1 & -2 \\ 3 & 1 & 4 \end{vmatrix} \begin{matrix} 1 & 2 \\ -2 & -1 \\ 3 & 1 \end{matrix}$$

$= [(1)(-1)(4) + (2)(-2)(3) + (3)(-2)(1) - (3)(-1)(3) - (1)(-2)(1) - (4)(-2)(2)]$

$= -4 - 12 - 6 + 9 + 2 + 16$

$D = 5$

738. $D = \begin{vmatrix} 1 & 6 & 0 & 1 & 6 \\ 4 & 2 & 7 & 4 & 2 \\ 0 & 5 & 3 & 0 & 5 \end{vmatrix}$

$D = [(1)(2)(3) + (6)(7)(0) + (0)(4)(5) - (0)(2)(0) - (5)(7)(1) - (3)(4)(6)]$
$= 6 + 0 + 0 - 0 - 35 - 72$
$D = -101$

739. $D = \begin{vmatrix} 2 & 14 & 3 & 1 \\ \textcircled{1} & 5 & -1 & 3 \\ 1 & -2 & 2 & -3 \\ 3 & -4 & -3 & -4 \end{vmatrix}$

Using Pivotal element method: (Use the second row, first column element as the pivot number).

$$D = (1)\begin{vmatrix} 14-2(5) & 3-2(-1) & 1-2(3) \\ -2-1(5) & 2-1(-1) & -3-1(3) \\ -4-3(5) & -3-3(-1) & -4-3(3) \end{vmatrix}(-1)^{2+1}$$

$$D = (-1)\begin{vmatrix} 4 & 5 & -5 & 4 & 5 \\ -7 & 3 & -6 & -7 & 3 \\ -19 & 0 & -13 & -19 & 0 \end{vmatrix}$$

$D = (-1)\,[(4)(3)(-13) + (5)(-6)(-19) + (-5)(-7)(0) - (-19)(3)(-5)$
$- (0)(-6)(4) - (-13)(-7)(5)]$

$= (-1)\,[-156 + 570 + 0 - 285 + 0 - 455] = (-1)(-326)$
$D = 326$

740. $x = \begin{vmatrix} 4 & -1 & 2 & 3 \\ 2 & 0 & 2 & 1 \\ 10 & 3 & 0 & 1 \\ 14 & 2 & 4 & 5 \end{vmatrix}$

Multiply column 4 by –2 and add it to column 1:
$3(-2) + 4 = -2$
$1(-2) + 2 = 0$
$1(-2) + 10 = 8$
$5(-2) + 14 = 4$

Multiply column 4 by –2 and add it to column 3:
$3(-2) + 2 = -4$
$1(-2) + 2 = 0$
$1(-2) + 0 = -2$

$5(-2) + 4 = -6$

The new matrix becomes,

$$x = \begin{vmatrix} -2 & -1 & -4 & 3 \\ 0 & 0 & 0 & 1 \\ 8 & 3 & -2 & 1 \\ 4 & 2 & -6 & 5 \end{vmatrix} \qquad x = (1)\begin{vmatrix} -2 & -1 & -4 \\ 8 & 3 & -2 \\ 4 & 2 & -6 \end{vmatrix}(-1)^{2+4}$$

$$x = \begin{vmatrix} -2 & -1 & -4 & -2 & -1 \\ 8 & 3 & -2 & 8 & 3 \\ 4 & 2 & -6 & 4 & 2 \end{vmatrix}$$

$$x = [(-2)(3)(-6) + (-1)(-2)(4) + (-4)(8)(2) - (4)(3)(-4) - (2)(-2)(-2) - (-6)(8)(-1)]$$

$$= [36 + 8 - 64 + 48 - 8 - 48]$$

$$x = -28$$

741. $x + y + z = 2$

$3x - y - 2z = 4$

$5x - 2y + 3z = -7$

By Cramer's rule; $y = \dfrac{Dy}{D}$

$$D = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & -1 & -2 & 3 & -1 \\ 5 & -2 & 3 & 5 & -2 \end{vmatrix}$$

$$D = [(1)(-1)(3) + (1)(-2)(5) + (1)(3)(-2) - (5)(-1)(1) - (-2)(-2)(1) - (3)(3)(1)]$$

$$= [-3 - 10 - 6 + 5 - 4 - 9]$$

$$D = -27$$

$$Dy = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 3 & 4 & -2 & 3 & 4 \\ 5 & -7 & 3 & 5 & -7 \end{vmatrix}$$

$$Dy = [(1)(4)(3) + (2)(-2)(5) + (1)(3)(-7) - (5)(4)(1) - (-7)(-2)(1) - (3)(3)(2)]$$

$$= [12 - 20 - 21 - 20 - 14 - 18]$$

$$Dy = -81$$

$$y = \frac{Dy}{D} = \frac{-81}{-27}$$

$$y = 3$$

742. $2x - y + 3z = -3$
$3x + 3y - z = 10$
$-x - y + z = -4$

$$D = \begin{vmatrix} 2 & -1 & 3 & 2 & -1 \\ 3 & 3 & -1 & 3 & 3 \\ -1 & -1 & 1 & -1 & -1 \end{vmatrix}$$

$D = [(2)(3)(1) + (-1)(-1)(-1) + (3)(3)(-1) - (-1)(3)(3)$
$- (-1)(-1)(2) - (1)(3)(-1)]$

$D = [6 - 1 - 9 + 9 - 2 + 3] = 6$

$$Dx = \begin{vmatrix} -3 & -1 & 3 & -3 & -1 \\ 10 & 3 & -1 & 10 & 3 \\ -4 & -1 & 1 & -4 & -1 \end{vmatrix}$$

$Dx = [(-3)(3)(1) + (-1)(-1)(-4) + (3)(10)(-1) - (-4)(3)(3)$
$- (-1)(-1)(-3) - (1)(10)(-1)]$

$Dx = [-9 - 4 - 30 + 36 + 3 + 10] = 6$

$$x = \frac{Dx}{D} = \frac{6}{6} = 1$$

$$Dy = \begin{vmatrix} 2 & -3 & 3 & 2 & -3 \\ 3 & 10 & -1 & 3 & 10 \\ -1 & -4 & 1 & -1 & -4 \end{vmatrix}$$

$Dy = [(2)(10)(1) + (-3)(-1)(-1) + (3)(3)(-4) - (-1)(10)(3)$
$- (-4)(-1)(2) - (1)(3)(-3)]$

$Dy = [20 - 3 - 36 + 30 - 8 + 9] = 12$

$$y = \frac{Dy}{D} = \frac{12}{6} = 2$$

$$Dz = \begin{vmatrix} 2 & -1 & -3 & 2 & -1 \\ 3 & 3 & 10 & 3 & 3 \\ -1 & -1 & -4 & -1 & -1 \end{vmatrix}$$

$Dz = [(2)(3)(-4) + (-1)(10)(-1) + (-3)(3)(-1) - (-1)(3)(-3)$
$- (-1)(10)(2) - (-4)(3)(-1)]$

$Dz = [-24 + 10 + 9 - 9 + 20 - 12] = -6$

$$z = \frac{Dz}{D} = \frac{-6}{6} = -1$$

Thus, the answer is $(1, 2 - 1)$.

743. $A = \begin{vmatrix} 2 & 3 & 1 \\ -1 & 2 & \textcircled{4} \\ 0 & 5 & 7 \end{vmatrix}$

Let: x = cofactor matrix of A

$$x = \begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} (-1)^{2+3} = -\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix}$$

744. $A = \begin{vmatrix} 3 & \textcircled{1} & 2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{vmatrix}$

Let: x = cofactor matrix of A

$$x = \begin{vmatrix} -2 & 0 \\ 0 & -1 \end{vmatrix} (-1)^{1+2} = -\begin{vmatrix} -2 & 0 \\ 0 & -1 \end{vmatrix}$$

745. Let: A = 3 x 3 matrix
A^{-1} = inverse of matrix A

$A(A^{-1}) = A(\frac{1}{A}) = 1$(unity or identity matrix)

Note: A unity matrix is a matrix whose elements in the main diagonal are all number 1.

$$\text{Unity matrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

746. $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

By inspection, since the resulting product is zero then, x = y = z = 0.

747. $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$1(x) + 1(y) = 2$

$x = 2 - y$ ☞ ❶

$3(x) + 2(y) = 0$ ☞ ❷

Substitute (1) in (2):

$3(2 - y) + 2y = 0$

$6 - 3y + 2y = 0$

$y = 6$

$x = 2 - 6 = -4$

748. $\begin{bmatrix} 1 & 4 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

By inspection, since the resulting product is zero, $x = y = 0$.

749. Since matrix B is a unity matrix, then A x B is equal to matrix A.

$$\begin{vmatrix} 4 & 5 & 0 \\ 6 & 7 & 3 \\ 1 & 2 & 5 \end{vmatrix} \times \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 4 & 5 & 0 \\ 6 & 7 & 3 \\ 1 & 2 & 5 \end{vmatrix}$$

750. $\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} + 2\begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} -2 & 4 \\ 2 & 2 \end{vmatrix}$

$$= \begin{vmatrix} 2-2 & 1+4 \\ -1+2 & 3+2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 5 \\ 1 & 5 \end{vmatrix}$$

751. $\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix}\begin{bmatrix} 3 & 6 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 1(3)+2(4) & 1(6)+2(1) \\ 0(3)+(-5)(4) & 0(6)+(-5)(1) \end{bmatrix}$

$$= \begin{bmatrix} 11 & 8 \\ -20 & -5 \end{bmatrix}$$

752. Note: The transpose of a given matrix is formed by interchanging the rows and columns.

$$A = \begin{vmatrix} 3 & 1 & 2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{vmatrix} \quad A_{transpose} = \begin{vmatrix} 3 & -2 & 0 \\ 1 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix}$$

753. Solving the determinant of the given matrix:

$$D = \begin{vmatrix} 1 & 5 \\ 2 & 9 \end{vmatrix} = 9 - 10 = -1$$

Note: For a 2 x 2 matrix, say, matrix $A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, its inverse is given by:

$$A^{-1} = \frac{1}{D}\begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$$

Thus, $A^{-1} = \dfrac{1}{(-1)}\begin{vmatrix} 9 & -5 \\ -2 & 1 \end{vmatrix} = \begin{vmatrix} -9 & 5 \\ 2 & -1 \end{vmatrix}$

754. $\sin kt = \dfrac{k}{s^2 + k^2}$ } *Formula only*

755. $\cos\ wt = \dfrac{s}{s^2 + w^2}$ } *Formula*

756. Note: $e^{-at} = \dfrac{1}{s+a}$ } *Formula*

Thus, $2e^{-t} - 4e^{-3t} = \dfrac{2}{s+1} - \dfrac{4}{s+3}$

757. Note: $e^{-at}\sin kt = \dfrac{k}{(s+a)^2 + k^2}$ } *Formula*

Thus, $\dfrac{200}{s^2 - 50s + 10625} = 2\left(\dfrac{100}{(s+25)^2 + (100)^2}\right) = 2e^{-25t}\sin 100t$

758. $\dfrac{s}{s^2 + w^2} = \cos wt$ } *Formula*

759. $\dfrac{2s-18}{s^2+9} = \dfrac{2s}{s^2+9} - \dfrac{18}{s^2+9} = 2\left[\dfrac{s}{s^2+9}\right] - 6\left[\dfrac{3}{s^2+3^2}\right]$

Note: $\cos at = \dfrac{s}{s^2 + a^2}$, $\sin at = \dfrac{a}{s^2 + a^2}$ } *Formulas*

Thus, $\dfrac{2s-18}{s^2+9} = 2\cos 3x - 6\sin 3x$

760. By completing the square of the denominator:

$$4s^2 - 8s = 4(s^2 - 2s)$$
$$= 4(s^2 - 2s + 1) - 4$$
$$= 4[(s-1)^2 - 1]$$

$$\frac{1}{4s^2 - 8s} = \frac{1}{4}\left[\frac{1}{(s-1)^2 - 1}\right]$$

Note: $e^{at}\sinh kt = \dfrac{k}{(s-a)^2 - k^2}$ } *Formula*

Thus, $\dfrac{1}{4s^2 - 8s} = \dfrac{1}{4}e^t \sinh t$

DAY 18

PHYSICS

VECTOR & SCALAR QUANTITIES

Vector quanities – are quantities whose measurement is specified by *magnitude* and *direction*. The following are examples of vector quantities: Weight, momentum, torque, velocity, displacement, acceleration, electric field intensity, etc.

Scalar quantities – are those quantities which have only *magnitudes*. The following are examples of scalar quantities: Speed, mass, volume, energy, length, temperature, etc.

Vector – is the line whose length indicates to scale the magnitude of the vector quantity and whose direction indicates the direction of the quantity. The term "vector" comes from Latin "vehere" which means "to carry".

Classifications of vectors:

1. **Free vector** – is one whose action is not confined to or associated with a unique line in space.
2. **Sliding vector** – is one for which a unique line in space must be maintained along which the quantity acts.
3. **Fixed vector** – is one for which a unique point of application is specified and therefore the vector occupies a particular position in space.

VELOCITY AND ACCELERATION

Displacement – is the change in position, specified by a length and a direction.

Speed – is the distance per unit time. Speed is a scalar quantity.

Velocity – is the displacement per unit time. Velocity is a vector quantity.

Acceleration – is the change of velocity per unit time.

Instantaneous acceleration – is the time rate of change of velocity.

Uniformly accelerated motion – is defined as the motion in a straight line in which the direction is always the same and the speed changes at a constant rate.

FORCE AND MOTION

Three Laws of Motion: These laws are commonly known as **"Newton's Laws of Motion".**

Newton's First Law: (The law of inertia)

" There is no change in the motion of a body unless an unbalanced external force is acting upon it."

Newton's Second Law: (The law of acceleration)

" Whenever a net (resultant) force acts on a body, it produces an acceleration in the direction of the resultant force that is directly proportional to the resultant force and inversely proportional to the mass of the body."

$$F = ma$$

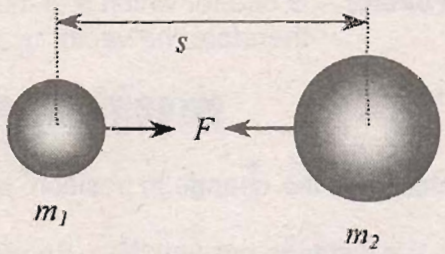
Newton's Third Law: (The law of reaction)

" For every force that acts on one body there is a second force equal in magnitude but opposite in direction that acts upon another body."

Law of Universal Gravitation:

"Every particle in the universe attracts every other particle with a force that is directly proportional to the product of the masses of the two particles and inversely proportional to the square of the distance between their centers of mass."

$$F = \frac{Gm_1m_2}{s^2}$$



where: G = gravitational constant = 6.670×10^{-11} N-m^2/kg^2

Inertia – is the property of the body by virtue of which a resultant force is required to change its motion.

Weight (of a body) – is the resultant gravitational force acting on the body due to all other bodies in space. It is always a vertical force acting downward.

Newton (N) – is the force that will give to a mass of one kilogram an acceleration of one meter per second per second.

Dyne (dyn) – is the force that will give to a mass of one gram an acceleration of one centimeter per second per second.

Poundal – is the force that will give to a mass of one pound an acceleration of one foot per second per second.

Slug – is the mass to which a force of one pound will give an acceleration of one foot per second per second.

Gram force – is one-thousandth the pull of the earth upon a standard kilogram at a place where g has a value of 980.665 cm/s^2.

WORK, ENERGY AND POWER

Work – is the product of force and the displacement in the direction of the force.

$$W = \text{Force} \times \text{distance}$$

Energy – is the property of the body or system of bodies by virtue of which work can be done. It is also defined as the ability to do work. Energy is a scalar quantity.

Potential energy – is also known as the energy of position or configuration.

$$E_p = Wh = mgh$$

Kinetic energy – is the energy in motion.

$$E_k = \frac{1}{2}mv^2$$

Frictional force – a force acting on the body whenever it moves while in contact with another body. This force always opposes the direction of the motion. The frictional force is proportional to the normal force and is directed parallel to the surface.

$$F = \mu N$$ where: μ = coefficient of friction

Coefficient of kinetic friction – is the ratio of the frictional force to the perpendicular force pressing the two surfaces together

$$\mu_k = \frac{F}{N}$$

Coefficient of static friction – is the ratio of the limiting frictional force to the normal force.

$$\mu_s = \frac{F_{max}}{N}$$

Law of Conservation of Energy:

" Energy can neither be created nor destroyed; it merely changes from one form to another."

Transformation of Kinetic & Potential Energy:

Potential Energy = Kinetic Energy

$$mgh = \frac{1}{2}mv^2 \quad \text{or} \quad v = \sqrt{2gh}$$

Transformation of Work & Kinetic Energy:

Work = Kinetic Energy

$$Fs = \frac{1}{2}mv^2$$

Power – is the time rate of doing work.

$$P = \frac{W}{t}$$

IMPULSE AND MOMENTUM

Momentum – is the product of the mass and velocity of a body. Momentum is a vector quantity.

$$p = mv$$

Impulse - is the product of the force and the time during which it acts. Impulse is equal to the change in momentum.

$$\text{Impulse} = F\Delta t = p_2 - p_1 = mv_{final} - mv_{initial}$$

Law of Conservation of Momentum:

"If there is no net external force acting upon a system of bodies, the momentum of the system does not change."

Elastic collision – is a collision of two bodies in which kinetic energy as well as momentum is conserved.

Inelastic collision – is a collision of two bodies in which only the momentum is conserve but not the kinetic energy.

Coefficient of restitution - is the negative ratio of the relative velocity after collision to the relative velocity before collision.

$$e = -\frac{v_{2A} - v_{2B}}{v_{1A} - v_{1B}} = \frac{v_{2A} - v_{2B}}{v_{1B} - v_{1A}}$$

If e = 1, the collision is perfectly elastic while if e = 0, the collision is completely inelastic.

GAS LAWS

Boyle's Law: " If the temperature remains constant, the product of the pressure and volume is constant." This was named after Irish physicist Robert Boyle (1627 – 1691).

$$PV = k$$

Charles' Laws: " If the volume of a confined gas is constant, the pressure is directly proportional to the absolute temperature."

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

" If the pressure of a confined gas is constant, the volume is directly proportional to the absolute temperature."

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

General Gas Law: The combination of the Boyle's Law and the Charles' Laws may be regarded as the general gas law. This is expressed mathematically as

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \text{or} \quad PV = nRT$$

where: $n = \frac{m}{W} = \frac{\text{mass of the gas}}{\text{atomic mass of the gas}}$

R = universal gas constant

FLUIDS AT REST

Density – is the mass per unit volume. This is expressed mathematically as

$$\rho = \frac{m}{V}$$

Weight density – is the weight per unit volume. The relationship between density and weight density is expressed in the following equation.

$$D = \frac{W}{V} \qquad D = \rho g$$

Specific gravity – is the density of the substance relative to that of water. This is also known as **relative density.** Water is considered the standard substance which has a maximum density at 4°C.

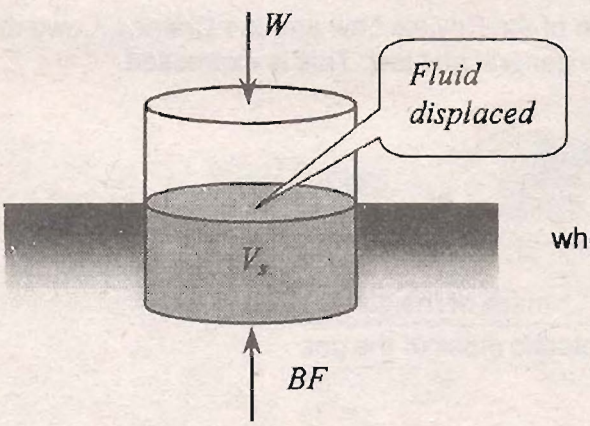
$$\text{specific gravity} = \frac{\text{density}}{\text{density of water}}$$

The values for the density of water are as follows:

62.4 pounds / ft^3
1000 kg / m^3
9.81 kN/m^3
9810 N/m^3
1 gram/cc

The specific gravity of water at densed condition (4°C) is 1.0

Archimedes' Principle: "When a body is immersed (partially or wholly) in a fluid, it is subjected to an upward force (buoyant force) which is equal to the weight of the fluid displaced." This was discovered by Greek mathematician and inventor, Archimedes (287 – 212 B.C.).



$$BF = \rho V_s$$

where: Vs = volume submerged
ρ = density of the fluid

Also by equilibrium, the buoyant force is equal to the weight of the body, thus

$$BF = W$$

Note: The buoyant force is always acting at the centroid (center of gravity) of the submerged volume.



Did you know that... the most difficult problem in Mathematics is the "Fermat's Last Theorem"! The search for the proof of this theorem begun right after Fermat's death in 1665 and remained an unsolved theorem through centuries of hopeless search. A British-born professor in Mathematics at Princeton University, Andrew Wiles brought an end to the search of the proof in 1995, i.e. 330 years later. Because of this, Fermat's Last Theorem was regarded as the Mount Everest of Mathematics!

Proceed to the next page for your 18th test. GOODLUCK ! ☞

Test 18

Time element: 3.0 hours

Problem 761: ME Board October 1994

The weight of a mass of 10 kg at a location where the acceleration of gravity is 9.77 m/s^2 is

A. 79.7 N
B. 77.9 N
C. 97.7 N
D. 977 N

Problem 762: ME Board April 1998

How much does a 30 lbf weigh on the moon? Gravitational acceleration in the moon is 5.47 ft/s^2 and in earth is 32.2 ft/s^2.

A. 2.0 lbf
B. 3.2 lbf
C. 3.4 lbf
D. 5.096 lbf

Problem 763: ME Board October 1994, ME Board April 1998

The mass of air in a room 3m x 5m x 20m is known to be 350 kg. Find its density.

A. 1.167
B. 1.176
C. 1.617
D. 1.716

Problem 764: ME Board April 1996

An iron block weighs 5 N and has a volume of 200 cubic centimeters. What is the density of the block?

A. 988 kg/m^3
B. 1255 kg/m^3
C. 2550 kg/m^3
D. 800 kg/m^3

Problem 765: ME Board October 1997, ME Board April 1998

100 g of water is mixed to 150 g of alcohol (ρ = 790 kg/m^3). Calculate the specific volume of the solution, assuming that it is mixed completely.

A. 0.82 cm^3/g

B. 0.88 cm^3/g
C. 0.63 cm^3/g
D. 1.20 cm^3/g

Problem 766: ME Board October 1997

100 g of water is mixed to 150 g of alcohol (ρ = 790 kg/m^3). Calculate the specific gravity of the total mixture.

A. 1.862
B. 0.963
C. 0.286
D. 0.862

Problem 767: ME Board April 1996

The specific gravity of mercury relative to water is 13.55. What is the specific weight of mercury? The specific weight of water is 62.4 lbf/ft^3.

A. 102.3 kN/m^3
B. 132.9 kN/m^3
C. 150.9 kN/m^3
D. 82.2 kN/m^3

Problem 768: ECE Board November 1998

A 16 gram mass is moving at 30 cm/sec while a 4 gram mass is moving in an opposite direction at 50 cm/sec. They collide head on and stick together. Their velocity after collision is

A. 0.14 m/s
B. 0.21 m/s
C. 0.07 m/s
D. 0.28 m/s

Problem 769: ME Board October 1996

A 60 ton rail car moving at 1 mile/hr is instantaneously coupled to a stationary 40 ton rail car. What is the speed of the coupled cars?

A. 0.88 mph
B. 1 mph
C. 0.6 mph
D. 0.4 mph

Problem 770: EE Board October 1996

A 10 g block slides with a velocity of 20 cm/s on a smooth level surface and makes a collision with a 30 g block moving in the opposite direction with a velocity of 10 cm/s. If the collision is perfectly elastic, what is the velocity of the 30 g block after the collision?

A. 15 cm/s
B. 10 cm/s

C. 25 cm/s
D. 5 cm/s

Problem 771: ME Board April 1997

A 60 ton rail car moving at 1 mile/hr is coupled to a second stationary rail car. If the velocity of the two cars after coupling is 1ft/s (in the original direction of motion) and the coupling is completed in 0.5 second, what is the average impulsive force on the 60 ton rail car?

A. 50 lbf
B. 3500 lbf
C. 1200 lbf
D. 60 lbf

Problem 772: ME Board April 1997

What momentum does a 40 lbm projectile posses if the projectile is moving at 420 mph?

A. 24,640 lbf-sec
B. 16,860 lbf-sec
C. 765 lbf-sec
D. 523.6 lbf-sec

Problem 773: ME Board April 1995, ME Board April 1998

A 10-kg block is raised vertically 3 meters. What is the change in potential energy? Answer in SI units closest to:

A. 320 J
B. 350 kg-m^2/s^2
C. 294 J
D. 350 N-m

Problem 774: ECE Board April 1997

An aircraft engine develops a forward thrust of 15,000 N. If the gross mass of the aircraft is 100 tons, what horsepower does the engine develop if it is flying at 1000 kph?

A. 150,000
B. 5585
C. 5400
D. 3108

Problem 775: EE Board October 1994

If a 10 kg piece of copper falls 100 m, how much heat might be produced?

A. 3.81 kCal
B. 32 BTU
C. 2300 Cal
D. 9.41 kJ

Problem 776: ME Board October 1997

A rocket is moving through a vacuum. It changes its velocity from 9020 ft/sec to 5100 ft/sec in 48 seconds. How much power is required to accomplished this if the rocket's mass is 13,000 slugs?

A. 1.63×10^7 hp
B. 3.16×10^7 hp
C. 3.61×10^7 hp
D. 1.36×10^7 hp

Problem 777: ME Board April 1998

A force of 200 lbf acts on a block at an angle of 28° with respect to horizontal. The block is pushed 2 feet horizontally. What is the work done by this force?

A. 320 J
B. 480 J
C. 540 J
D. 215 J

Problem 778:

What average force is necessary to stop a .45 caliber bullet of mass of 15 grams and speed of 300 m/s as it penetrates a block to a distance of 5 cm?

A. 12.5 kN
B. 13.0 kN
C. 13.5 kN
D. 12.0 kN

Problem 779: ME Board October 1995

What is the water pressure if manometer is 0.6 m Hg? Mercury is 13.6 times heavier than water.

A. 27.4 kPa
B. 47.2 kPa
C. 79.97 kPa
D. 72.4 kPa

Problem 780: EE Board October 1996

A mercury barometer at the base of Mt. Makiling reads 654 mm and at the same time another barometer at the top of the mountain reads 480 mm. Assuming specific weight of air to be constant at 12 N/m^3, what is the approximate height of Mt. Makiling?

A. 1,934.5 m.
B. 3,508.4 m.
C. 4,168.2 m.
D. 2,647.7 m.

Problem 781: EE Board October 1994

Assuming the barometer reads 760 mm Hg, what is the absolute pressure for 900 mm Hg gauge?

A. 74.213 kPa
B. 221.24 kPa
C. 48 kPa
D. 358 kPa

Problem 782: CE Board May 1994

A barometer reads 760 mm Hg and a pressure gage attached to a tank reads 850 cm of oil (sp.gr. 0.80). What is the absolute pressure in the tank in kPa?

A. 168.1 kPa
B. 186.1 kPa
C. 118.6 kPa
D. 161.8 kPa

Problem 783: EE Board April 1996

A sealed tank contains oxygen at 27°C at a pressure of 2 atm. If the temperature increases to 100°C, what will be the pressure inside the tank?

A. 4.92 atm
B. 4.29 atm
C. 2.49 atm
D. 9.24 atm

Problem 784: ME Board April 1996

A volume of 400 cc of air is measured at a pressure of 740 mm Hg abs and a temperature of 18°C. What will be the volume at 760 mm Hg and 0°C?

A. 376 cc
B. 326 cc
C. 356 cc
D. 366 cc

Problem 785: EE Board October 1995

The pressure of the nitrogen gas thermometer is 76 cm at 0°C. What is the temperature of a liquid in which the bulb of the thermometer is immersed when the pressure is seen to be 87.7 cm.

A. 34°C
B. 45°C
C. 60°C
D. 90°C

Problem 786: ME Board April 1998

A transportation company specializes in the shipment of pressurized gaseous materials. An order is received for 100 liters of a particular gas at STP (32°F and 1 atm). What minimum volume tank is necessary to transport the gas at 80°F and a maximum pressure of 8 atm.

A. 16 liters
B. 14 liters
C. 10 liters
D. 12 liters

Problem 787: EE Board April 1996

A 20 liter sample of gas exerts a pressure of 1 atm at 25°C. If it is expanded into a 40 liter vessel that is held at 100°C, what will be its final pressure?

A. 0.50 atm
B. 1.0 atm
C. 0.315 atm
D. 0.63 atm

Problem 788: ME Board October 1997

A 10 lbm object is acted upon by a 4.4 lbf force. What acceleration in ft/s^2 does the object possess?

A. 12.4
B. 10.0
C. 14.2
D. 13.0

Problem 789:

A 50 kN truck traveling with a speed of 50 kph hits a lamp post and is brought to rest in 0.1 s. What is the average force of the truck?

A. – 408 kN
B. – 508 kN
C. – 608 kN
D. – 708 kN

Problem 790:

A tennis ball moving horizontally to the left at 40 m/s hits a racket and rebounds horizontally to the right at 30 m/s. If the mass of the ball is 100 grams, find the impulse of the force (in kg-m/s) exerted on the ball by the racket.

A. 1
B. –1
C. 7
D. 12

Problem 791:

Two steel balls of masses 500 kg and 50 kg, respectively are placed with their centers 0.5 m apart. The two balls attract with a force of

A. 6.67×10^{-10} N
B. 6.67×10^{-7} N
C. 6.67×10^{-6} N
D. 6.67×10^{-3} N

Problem 792: EE Board October 1995

A 50 g mass hangs at the end of the spring. When 20 grams more are added to the end of the spring, it stretches 7 cm more. Find the spring constant.

A. 2.8
B. 2.9
C. 4.3
D. 2.5

Problem 793: EE Board April 1996

Determine the submerged depth of a cube of steel 0.3 m on each side floating in mercury. The specific gravities of steel and mercury are 7.8 and 13.6 respectively.

A. 0.155 m.
B. 0.165 m.
C. 0.134 m.
D. 0.172 m.

Problem 794: EE Board October 1995

A block of wood floats in water with 5 cm projecting above the water surface. When placed in glycerine of specific gravity of 1.35, the block projects 7.5 cm above the liquid. Determine its specific gravity.

A. 0.514
B. 0.704
C. 0.836
D. 0.658

Problem 795: EE Board October 1996

A solid cube material is 0.75 cm on each side. If it floats in oil of density 800 kg/m^3 with one-third of the block out of the oil. What is the density of the material of the cube?

A. 533 kg/m^3
B. 523 kg/m^3
C. 513 kg/m^3
D. 543 kg/m^3

Problem 796: CE Board November 1993

A hollow cylinder 1 m in diameter and 2 m high weighs 2825 N. How many kN of lead weighing 110 kN/m^3 must be fastened to the outside bottom of the cylinder to make it float with 1.5 m submerged in water?

A. 8.5 kN
B. 6.5 kN
C. 10.5 kN
D. 9.5 kN

Problem 797: ME Board October 1995, ME Board October 1996

How long must a current of 5 amperes pass through a 10-ohm resistor until a charge of 1200 coulombs passes through?

A. 3 min.
B. 1 min.
C. 4 min.
D. 2 min.

Problem 798: ME Board April 1996

What is the power required to transfer 97,000 coulombs of charge through a potential rise of 50 volts in one hour?

A. 0.5 kW
B. 1.3 kW
C. 0.9 kW
D. 2.8 kW

Problem 799: EE Board October 1996

How much oil at 200°C must be added to 50 grams of the same oil at 20°C to heat it to 70°C?

A. 12.39 grams
B. 29.12 grams
C. 19.23 grams
D. 23.91 grams

Problem 800: EE Board October 1996

The temperatures of three different liquids are maintained at 15°C, 20°C and 25°C respectively. When equal masses of the first two liquids are mixed, the final temperature is 18°C and when equal masses of the last two are mixed, the final temperature is 24°C. What temperature will be achieved by mixing equal masses of the first and the last liquid?

A. 15.87°C
B. 10.30°C
C. 8.65°C
D. 23.57°C

ANSWER KEY

761. C	771. B	781. B	791. C
762. D	772. C	782. A	792. A
763. A	773. C	783. C	793. D
764. C	774. B	784. D	794. D
765. D	775. C	785. A	795. A
766. D	776. D	786. B	796. A
767. B	777. B	787. D	797. C
768. A	778. C	788. C	798. B
769. C	779. C	789. D	799. C
770. D	780. A	790. C	800. D

RATING

- ☐ 34-40 Topnotcher
- ☐ 26-33 Passer
- ☐ 20-25 Conditional
- ☐ 0-19 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 18

761. $W = mg$

$W = (10\ kg)(9.77\ m/s^2) = 97.7\ N$

762. Mass in earth = Mass in moon

$$\frac{30}{32.2} = \frac{W}{5.47}$$

$W = 5.096$ lbf.

763. Note: Volume of air is the same as the volume of the room

$V = 3(5)(20) = 300\ m^3$

$$\rho = \frac{m}{V} = \frac{350\,kg}{300\,m^3}$$

$\rho = 1.167\ kg/m^3$

764. $W = mg$

$5 = m(9.81)$

$m = 0.5096$ kg.

$$V = 200\ cm^3 \times \left(\frac{1m}{100\,cm}\right)^3 = 0.0002\ m^3$$

$$\rho = \frac{m}{V} = \frac{0.5096\,kg}{0.0002\,m^3}$$

$\rho = 2548\ kg/m^3$

Note: From the choices, the nearest answer is $2550\ kg/m^3$

765. $m_t = m_1 + m_2$

$m_t = 100 + 150 = 250$ g.

$$V_t = V_1 + V_2 = \frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}$$

Note: Density of water (ρ_1) is $1000\ kg/m^3$

$$V_t = \frac{0.100}{1000} + \frac{0.150}{790} = 2.8987 \times 10^{-4}\ m^3 \times \left(\frac{100\,cm}{1m}\right)^3$$

$V_t = 289.87\ cm^3$

$$v = \frac{V_t}{m_t} = \frac{289.87}{250} = 1.2\ cm^3/g$$

766. $m_t = m_1 + m_2$

$m_t = 0.100 + 0.150 = 0.250$ kg.

$$V_t = V_1 + V_2 = \frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}$$

Note: Density of water (ρ_1) is 1000 kg/m^3

$$V_t = \frac{0.100}{1000} + \frac{0.150}{790} = 2.8987 \times 10^{-4}\ m^3$$

$$\rho = \frac{m}{V} = \frac{0.250\ kg}{2.8987 \times 10^{-4}\ m^3} = 862.45\ kg/m^3$$

$$\text{sp. gr.} = \frac{\rho}{\rho_{water}} = \frac{862.45}{1000} = 0.862$$

767. Note: Specific weight of water is 62.4 lbs/ft^3 or 9.81 kN/m^3

$$\text{sp.gr.} = \frac{\omega_m}{\omega_{water}}$$

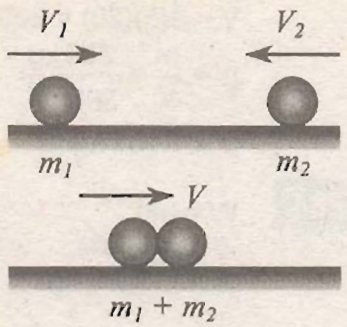
$$\omega_m = 13.55\ (9.81) = 132.9\ kN/m^3$$

768. Initial momentum = Final momentum

$$m_1V_1 + m_2V_2 = (m_1 + m_2)V$$

$$16(0.3) + 4(-0.5) = (16 + 4)V$$

$$V = 0.14\ m/s$$



769. Initial momentum = Final momentum

$$m_1V_1 + m_2V_2 = (m_1 + m_2)V$$

$$60(1) + 40(0) = (60 + 40)V$$

$$V = 0.6\ mph$$

Note: Since the second car is stationary, its velocity (V_2) is zero.

770. Initial momentum = Final momentum

$$m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$$

$$10(20) + 30(-10) = 10(-V_1') + 30V_2'$$

$$-100 = -10V_1' + 30V_2' \quad ☞ ❶$$

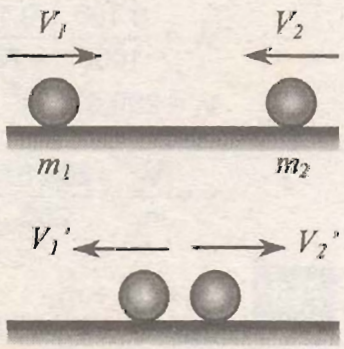
Note: For a perfectly elastic collision,
coefficient of restitution (e) is equal to 1.

$$e = \frac{V_2' - V_1'}{V_1 - V_2} = 1$$

$$V_1 - V_2 = V_2' - V_1'$$

$$20 - (-10) = V_2' - (-V_1')$$

$$V_1' = 30 - V_2' \quad ☞ ❷$$



Substitute (2) in (1):

$$-100 = -10(30 - V_2') + 30V_2'$$

$$-100 = -300 + 10V_2' + 30V_2'$$

$$V_2' = 5\ cm/s$$

771. $V_1 = \frac{1\text{mi}}{\text{hr}} \times \frac{1\text{hr}}{3600\text{ s}} \times \frac{5280\text{ ft}}{1\text{ mi}} = 1.4667\text{ ft/s}$

$F(t) = m(V_1 - V_1')$

$F(0.5) = \frac{60(2000)}{32.2}(1.4667 - 1)$

$F = 3{,}478.509\text{ lbf}$

Note: From the choices, the nearest answer is 3500 lbf.

772. $V = \frac{420\text{ mi}}{\text{hr}} \times \frac{1\text{hr}}{3600\text{ s}} \times \frac{5280\text{ ft}}{1\text{ mi}} = 616\text{ ft/s}$

Momentum = mV

$= 40(616) = 24640\text{ lbm-ft/s}$

$\text{Momentum} = \frac{24640}{32.2} = 765\text{ lbf-sec}$

773. PE = mgh

$= 10(9.81)(3)$

$PE = 294\text{ J}$

774. $V = \frac{1000\text{ km}}{\text{hr}} \times \frac{1\text{hr}}{3600\text{s}} \times \frac{1000\text{ m}}{1\text{km}} = 277.778\text{ m/s}$

$P = FV$

$= 15{,}000 \times 277.778$

$= 4166670\text{ watts} \times \frac{1\text{hp}}{746\text{watts}}$

$P = 5585\text{ hp}$

775. PE = mgh

$= 10(9.81)(1000$

$PE = 9810\text{ J}$

Let: Q = heat

Assuming no losses, the total heat equals the initial PE of the copper.

$Q = 9810\text{ J} \times \frac{\text{Cal}}{4.2\text{ J}} = 2335.714\text{ Cal}$

$= 2335.714\text{ Cal} \times \frac{1\text{ BTU}}{252\text{ Cal}} = 9.268\text{ BTU}$

Note: from the choices, the nearest answer is 2300 Cal

776. $P = \dfrac{\Delta KE}{\Delta t}$

$\Delta KE = \dfrac{1}{2} m(V_0^2 - V^2)$

$= \dfrac{1}{2}(13{,}000)[\,(9020)^2 - (5100)^2\,]$

$\Delta KE = 3.597 \times 10^{11}$ lb-ft

$P = \dfrac{\Delta KE}{\Delta t} = \dfrac{3.597 \times 10^{11}}{48} = 7.49 \times 10^9$ lb-ft/s

$= 7.49 \times 10^9 \text{ lb-ft/s} \times \dfrac{1 \text{ hp}}{550 \text{ lb-ft/s}}$

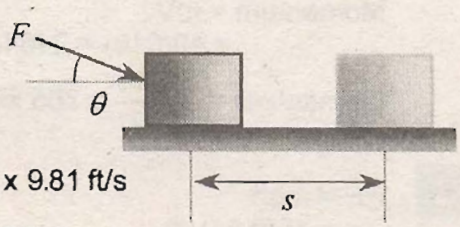
$P = 1.36 \times 10^7$ hp

777. $W = F\cos\theta\,(s)$

$= 200 \cos 28° \times 2$

$= 353.179 \text{ lb-ft} \times \dfrac{1 \text{ kg}}{2.202 \text{ lb}} \times \dfrac{1 \text{ m}}{3.281 \text{ ft}} \times 9.81 \text{ ft/s}$

$W = 479.55$ N-m or J, approximately 480 J



778. Work done by retarding force = initial kinetic energy of the bullet

$F(s) = \dfrac{1}{2}mV^2$

$F(0.05) = \dfrac{1}{2}(0.015)(300)^2$

$= 13{,}500$ N

$F = 13.5$ kN



779. Note: Pressure (P) = Specific weight (ω) x Height (h)

$P = (\omega_{Hg})\,h = (\text{sp.gr.}_{Hg})(\omega_{water})h$

$= 13.6(9.8)(0.6)$

$P = 79.97$ kPa

780. Let: h = height of Mt. Makiling

$P_{bottom} = P_{top} + \omega_{air}\,h$

$(\omega_{Hg})\,h_b = (\omega_{Hg})\,h_t + \omega_{air}\,h$

$(\text{sp.gr.}_{Hg})(\omega_{water})\,h_b = (\text{sp.gr.}_{Hg})(\omega_{water})\,h_t + \omega_{air}\,h$

$13.6(9810)(0.654) = 13.6(9810)(0.48) + 12h$

$h = 1{,}934.5$ m

781. $P_{abs} = P_{gage} + P_{atm}$

$= 13.6(9.81)(0.9) + 13.6(9.81)(0.76)$

$P_{abs} = 221.47$ kPa

Note: From the choices, the nearest answer is 224.24 kPa

782. $P_{abs} = P_{gage} + P_{atm}$

$= 0.8(9.81)(8.5) + 13.6(9.81)(0.76)$

$P_{abs} = 168.1$ kPa

783. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$, Note: $V_1 = V_2$

$$\frac{2}{27+273} = \frac{P_2}{100+273}$$

$P_2 = 2.49$ atm

784. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$$\frac{740(400)}{18+273} = \frac{760(V_2)}{0+273}$$

$V_2 = 365.38$ cc

Note: From the choices, the nearest answer is 366 cc.

785. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$, Note: $V_1 = V_2$

$$\frac{78}{0+273} = \frac{87.7}{T_2}$$

$T_2 = 306.95°K$

$= 306.95 - 273$

$T_2 = 34°C$

786. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$$\frac{1(100)}{32+460} = \frac{8(V_2)}{80+460}$$

$V_2 = 13.7$ liters, approximately 14 liters

787. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$$\frac{1(20)}{25+273} = \frac{P_2(40)}{100+273}$$

$P_2 = 0.63$ atm

788. $F = ma$ ☞ *Newton's second law of motion*

$$4.4 = \left(\frac{10}{32.2}\right)a$$

$a = 14.2$ ft/s^2

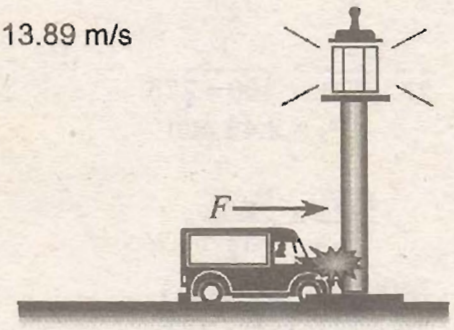
789. Using the relationship between impulse and momentum:
Impulse = change in momentum

$$F\,\Delta t = m\,V_1 - m\,V_0$$

$$m = \frac{50000N}{9.81} = 5096.84kg$$

$$V_0 = \frac{50\text{ km}}{1\text{ hr}} \times \frac{1\text{ hr}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}} = 13.89\text{ m/s}$$

$$F(0.1) = 5096.84(0 - 13.89)$$
$$= -707951\text{ N}$$
$$F = -708\text{ kN}$$



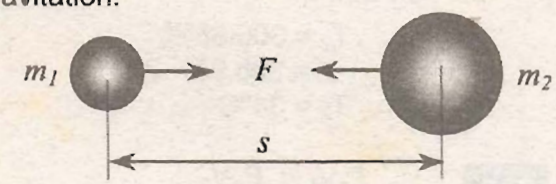
790. Momentum = mass x velocity

$$P_1 = mV_1 = (0.1)(-40)$$
$$= -4\text{ kg-m/s}$$
$$P_2 = mV_2 = (0.1)(30)$$
$$= 3\text{ kg-m/s}$$

Impulse = change in momentum
$$= P_2 - P_1$$
$$= 3 - (-4)$$
Impulse = 7 kg-m/s

791. Using the formula for universal gravitation:

$$F = \frac{Gm_1m_2}{s^2}$$



where : F = force of attraction in N.
m_1 and m_2 = respective masses of two particles in kg.
s = distance between the centers of the two particles in m.

$$G = \text{gravitational constant} = 6.67 \times 10^{-11}\ \frac{N-m^2}{kg^2}$$

$$F = \frac{(6.67x10^{-11})(500)(50)}{(0.5)^2} = 6.67 \times 10^{-6}\text{ N}$$

792. $F = ks$ ☞ *by Hooke's law*
where: F = tension in the spring due to the load attached to it
k = spring constant
s = elongation of the spring due to the load attached to it

$50 = ks$

$$s = \frac{50}{k}$$ ☞ 1

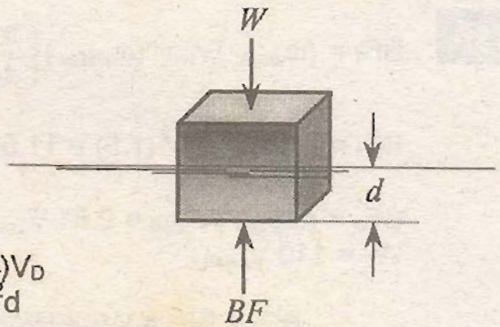
$$F = k(s + 7)$$
$$50 + 20 = k\,(s + 7)$$
$$70 = ks + 7k$$ ☞ 2

Substitute (1) in (2):

$$70 = k\frac{50}{k} + 7k$$
$$k = 2.8$$

793.

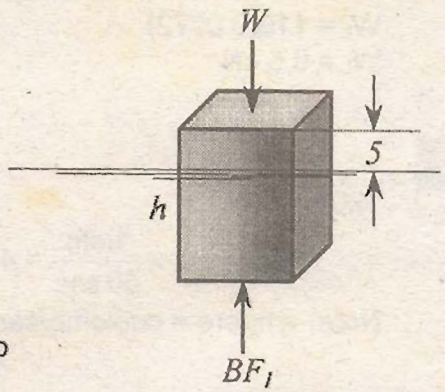
$$\sum F_V = 0$$
$$W = BF$$
$$\omega_s V_t = \omega_{Hg} V_D$$
$$(\text{sp.gr.}_s)(\omega_{water})V_t = (\text{sp.gr.}_{Hg})(\omega_{water})V_D$$
$$7.81(9.81)(0.3)^3 = 13.6(9.81)(0.3)^2 d$$
$$d = 0.172 \text{ m.}$$



794. Let: A = base area
h = height of the wood

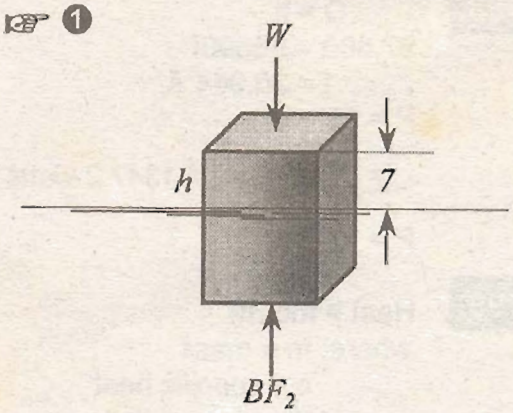
$$W = BF_1$$
$$(\text{sp.gr.}_w)(\omega_{water})(V_l) = (\omega_{water})V_D$$
$$\text{sp.gr.}_w\,(Ah) = A(h - 5)$$
$$\text{sp.gr.}_w = \frac{h - 5}{h}$$ ☞ 1

$$W = BF_2$$
$$(\text{sp.gr.}_w)(\omega_{water})(V_l) = (\text{sp.gr.}_g)(\omega_{water})V_D$$
$$\text{sp.gr.}_w\,(Ah) = (1.35)(A)(h - 7)$$
$$\text{sp.gr.}_w = \frac{1.35(h - 7.5)}{h}$$ ☞ 1



Equate (1) and (2):

$$\frac{h - 5}{h} = \frac{1.35(h - 7.5)}{h}$$
$$h - 5 = 1.35h - 10.125$$
$$h = 14.64 \text{ cm.}$$

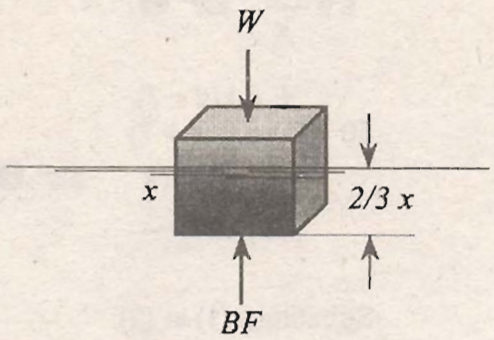


Substitute in (1):

$$\text{sp.gr.}_w = \frac{14.64 - 5}{14.64} = 0.658$$

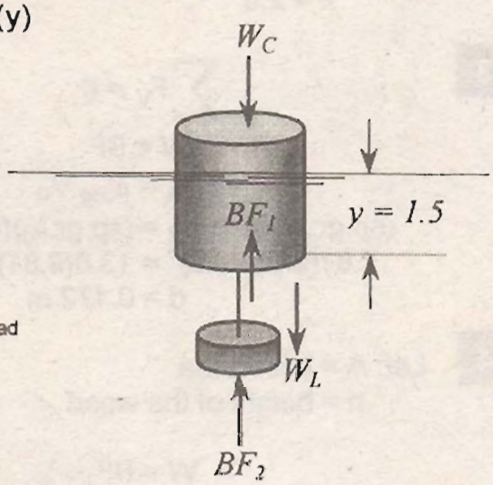
795.

$$W = BF$$
$$\rho_c V_t = \rho_{oil} V_D$$
$$\rho_c (x)^3 = 800\left(\frac{2}{3}x\right)(x)^2$$
$$\rho_c (0.0075)^3 = 800\left(\frac{2}{3}\right)(0.0075)^3$$
$$\rho_c = 533.33 \text{ kg/m}^3$$



796.

$$BF_1 = (\omega_{water})V_D = (\omega_{water})\left(\frac{\pi}{4}\right)(d)^2(y)$$
$$BF_1 = 9.81\left(\frac{\pi}{4}\right)(1)^2(1.5) = 11.56 \text{ kN}$$
$$BF_2 = (\omega_{water})V_{Lead} = 9.81\ V_{Lead}$$
$$W_L = 110\ V_{Lead}$$
$$BF_1 + BF_2 = W_C + W_L$$
$$11.56 + 9.81V_{Lead} = 3.825 + 110V_{Lead}$$
$$V_{Lead} = 0.0772 \text{ m}^3$$
$$W_L = 110(0.0772)$$
$$W_L = 8.5 \text{ kN}$$



797.

$$Q = It$$
$$1200 = 5(t)$$
$$t = 240 \text{ sec} \times \frac{1\text{min}}{60 \text{ sec}} = 4 \text{ minutes}$$

Note: ampere = coulomb/sec.

798.

$$Q = It$$
$$97{,}000 = I(3600)$$
$$I = 26.944 \text{ A.}$$
$$P = EI$$
$$= 50(26.944) = 1347.2 \text{ watts} \times \frac{1\text{kW}}{1000 \text{ watts}}$$
$$P = 1.3 \text{ kW}$$

799.

Heat = $mc(\Delta t)$
where: m = mass
c = specific heat
Δt = change in temperature

Heat gained = Heat loss
$50c(70 - 20) = mc(200 - 70)$
m = 19.23 grams

800.

Heat gained = Heat loss

$mc_1(18 - 15) = mc_2(20 - 18)$

$3c_1 = 2c_2$ ☞ ❶

Substitute (2) in (1):

$3c_1 = 2(0.25c_3)$

$c_1 = 0.1667c_3$

Heat gained = Heat loss

$mc_1(t - 15) = mc_3(25 - t)$

$0.1667c_3(t - 15) = c_3(25 - t)$

$0.1667t - 2.5 = 25 - t$

$t = 23.57°C$

Heat gained = Heat loss

$mc_2(24 - 20) = mc_3(25 - 24)$

$4c_2 = c_3$

$c_2 = 0.25\ c_3$ ☞ ❷

DAY 19

ENGINEERING MECHANICS
STATICS

Mechanics is the oldest branch of physical science which deals with the state of rest or motion of bodies under the action of forces.

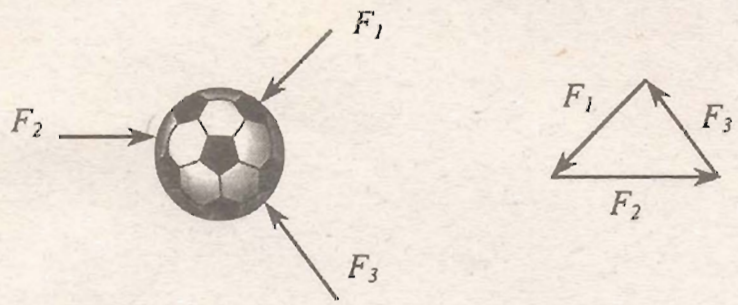
Branches of mechanics:

A. **Statics** – deals with bodies in the state of rest.
B. **Dynamics** or **Kinetics** – deals with bodies in motion under the action of forces.
C. **Kinematics** – refers to the study of motion without reference to the forces which causes the motion.

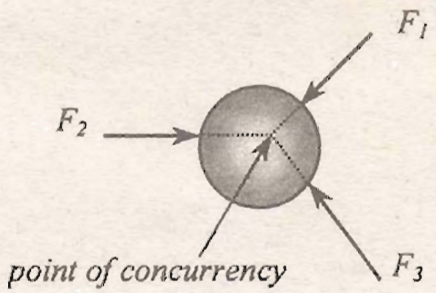
STATICS

Conditions for equilibrium:

1. **Graphical condition**: Under this condition, the forces or vectors are transformed into a force polygon. For equilibrium, the force polygon must close.



2. **Directional condition**: If three or more non-parallel forces or vectors are in equilibrium, then they must be concurrent.



3. **Analytical condition**: If forces or vectors are in equilibrium, then it must satisfy the three static equations; namely

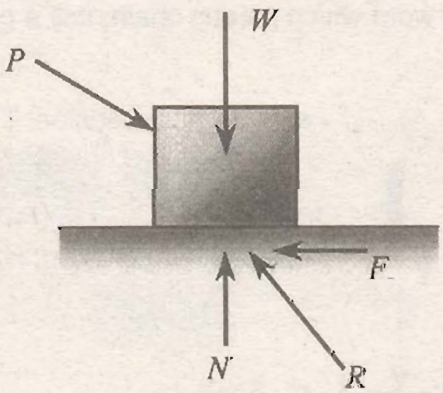
$\Sigma F_x = 0$ → = ←

$\Sigma F_y = 0$ ↑ = ↓

$\Sigma M_x = 0$ ↻ = ↺

FRICTION

Friction is defined as the limited amount of **resistance** to sliding between the surfaces of two bodies in contact.



$$F = \mu N$$

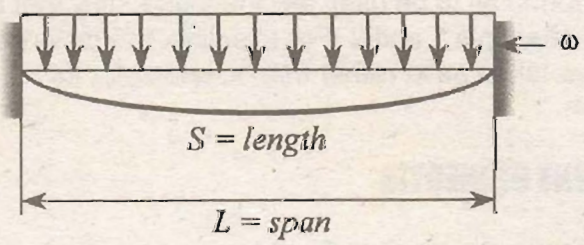
where: F = frictional force
μ = coefficient of friction
N = normal force
ϕ = angle of friction



$$\tan\phi = \mu$$

PARABOLIC CABLE & CATENARY

Parabolic cable: When the loading is uniformly distributed horizontally, the cable is analyzed as a parabolic cable.

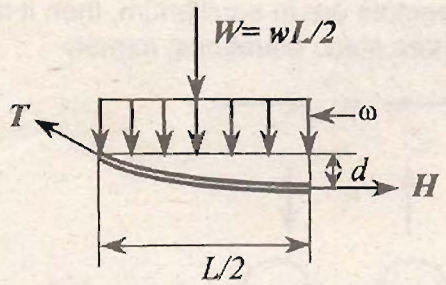


a) Tension at the lowest point, H:

$$H = \frac{\omega L^2}{8d}$$

b) Tension at the support, T:

$$T = \sqrt{H^2 + \left(\frac{\omega L}{2}\right)^2}$$

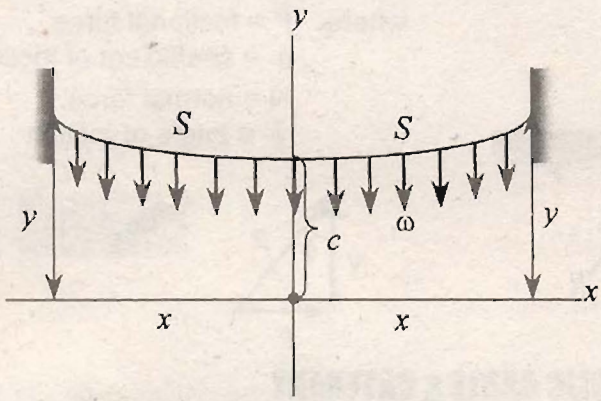


c) Length of parabolic cable, S:

$$S = L + \frac{8d^2}{3L} - \frac{32d^4}{5L^3}$$

where: S = length of parabolic cable
d = sag
L = span or distance between supports
ω = unit weight or load per unit length
T = maximum tension (usually at the support)
H = minimum tension (usually at the lowest point of the cable)

Catenary: When the loading is distributed along the cable, the cable is analyzed as a catenary. Catenary comes from the latin word which means chain. It is a graph of the equation y = cosh x.



a) Maximum tension, T:

$$T = \omega y \qquad y^2 = S^2 + c^2$$

b) Minimum tension, H:

$$H = \omega c$$

c) Span, L:

$$L = 2x \qquad x = c \ln \frac{S+y}{c}$$

d) Length $\text{length} = 2S$

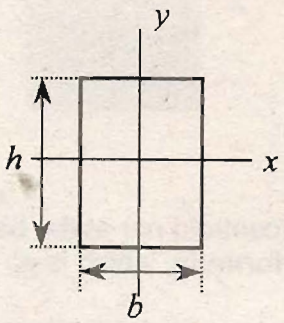
If the cable has uneven supports, the formulas to be used are the same, only that all unequal dimensions will now contain subscripts 1 and 2. For example, the distance from the origin to the left support is now taken as x_1 rather than x, and x_2 for its distance to the right support, and so on.

MOMENT OF INERTIA

Another term for moment of inertia is **second moment of area.**

1. Centroidal moment of inertia (with respect to an axis passing through the centroid):

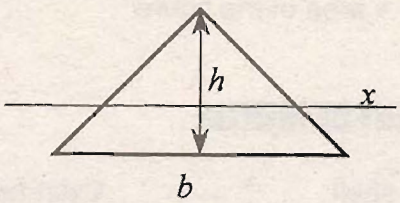
A. Rectangle



$$I_x = \frac{bh^3}{12}$$

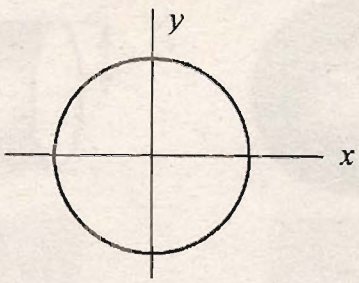
$$I_y = \frac{b^3h}{12}$$

B. Triangle



$$I_x = \frac{bh^3}{36}$$

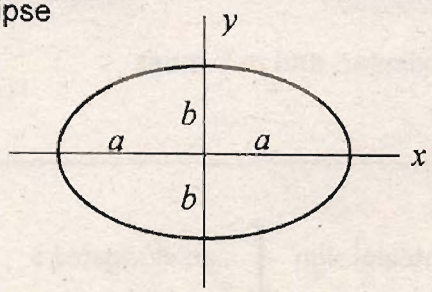
C. Circle



$$I_x = I_y$$

$$I_x = \frac{\pi r^4}{4} = \frac{\pi D^4}{64}$$

D. Ellipse

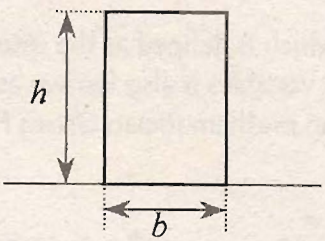


$$I_x = \frac{\pi ab^3}{4}$$

$$I_y = \frac{\pi a^3 b}{4}$$

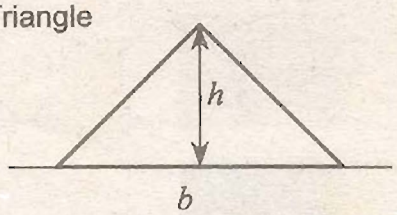
2. Moment of inertia with respect to an axis passing through the base:

A. Rectangle



$$I_b = \frac{bh^3}{3}$$

B. Triangle



$$I_b = \frac{bh^3}{12}$$

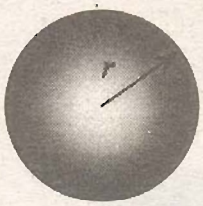
For composite figures and for axis not at the centroid nor at the base, moment of inertia may be calculated using the **transfer formula**, which is as follows;

$$I = I_g + Ad^2$$

where: d = distance from the centroid to the axis
A = area of the figure

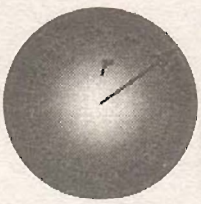
MASS MOMENT OF INERTIA

Sphere



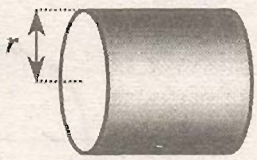
$$I = \frac{2}{5}mr^2$$

Spherical shell



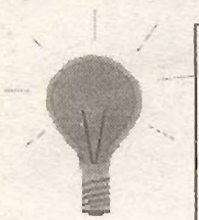
$$I = \frac{2}{3}mr^2$$

Cylinder



$$I = \frac{1}{2}mr^2$$

where: r = radius of sphere/cylinder, and m = mass



Did you know that... The integral sign $\int$, an elongated S denoting sum (Latin for "summa"), was introduced by Gottfried Wilhelm Leibniz, who named integral calculus "calculus summatorius" !

... The definite integral which is defined as the integral between two values of an independent variables is also known as "Riemann Integral" after the German mathematician Georg Friedrich Bernhard Riemann (1826 – 1866)!

Proceed to the next page for your 19th test. GOODLUCK ! ☞

Test 19

Time element: 3.0 hours & 30 minutes

Problem 801:
Three forces, 20 N, 30 N and 40 N are in equilibrium. Find the largest angle they make with each other.

A. 104.48°
B. 105.58°
C. 106.69°
D. 107.96°

Problem802: ME Board October 1996
Two forces of 20 units and 30 units act at right angle. What is the magnitude of the resultant force?

A. 36
B. 42
C. 40
D. 44

Problem803:
What is the magnitude of the resultant force of the two forces 200 N at 20° and 400 N at 144°?

A. 332.5 N
B. 323.5 N
C. 313.5 N
D. 233.5 N

Problem804: ECE Board November 1998
A load of 100 lb is hung from the middle of a rope, which is stretched between two rigid walls 30 ft. apart. Due to the load, the rope sags 4 feet in the middle. Determine the tension in the rope.

A. 165 lbs
B. 173 lbs
C. 194 lbs
D. 149 lbs

Problem805:

A boat moving at 12 kph is crossing a river 500 m wide in which a current is flowing at 4 kph. In what direction should the boat head if it is to reach a point on the other side of the river directly opposite its starting point?

A. 19.47° downstream
B. 19.47° upstream
C. 18.43° downstream
D. 18.43° upstream

Problem806: EE Board October 1997

A 100 kg weight rest on a 30° incline plane. Neglecting friction, how much pull must one exert to bring the weight up the plane?

A. 88.67 kg
B. 100 kg
C. 70.71 kg
D. 50 kg

Problem 807: ECE Board November 1998

A block weighing 500 kN rest on a ramp inclined at 25° with the horizontal. The force tending to move the block down the ramp is _____.

A. 121 kN
B. 265 kN
C. 211 kN
D. 450 kN

Problem 808: CE Board November 1994

A 200 kg crate impends to slide down a ramp inclined at an angle of 19.29° with the horizontal. What is the frictional resistance?

A. 612.38 N
B. 628.38 N
C. 648.16 N
D. 654.12 N

Problem 809: EE Board October 1993

A man can exert a maximum pull of 1000 N but wishes to lift a new stone door for his cave weighing 20,000 N. If he uses a lever, how much closer must the fulcrum be to the stone than to his hand?

A. 10 times nearer
B. 20 times farther
C. 10 times farther
D. 20 times nearer

Problem 810:

A beam rests on a fulcrum, 1.2 m from one end. A weight of 350 kg is suspended from this end causing the beam to balance. If the weight of 350 kg is suspended on the opposite end of the beam, it is necessary to suspend a 1000 kg weight on the first end in order to effect an even balance. Find the length of the beam.

A. 2.48 m
B. 3.24 m
C. 3.43 m
D. 4.21 m

Problem 811: EE Board October 1991

A simply supported beam is 5 meters in length. It carries a uniformly distributed load including its own weight of 300 N/m and a concentrated load of 100 N, 2 meters from the left end. Find the reactions if reaction A is at the left end and reaction B at the right end.

A. $R_A = 810$ N & $R_B = 700$ N
B. $R_A = 700$ N & $R_B = 800$ N
C. $R_A = 810$ N & $R_B = 780$ N
D. $R_A = 700$ N & $R_B = 810$ N

Problem 812:

A beam of span "x" meters with uniform loading of "w" kilograms per meter is supported at one end (A) and a distance of 2 m from the other end (B). Find the reaction at support A.

A. $\dfrac{wx^2}{2(x-2)}$ kg.

B. $\dfrac{wx(x-4)}{2(x-2)}$ kg.

C. $\dfrac{wx(x-2)}{2(x-2)}$ kg.

D. $\dfrac{wx}{2(x-2)}$ kg.

Problem 813:

When one boy is sitting 1.2 m from the center of a see-saw, another boy must to sit on the other side 1.5 m from the center to maintain an even balance. However, when the first boy carries an additional weight of 14 kg and sit 1.8 m from the center, the second boy must move to 3 m from the center to balance. Neglecting the weight of the see-saw, find the weight of the heavier boy.

A. 30 kg
B. 42 kg
C. 34 kg
D. 45 kg

Problem 814: CE Board November 1996

A 40 kg block is resting on an inclined plane making an angle of 20° from the horizontal. If the coefficient of friction is 0.60, determine the force parallel to the incline that must be applied to cause impending motion down the plane.

A. 82
B. 77
C. 87
D. 72

Problem 815: EE Board October 1997

A 250 lb block is initially at rest on a flat surface that is inclined at 30°. If the coefficient of kinetic friction 0.30 and the coefficient of static friction is 0.40, find the force required to start the block moving up the plane.

A. 190 lb
B. 212 lb
C. 125 lb
D. 75 lb

Problem 816:

A 600 N block rests in a surface inclined at 30°. Determine the horizontal force P required to prevent the block from sliding down. Angle of friction between the block and the inclined plane is 15°.

A. 160.75 N
B. 198.55 N
C. 164.60 N
D. 190.45 N

Problem 817: ME Board March 1998

Assume the three force vectors intersect at a single point.

$F_1 = 4i + 2j + 5k$
$F_2 = -2i + 7j - 3k$
$F_3 = 2i - j + 6k$

What is the magnitude of the resultant force vector, R?

A. 14
B. 12
C. 13
D. 15

Problem 818: EE Board March 1998

Given the 3-dimensional vectors:

$A = i\,(xy) + j\,(2yz) + k\,(3zx)$
$B = i\,(yz) + j\,(2zx) + k\,(3xy)$

Determine the magnitude of the vector sum $|A + B|$ at coordinates (3,2,1).

A. 32.92

B. 29.92
C. 27.20
D. 24.73

Problem 819:

At what angle does the force F = 6.23i – 2.38j + 4.92 k N makes with the x-axis?

A. 39.2°
B. 40.2°
C. 41.3°
D. 42.2°

Problem 820: ME Board October 1996

Assume the three force vectors intersect at a single point.

$F_1 = i + 3j + 4k$
$F_2 = 2i + 7j - k$
$F_3 = -i + 4j + 2k$

What is the magnitude of the resultant force vector, R?

A. 15
B. 13.23
C. 14.73
D. 16.16

Problem 821: EE Board October 1991

A certain cable is suspended between two supports at the same elevation and 500 ft apart, the load is 500 lbs per horizontal foot including the weight of the cable. The sag of the cable is 30 ft. Calculate the total length of the cable.

A. 503.76 ft.
B. 502.76 ft
C. 504.76 ft
D. 501.76 ft

Problem 822: EE Board April 1994

A cable supported at two points of same level has a unit weight , ω of 0.02 kg per meter of horizontal distance. The allowed sag is 0.02 m and a maximum tension at the lowest point of 1200 kg and a factor of safety of 2. Calculate the allowable spacing of the poles assuming a parabolic cable.

A. 64.02 m
B. 66.37 m
C. 67.76 m
D. 69.28 m

Problem 823:

A cable carries a horizontal load of 20 kg/m. Neglecting its own weight, find the maximum tension on the cable if the distance between the supports is 100 m and the sag is 5 m.

A. 5099 kg

B. 5059 kg
C. 5199 kg
D. 5215 kg

Problem 824: CE Board May 1993

Determine the sag of a flexible wire cable weighing 60 N/m over two frictionless pulleys 100 m apart and carrying one 10 kN weight at each end. Assume the weight of the cable to be uniformly distributed horizontally. The cable extends 5 m beyond each pulley to the point they are attached to the weights.

A. 7.2 m
B. 7.4 m
C. 7.6 m
D. 7.8 m

Problem 825: EE Board October 1993

A copper cable is suspended between two supports on the same level, spaced 600 m apart. The cable hangs under the influence of its own weight only. Under these conditions, it is desired to calculate the maximum sag (at the center of the span) when the maximum stress in the material is 1000 kg/cm^2. The cross-section of the cable is 1.77 sq. cm. Weight of cable = 1.6 kg/m. Use parabolic equation.

A. 42.26 m
B. 43.26 m
C. 44.26 m
D. 45.26 m

Problem 826:

A cable weighing 0.4 kg/m and 800 m long is to be suspended with a sag of 80 m. Determine the maximum tension.

A. 414 kg
B. 420 kg
C. 416 kg
D. 400 kg

Problem 827:

A cable weighing 60 N/m is suspended between two supports on the same level at 300 m apart. The sag is 60 m. Compute the distance of the lowest point of the cable from the ground level.

A. 205.5 m
B. 196.8 m
C. 200.5 m
D. 188.2 m

Problem 828:

Find the location of the centroid of the composite area consisting of a 10-inch square surmounted by a semi-circle. The centroid of a semicircle is located $4r/3\pi$ above the base (diameter) of the semi-circle of radius r.

A. 6.0 inches from the bottom
B. 6.2 inches from the bottom
C. 6.4 inches from the bottom
D. 7.0 inches from the bottom

Problem 829: EE Board March 1998

Electrical loads are arranged on horizontal x, y axes as follows:

Load	x-coordinate	y-coordinate	Kilowatt load
1	0	2	100
2	1	1	180
3	1	3	200
4	2	0	120
5	2	4	150
6	3	1	200
7	3	3	180
8	4	2	100

A. x = 2.000, y = 2.049
B. x = 2.163, y = 2.195
C. x = 1.854, y = 2.211
D. x = 2.146, y = 1.902

Problem 830:

A rectangle has a base of 3 cm and a height of 6 cm. What is its second moment of area (in cm^4) about an axis through the center of gravity and parallel to the base?

A. 64
B. 34
C. 44
D. 54

Problem 831: EE Board March 1998

A circle has a diameter of 20 cm. Determine the moment of inertia of the circular area relative to the axis perpendicular to the area through the center of the circle in cm^4.

A. 14,280
B. 15,708
C. 17,279
D. 19,007

Problem 832: ME Board October 1993

The moment of inertia of a section 2" wide x 2' 0" high about an axis 1'0" above the bottom edge of the section is:

A. 1834 in^4
B. 384 in^4
C. 9214 in^4
D. 2304 in^4

Problem 833: EE Board March 1998

An isosceles triangle has a 10 cm base and a 10 cm altitude. Determine the moment of inertia of the triangular area relative to a line parallel to the base and through the upper vertex in cm^4.

A. 2750
B. 3025
C. 2500
D. 2273

Problem 834: ECE Board April 1999

What is the moment of inertia of a cylinder of radius 5 m and mass of 5 kg?

A. 120 kg-m^2
B. 80 kg-m^2
C. 62.5 kg-m^2
D. 72.5 kg-m^2

Problem 835: ECE Board April 1998

What is the inertia of a bowling ball (mass = 0.5 kg) of radius 15 cm rotating at an angular speed of 10 rpm for 6 seconds?

A. 0.001 kg-m^2
B. 0.002 kg-m^2
C. 0.005 kg-m^2
D. 0.0045 kg-m^2

ANSWER KEY

801. A	811. A	821. C	831. B
802. A	812. B	822. D	832. D
803. A	813. B	823. A	833. C
804. C	814. C	824. C	834. C
805. B	815. B	825. A	835. D
806. D	816. A	826. C	
807. C	817. B	827. B	
808. C	818. B	828. D	
809. D	819. C	829. A	
810. C	820. A	830. D	

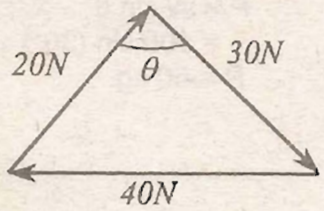
RATING

- ☐ 30-35 Topnotcher
- ☐ 21-29 Passer
- ☐ 17-20 Conditional
- ☐ 0-16 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 19

801. Note: For forces under equilibrium, the force polygon constructed from these forces must be closed.



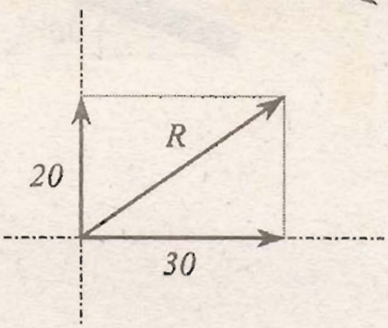
By cosine law:

$(40)^2 = (20)^2 + (30)^2 - 2(20)(30)\cos\theta$

$\theta = 104.48$

802. $R = \sqrt{(20)^2 + (30)^2}$

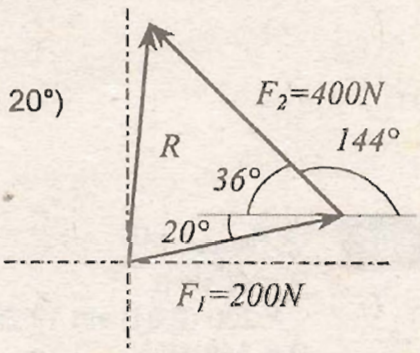
$R = 36$ units



803. By cosine law:

$R^2 = (200)^2 + (400)^2 - 2(200)(400)\cos(36° + 20°)$

$R = 332.5$ N



804. $\tan\theta = \frac{15}{4}$

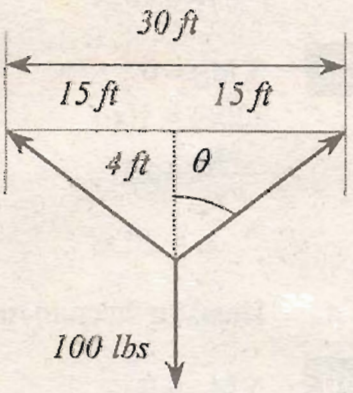
$\theta = 75.068°$

$\sum F_V = 0$

$2T\cos\theta = 100$

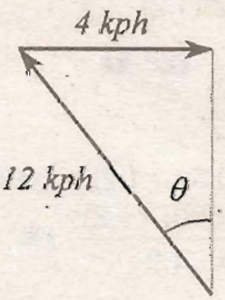
$T = \frac{100}{2\cos\theta}$

$T = \frac{100}{2\cos(75.068°)} = 194$ lbs.



805 $\sin\theta = \frac{4}{12}$

$\theta = 19.47°$, upstream.

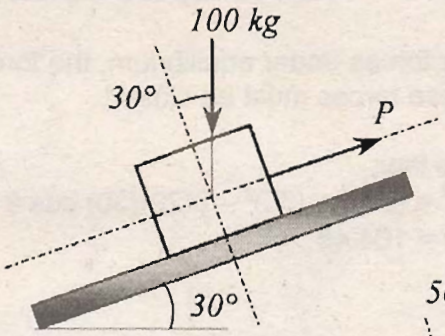


806. $\sum F_{inclined} = 0$

$P = W \sin \theta$

$= 100 \sin (30°)$

$P = 50$ kg.

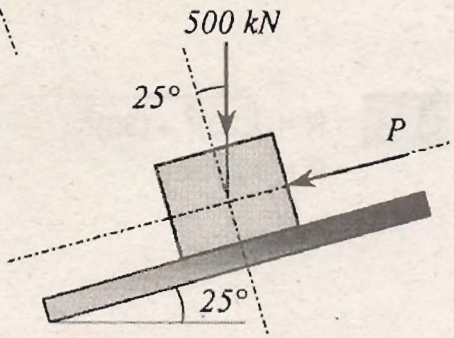


807. $\sum F_{inclined} = 0$

$P = W \sin \theta$

$= 500 \sin 25°$

$P = 211$ kN

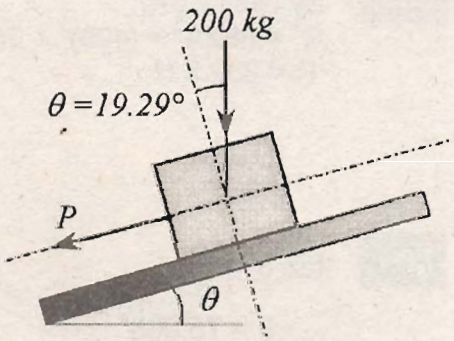


808. $\sum F_{inclided} = 0$

$P = W \sin \theta$

$= 200\ (9.81) \sin 19.29°$

$P = 648.15$ N

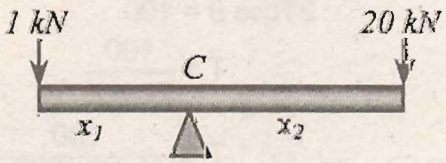


809. $\sum M_C = 0$

$20(x_2) = 1(x_1)$

$\frac{x_1}{x_2} = 20$

$x_1 = 20\, x_2$



Thus, the fulcrum must be placed 20 times nearer.

810. $\sum M_C = 0$

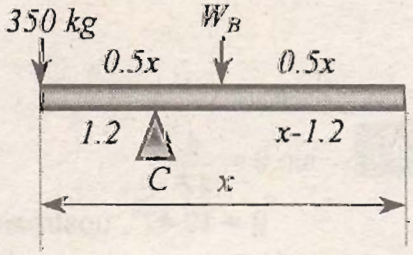
$W_B(0.5x - 1.2) = 350(1.2)$

$$W_B = \frac{420}{0.5x - 1.2}$$ ☞ ❶

$\sum M_C = 0$

$350(x - 1.2) + W_B(0.5x - 1.2) = 1000(1.2)$

$$W_B = \frac{1200 - 350(x - 1.2)}{0.5x - 1.2}$$ ☞ ❷



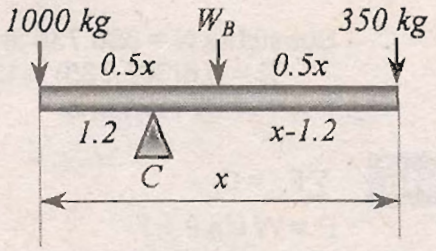
Equate (1) to (2):

$$\frac{420}{0.5x-1.2} = \frac{1200-350(x-1.2)}{0.5x-1.2}$$

$$420 = 1200 - 350x + 420$$

$$350x = 1200$$

$$x = 3.43 \text{ m.}$$



811. $\sum M_A = 0$

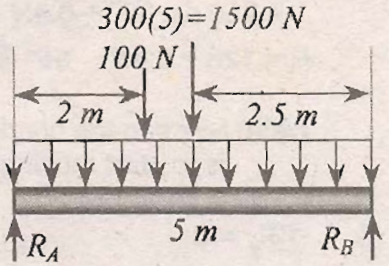
$$100(2) + 1500(2.5) - R_B(5) = 0$$

$$R_B = 700 \text{ N}$$

$$\sum M_B = 0$$

$$R_A(5) - 100(3) - 1500(2.5) = 0$$

$$R_A = 810$$

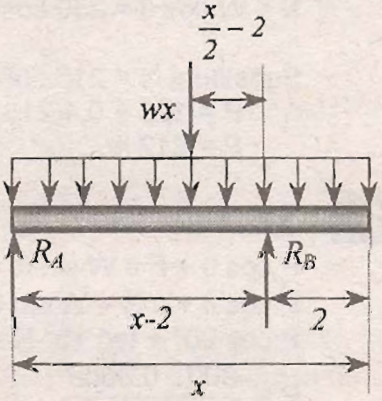


812. $\sum M_B = 0$

$$R_A(x-2) = wx\left(\frac{x}{2}-2\right)$$

$$= wx\left(\frac{x-4}{2}\right)$$

$$R_A = \frac{wx(x-4)}{2(x-2)} \text{ kg.}$$



813. $\sum M_C = 0$

$$W_B(1.5) - W_A(1.2) = 0$$

$$W_B = 0.8\, W_A$$

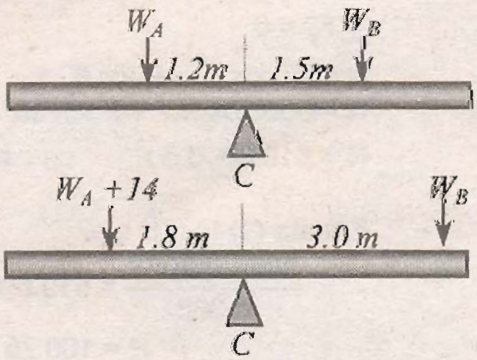
$$\sum M_C = 0$$

$$W_B(3) - (W_A + 14)(1.8) = 0$$

$$(0.8\, W_A)(3) - (W_A + 14)(1.8) = 0$$

$$2.4\, W_A - 1.8\, W_A - 25.2 = 0$$

$$W_A = 42 \text{ kg.}$$



814. $\sum F_x = 0$

$$F = W \sin\theta + P$$

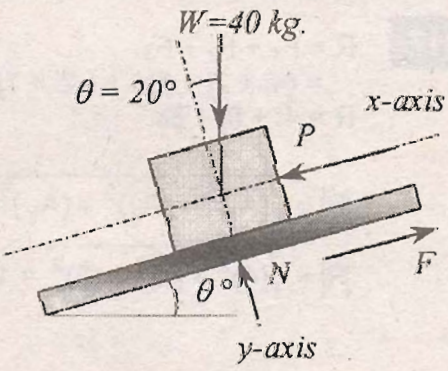
$$\mu N = W \sin\theta + P$$

$$0.6N = 40(9.81)\sin 20° + P$$

$$P = 0.6N - 134.208$$ ☞ ❶

$$\sum F_y = 0$$

$$N = W \cos 20°$$



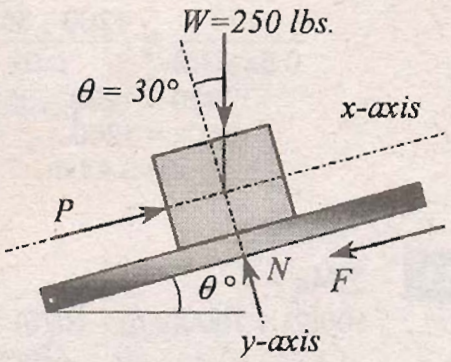
N = 40(9.81) cos 20° = 368.735

Substitute N = 368.735 in (1):
P = 0.6(368.735) – 134.208
P = 87 Newtons

815. $\sum F_x = 0$

$P = W \sin\theta + F$
$= W \sin\theta + \mu N$
$= 250 \sin 30° + 0.4N$

$P = 125 + 0.4N$ ☞ ❶



Note: Since in the condition of the problem, the block is to start moving, then use the coefficient of static friction.

$\sum F_y = 0$

$N = W \cos\theta = 250 \cos 30° = 216.506$ lb.

Substitute N = 216.506 in (1):
P = 125 + 0.4(216.506)
P = 212 lb.

816. $\sum F_x = 0$

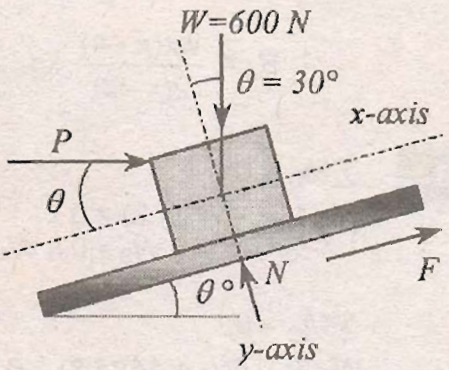
$P \cos\theta + F = W \sin\theta$
$P \cos\theta + \mu N = W \sin\theta$
$P \cos 30° + \tan 15° N = 600 \sin 30°$

$$P = \frac{300 - 0.866P}{0.268}$$ ☞ ❶

$\sum F_y = 0$

$N = W \cos\theta + P \sin\theta$
$N = 600 \cos 30° + P \sin 30°$

$N = 519.6 + 0.5P$ ☞ ❷



Equate (1) to (2):

$$\frac{300 - 0.866P}{0.268} = 519.6 + 0.5P$$

P = 160.75 Newtons

817. $R = F_1 + F_2 + F_3$
$= (4i + 2j + 5k) + (-2i + 7j - 3k) + (2i - j + 6k)$
$R = 4i + 8j + 8k$

$$|R| = \sqrt{(A_i)^2 + (A_j)^2 + (A_k)^2}$$

$$|R| = \sqrt{(4)^2 + (8)^2 + (8)^2} = 12 \text{ units}$$

818. Substitute the values of x,y and z to the two vectors:

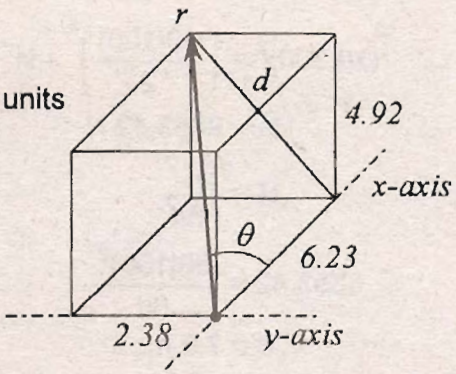
$A = i\,(xy) + j\,(2yz) + k\,(3zx)$
$= i\,(3)(2) + j\,(2)(2)(1) + k\,(3)(1)(3)$
$A = 6i + 4j + 9k$

$B = i\,(yz) + j\,(2zx) + k\,(3xy)$
$= i\,(2)(1) + j\,(2)(1)(3) + k\,(3)(3)(2)$
$B = 2i + 6j + 18k$

$A + B = (6 + 2)i + (4 + 6)j + (9 + 18)k$
$A + B = 8i + 10j + 27k$

$$|A+B| = \sqrt{(A_i)^2 + (A_j)^2 + (A_k)^2}$$

$$|A+B| = \sqrt{(8)^2 + (10)^2 + (27)^2} = 29.88 \text{ units}$$



819. $r = \sqrt{(6.23)^2 + (2.38)^2 + (4.92)^2} = 8.287$

$d = \sqrt{(2.38)^2 + (4.92)^2} = 5.465$

By cosine law:

$$d^2 = r^2 + x^2 - 2(r)(x)\cos\theta$$
$$(5.465)^2 = (8.287)^2 + (6.23)^2 - 2(8.287)(6.23)\cos\theta$$
$$\cos\theta = 0.7517339$$
$$\theta = 41.3°$$

820. $R = F_1 + F_2 + F_3$
$= (i + 3j + 4k) + (2i + 7j - k) + (-i + 4j + 2k)$
$R = 2i + 14j + 5k$

$$|R| = \sqrt{(A_i)^2 + (A_j)^2 + (A_k)^2}$$

$$|R| = \sqrt{(2)^2 + (14)^2 + (5)^2} = 15 \text{ units}$$

821. Let: S = length of the cable

$$S = L + \frac{8d^2}{3L} - \frac{32d^4}{5L^3} = 500 + \frac{8(30)^2}{3(500)} - \frac{32(30)^4}{5(500)^3} = 504.76 \text{ ft.}$$

822. $$H = \frac{\omega L^2}{8d}$$

$$L^2 = H\left[\frac{8d}{\omega}\right] = \frac{1200}{2}\left[\frac{8(0.02)}{0.02}\right] = 4{,}800$$

$L = 69.28$ m.

823. $H = \dfrac{\omega L^2}{8d} = \dfrac{20(100)^2}{8(5)} = 5000$ kg.

$$T = \sqrt{\left(\frac{\omega L}{2}\right)^2 + H^2} = \sqrt{\left[\frac{20(100)}{2}\right]^2 + (5000)^2}$$

$T = 5099$ kg.

824. $T = 10{,}000 + \dfrac{60N}{m}(5\text{ m}) = 10{,}300$ N

$$T^2 = \left(\frac{\omega L}{2}\right)^2 + H^2$$

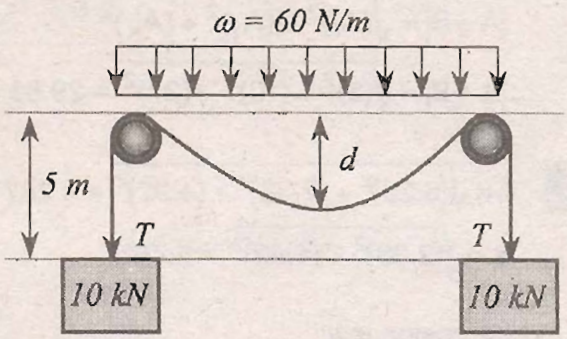
$$(10{,}300)^2 = \left(\frac{60(100)}{2}\right)^2 + H^2$$

$H = 9853.42$ N

$$H = \frac{\omega L^2}{8d}$$

$$9853.42 = \frac{60(100)^2}{8d}$$

$d = 7.6$ m.



825. $S = \dfrac{F}{A}$

$$F = SA = \frac{1000\text{kg}}{\text{cm}^2} \times 1.77\text{cm}^2 = 1770\text{ kg.}$$

Note: This force is equivalent to the tension (T) at each support.

$$T^2 = \left(\frac{\omega L}{2}\right)^2 + H^2$$

$$(1770)^2 = \left(\frac{1.6(600)}{2}\right)^2 + H^2$$

$H = 1703.67$ kg.

$$H = \frac{\omega L^2}{8d}$$

$$1703.67 = \frac{(1.6)(600)^2}{8d}$$

$d = 42.26$ m.

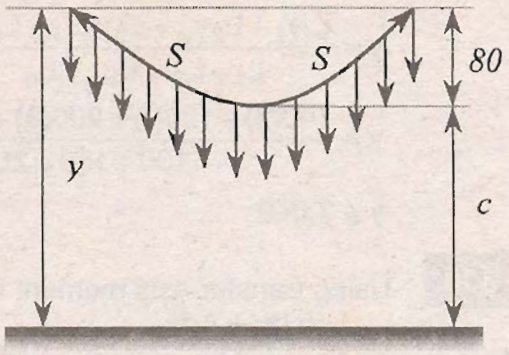
826. $T = \omega y$

$T = 0.4(80 + c)$ ☞ ❶

$2S = 800$

$S = 400$ m.

$$y^2 = S^2 + c^2$$
$$(80 + c)^2 = (400)^2 + c^2$$
$$6400 + 2c + c^2 = (400)^2 + c^2$$
$$c = 960 \text{ m.}$$



Substitute c = 960 in (1):

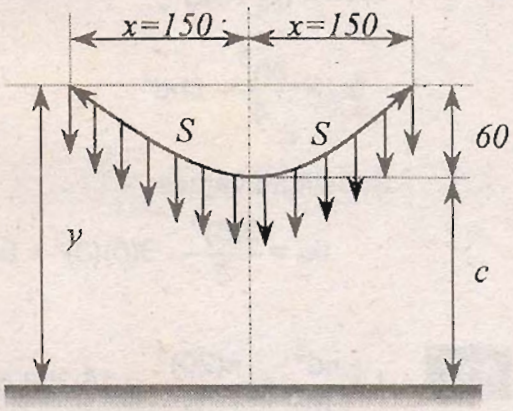
$T = 0.4(80 + 960)$

$T = 416$ kg.

827. $x = c \ln \dfrac{S + y}{c}$

$150 = c \ln \dfrac{S + (60 + c)}{c}$ ☞ ❶

$$y^2 = S^2 + c^2$$
$$(60 + c)^2 = S^2 + c^2$$
$$3600 + 120c + c^2 = S^2 + s^2$$
$$S = \sqrt{3600 + 120c}$$ ☞ ❷



Substitute (2) in (1):

$$150 = c \ln \frac{\sqrt{3600 + 120c} + (60 + c)}{c}$$

By trial and error:

$c = 196.8$ m

828. $A_T = A_{square} + A_{semi\text{-}circle}$

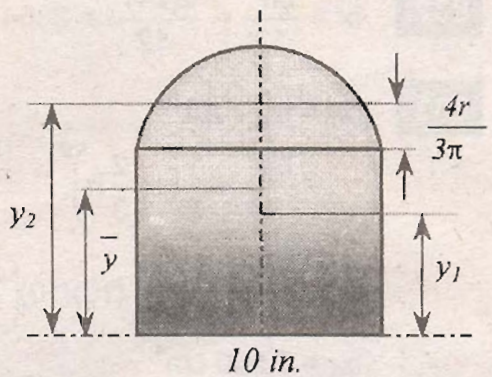
$$= (10)^2 + \left(\frac{1}{2}\right)\left(\frac{\pi}{4}\right)(10)^2$$

$A_T = 139.27 \text{ in}^2$

$A_T \bar{y} = A_1 y_1 + A_2 y_2$ ☞ ❶

where: $y_1 = 5$ in.

$$y_2 = 10 + \frac{4(5)}{3\pi} = 12.12 \text{ in.}$$



Substitute y_1 and y_2 in (1):

$139.27\ \bar{y} = 100(5) + 39.2\ (12.12)$

$\bar{y} = 7$ inches (from the bottom)

829. $$\bar{x} = \frac{L_1x_1 + L_2x_2 + L_3x_3 \ldots\ldots L_nx_n}{L_1 + L_2 + L_3 \ldots\ldots L_n}$$

$$\bar{x} = \frac{100(0) + 180(1) + 200(1) + 120(2) + 150(2) + 200(3) + 180(3) + 100(4)}{100 + 180 + 200 + 120 + 150 + 200 + 180 + 100}$$

$\bar{x} = 2$

$$\bar{y} = \frac{L_1y_1 + L_2y_2 + L_3y_3 \ldots\ldots L_ny_n}{L_1 + L_2 + L_3 \ldots\ldots L_n}$$

$$\bar{y} = \frac{100(2) + 180(1) + 200(3) + 120(0) + 150(4) + 200(1) + 180(3) + 100(2)}{100 + 180 + 200 + 120 + 150 + 200 + 180 + 100}$$

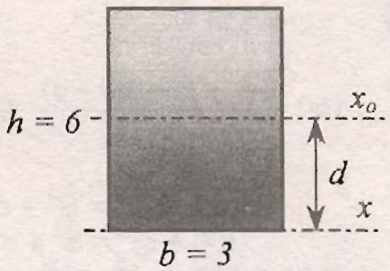
$$\bar{y} = 2.049$$

830. Using transfer-axis moment formula:

$I_x = I_{x_o} + Ad^2$

$$\frac{bh^3}{4} = I_{x_o} + Ad^2$$

$$I_{x_o} = \frac{bh^3}{4} - bhd^2$$



Substitute values:

$$I_{x_o} = \frac{3(6)^3}{3} - 3(6)(3)^2 = 54 \text{ m}^4.$$

831. $$J = \frac{\pi d^4}{32} = \frac{\pi(20)^4}{32} = 15{,}708 \text{ cm}^4$$

where: J = polar moment of inertia

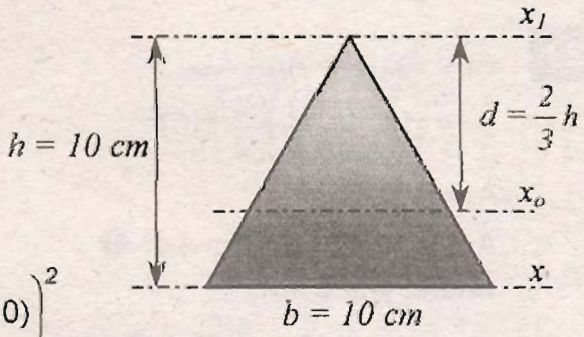
832. $$I_x = \frac{bh^3}{12} = \frac{2(24)^3}{12} = 2304 \text{ in}^4$$

833. $I_{x_1} = I_{x_o} + Ad^2$

$$= \frac{bh^3}{36} + \frac{1}{2}bh\left(\frac{2}{3}h\right)^2$$

$$= \frac{10(10)^3}{36} + \frac{1}{2}(10)(10)\left(\frac{2}{3}(10)\right)^2$$

$I_{x_1} = 2{,}500 \text{ cm}^4$



834. $$I = \frac{1}{2}mr^2 = \frac{1}{2}(5)(5)^2 = 62.5 \text{ kg-m}^2$$

where: I = mass moment of inertia

835. $I = \frac{2}{5}mr^2$ ☞ *Formula for mass moment of inertia of a solid sphere*

$$I = \frac{2}{5}(0.5)(0.15)^2 = 0.0045 \text{ kg-m}^2$$

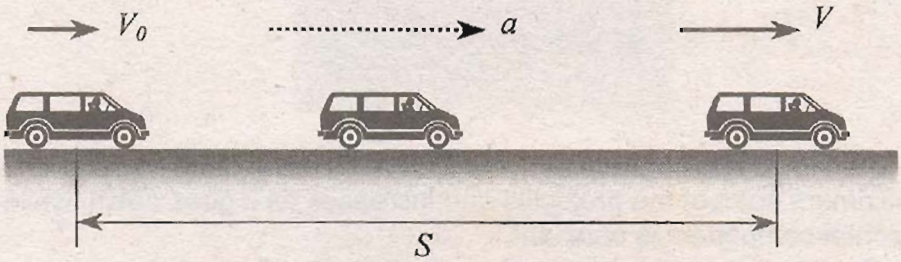
DAY 20

ENGINEERING MECHANICS
DYNAMICS

DYNAMICS

A. Rectilinear Translation:

1. Horizontal motion

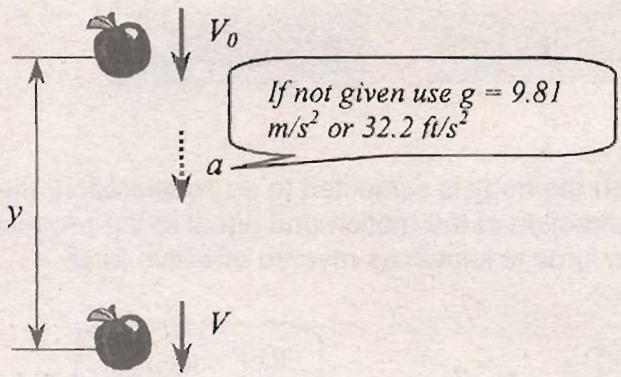


$$S = V_0 t \pm \frac{1}{2} a t^2$$

$$V = V_0 \pm at$$

$$V^2 = V_0^2 \pm 2aS$$

2. Vertical motion



$$y = V_0 t \pm \frac{1}{2} g t^2$$

$$V = V_0 \pm gt$$

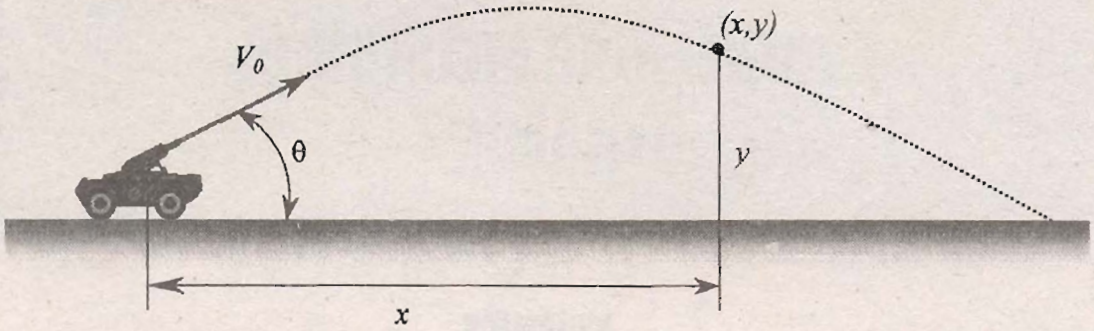
$$V^2 = V_0^2 \pm 2gy$$

If initial velocity, $V_0 = 0$, it is said to be a free falling body, thus,

$$y = \frac{1}{2} g t^2$$

B. Curvilinear translation:

1. Projectile or trajectory

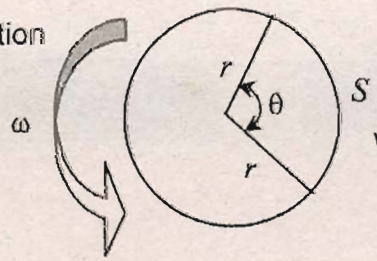


Projectile has an equation that of a parabola. The general equation of a projectile is,

$$y = x\tan\theta - \frac{gx^2}{2V_0^2\cos^2\theta}$$

The vertical component of the velocity decreases as it goes up and is zero at maximum point of the projectile and increases as it goes down, while the horizontal component is constant.

2. Rotation



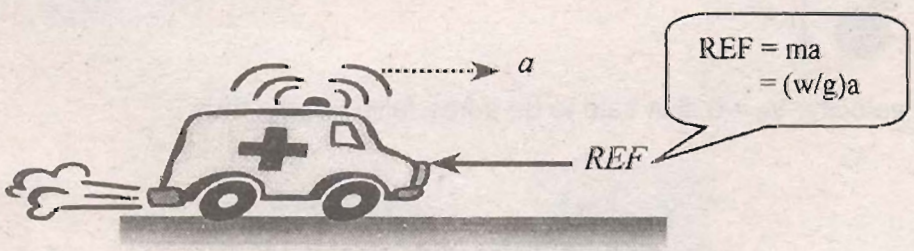
$S = r\theta$ $V = r\omega$ $a = r\alpha$

where: S, V and a are linear dimensions, θ, ω and α are angular distance, velocity and acceleration, respectively.

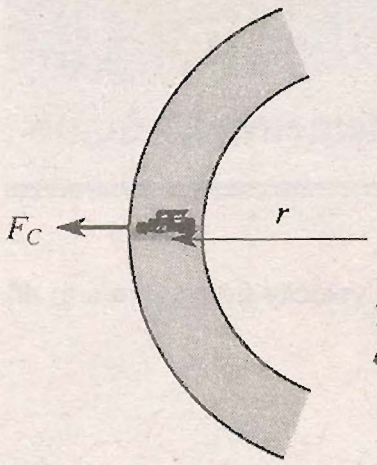
Also,

$$\theta = \omega_0 t \pm \frac{1}{2}\alpha t^2 \qquad \omega = \omega_0 \pm \alpha t \qquad \omega^2 = \omega_0^2 \pm 2\alpha\theta$$

C. D'Alembert's Principle: When the body is subjected to an acceleration, there exists a force opposite the direction of the motion and equal to the product of mass and acceleration. This force is known as reverse effective force.



D. Centifugal force:



$$F_C = \frac{WV^2}{gr}$$

$$F_C = F$$

Top View of the circular track



Elevation view



Did you know that... About 500 B.C., the Pythagorean Brotherhood was originally aware of the four regular polyhedrons and considered them to represent the four basic elements namely, tetrahedron – "fire", octahedron – "air", hexahedron – "earth", icosahedron – "water". When the Pythagoreans learned the existence of the fifth regular polyhedron, dodecagon, they considered it to represent the fifth element – "universe"!

Proceed to the next page for your 20th test. GOODLUCK ! ☞

Test 20

Time element: 4.0 hours

Problem 836: ME Board April 1996

What is the acceleration of a body that increases in velocity from 20 m/s tp 40 m/s in 3 seconds?

A. 5.00 m/s^2
B. 6.67 m/s^2
C. 7.00 m/s^2
D. 8.00 m/s^2

Problem 837: ECE Board November 1998

How far does an automobile move while its speed increases uniformly from 15 kph to 45 kph in 20 seconds?

A. 185 m
B. 167 m
C. 200 m
D. 172 m

Problem 838: CE Board November 1996

A train passing point A at a speed of 72 kph accelerates at 0.75 m/s^2 for one minute along a straight path then decelerates at 1.0 m/s^2. How far in km from point A will it be 2 minutes after passing point A?

A. 3.60 km
B. 4.65 km
C. 6.49 km
D. 7.30 km

Problem 839: CE Board May 1996

From a speed of 75 kph, a car decelerates at the rate of 500 m/min^2 along a straight path. How far in meters will it travel in 45 seconds?

A. 790.293 m
B. 791.357 m
C. 793.238 m
D. 796.875 m

Problem 840: CE Board November 1997

A train starting at initial velocity of 30 kph travels a distance of 21 km in 18 minutes. Determine the acceleration of the train at this instant.

A. 0.0043 m/s^2
B. 0.0206 m/s^2
C. 0.0865 m/s^2
D. 0.3820 m/s^2

Problem 841: EE Board October 1996

An automobile moving at a constant velocity of a 15 m/sec passes a gasoline station. Two seconds later, another automobile leaves the gasoline station and accelerates at a constant rate of 2 m/sec^2. How soon will the second automobile overtake the first?

A. 15.3 sec
B. 16.8 sec
C. 13.5 sec
D. 18.6 sec

Problem 842: EE Board October 1996

If a particle position is given by the expression $x(t) = 3.4t^3 - 5.4t$ meters, what is the acceleration of the particle after t = 5 seconds?

A. 1.02 m/s^2
B. 102 m/s^2
C. 3.4 m/s^2
D. 18.1 m/s^2

Problem 843: ME Board October 1995

The distance a body travels is a function of time and is given by $x(t) = 18t + 9t^2$. Find its velocity at t = 2.

A. 36
B. 54
C. 24
D. 20

Problem 844: CE Board May 1998

Determine the velocity of progress with the given equation: $D = 20t + \frac{5}{t+1}$ when t = 4 seconds.

A. 18.6 m/s
B. 19.8 m/s
C. 21.2 m/s
D. 22.4 m/s

Problem 845: ECE Board April 1999

A ball is dropped from a building 100 m high. If the mass of the ball is 10 gm after what time will the ball strike the earth?

A. 4.52 s
B. 4.42 s
C. 5.61 s
D. 2.45 s

Problem 846: ME Board April 1995

A ball is dropped from the roof of a building 40 meters tall will hit the ground with a velocity of:

A. 50 m/sec
B. 28 m/sec
C. 19.8 m/sec
D. 30 m/sec

Problem 847: ME Board April 1992

Using a powerful air gun, a steel ball is shot vertically upward with a velocity of 80 meters per second, followed by another shot after 5 seconds. Find the initial velocity of the second ball in order to meet the first ball 150 meters from the ground.

A. 65.3 m/sec
B. 45.1 m/sec
C. 56.2 m/sec
D. 61.3 m/sec

Problem 848: EE Board October 1995

A ball is thrown vertically upward from the ground and a student gazing out of the window sees it moving upward pass him at 5 m/sec. The window is 10 m above the ground. How high does the ball go above the ground?

A. 15.25 m
B. 14.87 m
C. 9.97 m
D. 11.30 m

Problem 849: EE Board October 1996

A ball is dropped from a height of 60 meters above the ground. How long does it take to hit the ground?

A. 2.1 sec
B. 3.5 sec
C. 5.5 sec
D. 1.3 sec

Problem 850: ECE Board April 1998

A baseball is thrown from a horizontal plane following a parabolic path with an initial velocity of 100 m/s at an angle of 30° above the horizontal. How far from the throwing point will the ball attain its original level?

A. 890 m
B. 883 m
C. 880 m
D. 875 m

Problem 851: ME Board April 1991

A plane dropped a bomb at an elevation of 1000 meters from the ground intended to hit the target at an elevation of 200 meters from the ground. If the plane was flying at a velocity of 300 km/hr, at hat distance from the target must the bomb be dropped to hit the target. Wind velocity and atmospheric pressure to be disregarded.

A. 1024.2 m
B. 1055.6 m
C. 1075.5 m
D. 1064.2 m

Problem 852: ME Board October 1997

The muzzle velocity of a projectile is 1500 fps and the distance of the target is 10 miles. The angle of elevation of the gun must be:

A. 21°59'
B. 22°41'
C. 24°33'
D. 25°18'

Problem 853: ME Board April 1995, ME Board October 1996

A shot is fired at an angle of 45° with the horizontal and a velocity of 300 fps. Calculate, to the nearest value, the range of the projectile.

A. 932 yards
B. 1200 yards
C. 3500 yards
D. 4000 yards

Problem 854: CE Board May 1995

A projectile leaves a velocity of 50 m/s at an angle of 30° with the horizontal. Find the maximum height that it could reach.

A. 31.86 m
B. 31.28 m
C. 30.63 m
D. 30.12 m

Problem 855: ME Board October 1997

A shot is fired with an angle of 45° with the horizontal with a velocity of 300 ft/s. Find the maximum height and range that the projectile can cover, respectively.

A. 800 ft, 1600 ft
B. 923 ft, 3500 ft
C. 700 ft, 2800 ft
D. 1800 ft, 3000 ft

Problem 856: CE Board November 1996

A ball is thrown from a tower 30 m high above the ground with a velocity of 300 m/s directed at 20° from the horizontal. How long will the ball hit the ground?

A. 21.2 s
B. 22.2 s
C. 23.2 s
D. 24.2 s

Problem 857:

In the last 2 seconds of NBA finals featuring Chicago Bulls VS Utah Jazz, with the latter ahead by 2 points with the former at 94-92 count. Bulls Michael Jordan decides to shoot from a certain point on the rainbow territory which counts 3 points if converted. During the play, if Jordan releases the ball at 7 m from the basket and 2.15 m above the ground and an inclination of 40° with the horizontal and assuming no block was made by the opponents, at what velocity will the ball be given to cast the winning basket? The basket is 10 feet from the ground.

A. 8.57 m/s
B. 8.86 m/s
C. 9.03 m/s
D. 9.27 m/s

Problem 858: CE Board May 1995

A projectile is fired with a muzzle velocity of 300 m/s from a gun aimed upward at an angle of 20° with the horizontal, from the top of a building 30 m high above a level ground. With what velocity will it hit the ground in m/s?

A. 298 m/s
B. 299 m/s
C. 300 m/s
D. 301 m/s

Problem 859: CE Board May 1995

A stone is thrown upward at an angle of 30° with the horizontal. It lands 60 m measured horizontally and 2 m below measured vertically from its point of release. Determine the initial velocity of the stone in m/s.

A. 22.35 m/s
B. 23.35 m/s
C. 24.35 m/s

D. 25.35 m/s

Problem 860: CE Board November 1992

A wooden block having a weight of 50 N is placed at a distance of 1.5 m from the center of a circular platform rotating at a speed of 2 radians per second. Determine the minimum coefficient of friction of the blocks so that it will not slide. Radius of the circular platform is 3 m.

A. 0.55
B. 0.58
C. 0.61
D. 0.65

Problem 861: ME Board October 1991

The flywheel of a puncher is to be brought to a complete stop in 8 seconds from a speed of 60 revolutions per minute. Compute the number of turns the flywheel will still make if its deceleration is uniform.

A. 5 turns
B. 3 turns
C. 4 turns
D. 6 turns

Problem 862: ECE Board April 1998

What is the speed of a synchronous earth's satellite situated 4.5×10^7 m from the earth?

A. 11,070.0 kph
B. 12,000.0 kph
C. 11,777.4 kph
D. 12,070.2 kph

Problem 863: ECE Board November 1998

A rotating wheel has a radius of 2 feet and 6 inches. A point on the rim of the wheel moves 30 feet in 2 sec. Find the angular velocity of the wheel.

A. 2 rad/s
B. 4 rad/s
C. 5 rad/s
D. 6 rad/s

Problem 864: CE Board November 1997

A turbine started from rest to 180 rpm in 6 minutes at a constant acceleration. Find the number of revolutions that it makes within the elapsed time.

A. 500
B. 540
C. 550
D. 630

Problem 865:

A flywheel is 15 cm in diameter accelerates uniformly from rest to 500 rpm in 20 seconds. What is its angular acceleration?

A. 2.62 rad/s^2
B. 3.45 rad/s^2
C. 3.95 rad/s^2
D. 4.42 rad/s^2

Problem 866: ME Board April 1991

A boy tied a 80 grams stone to a string which he rotated to form a circular motion with a diameter of 1000 mm. Compute for the pull exerted on the string by the stone if it got loose leaving at a velocity of 25 m/sec.

A. 120 N
B. 100 N
C. 150 N
D. 135 N

Problem 867: EE Board April 1997

A man keeps a 1kg toy airplane flying horizontally in a circle by holding onto a 1.5 m long string attached to its wing tip. The string is always in the plane of the circular path. If the plane flies at 10 m/sec, find the tension in the string.

A. 28 N
B. 15 N
C. 67 N
D. 18 N

Problem 868: ME Board October 1996

An automobile travels on a perfectly horizontal, unbanked circular track of radius R. The coefficient of friction between the tires and track is 0.3. If the car's velocity is 15 m/s, what is the smallest radius it may travel without skidding?

A. 68.4 m
B. 69.4 m
C. 71.6 m
D. 76.5 m

Problem 869: CE Board November 1998

A hi-way curve has a super elevation of 7degrees. What is the radius of the curve such that there will be no lateral pressure between the tires and the roadway at a speed of 40 mph?

A. 265.71 m
B. 438.34 m
C. 345.34 m
D. 330.78 m.

Problem 870: ME Board April 1998

Traffic travels at 65 mi/hr around a banked highway curve with a radius of 3000 feet. What banking angle is necessary such that friction will not be required to resist the centrifugal force?

A. 3.2°
B. 2.5°
C. 5.4°
D. 18°

Problem 871: ECE Board April 1999

Determine the angle of the super elevation for a 200 m hi-way curve so that there will be no side thrust at a speed of 90 kph.

A. 19.17°
B. 17.67°
C. 18.32°
D. 20.11°

Problem 872: ECE Board April 1998

The inclination of ascend of a road having a 8.25% grade is _____.

A. 4.72°
B. 4.27°
C. 5.12°
D. 1.86°

Problem 873: ME Board April 1996

A cyclist on a circular track of radius r = 800 feet is traveling at 27 fps. His speed in the tangential direction increases at the rate of 3 fps^2. What is the cyclist's total acceleration?

A. 2.8 fps^2
B. 3.1 fps^2
C. 3.8 fps^2
D. 4.2 fps^2

Problem 874: ME Board October 1997

A concrete hi-way curve with a radius of 500 ft is banked to give lateral pressure equivalent to f = 0.15. For what coefficient of friction will skidding impend for a speed of 60 mph?

A. $\mu > 0.360$
B. $\mu < 0.310$
C. $\mu > 0.310$
D. $\mu < 0.360$

Problem 875: EE Board April 1993

What force is necessary to accelerate a 30,000 pounds railway electric car at the rate of 1.25 ft/sec^2, if the force required to overcome frictional resistance is 400 pounds?

A. 1565 pounds
B. 1585 pounds
C. 1595 pounds
D. 1575 pounds

Problem 876: ME Board October 1995

A car weighing 40 tons is switched to a 2 percent upgrade with a velocity of 30 mph. If the train resistance is 10 lb/ton, how far up the grade will it go?

A. 1124 ft on slope
B. 2014 ft on slope
C. 1204 ft on slope
D. 1402 ft on slope

Problem 877: EE Board April 1996

A car moving at 70 km/hr has a mass of 1700 kg. What force is necessary to decelerate it at a rate of 40 cm/s^2?

A. 4250 N
B. 0.68 N
C. 680 N
D. 42.5 N

Problem 878: ME Board April 1998

An elevator weighing 2,000 lb attains an upward velocity of 16 fps in 4 seconds with uniform acceleration. What is the tension in the supporting cables?

A. 2,150 lb
B. 2,495 lb
C. 1,950 lb
D. 2,250 lb

Problem 879: ME Board April 1998

A body weighing 40 lb starts from rest and slides down a plane at an angle of 30° with the horizontal for which the coefficient of friction $\mu = 0.30$. How far will it move during the third second?

A. 19.63 feet
B. 19.33 feet
C. 18.33 feet
D. 19.99 feet

Problem 880: ME Board April 1997

A pick-up truck is traveling forward at 25 m/s. The bed is loaded with boxes whose coefficient of friction with the bed is 0.4. What is the shortest time that the truck can be brought to a stop such that the boxes do not shift?

A. 2.35 s
B. 4.75 s
C. 5.45 s
D. 6.37 s

ANSWER KEY

836. B	848. D	860. C	872. A
837. B	849. B	861. C	873. B
838. B	850. B	862. C	874. B
839. D	851. D	863. D	875. A
840. B	852. C	864. B	876. C
841. B	853. A	865. A	877. C
842. B	854. A	866. B	878. D
843. B	855. C	867. C	879. B
844. B	856. A	868. D	880. D
845. A	857. C	869. A	
846. B	858. D	870. C	
847. D	859. D	871. B	

RATING

- ☐ 38-45 Topnotcher
- ☐ 27-37 Passer
- ☐ 22-26 Conditional
- ☐ 0-21 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 20

836. $V = V_0 + at$

$40 = 20 + a(3)$

$a = 6.67\ m/s^2$

837. $V_0 = \frac{15\,km}{hr} \times \frac{1000\,m}{1\,km} \times \frac{1\,hr}{3600\,sec} = 4.167\ m/sec$

$V = 45 \times \frac{1000}{3600} = 12.5\ m/sec.$

$V = V_0 + at$

$12.5 = 4.167 + a(20)$

$a = 0.41665\ m/s^2$

$S = V_0 t + \frac{1}{2}at^2 = 4.167(20) + \frac{1}{2}(0.416665)(20)^2$

$S = 167\ m.$

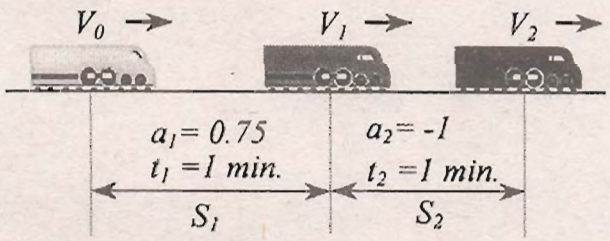
838. $V_0 = \frac{72\,km}{hr} \times \frac{1\,hr}{60\,min} \times \frac{1000m}{sec}$

$V_0 = 20\ m/sec$

$S_1 = V_0 t_1 + \frac{1}{2}at_1^2$

$= 20(60) + \frac{1}{2}(0.75)(60)^2$

$S_1 = 2550\ m = 2.55\ km.$



$V_1 = V_0 + at_1$

$= 20 + 0.75(60)$

$V_1 = 65\ m/sec$

$S_2 = V_1 t_2 - \frac{1}{2}at_2^2 = 65(60) - \frac{1}{2}(1)(60)^2$

$S_2 = 2100\ m = 2.1\ km.$

Solving for total distance:

$S = S_1 + S_2 = 2.55 + 2.1$

$S = 4.65\ km.$

839. $V_0 = \frac{75\,km}{hr} \times \frac{1000m}{1km} \times \frac{1\,hr}{60\,min} = 1250\ m/min$

$S = V_0 t - \frac{1}{2}at^2$

$= 1250\left(\frac{45}{60}\right) - \frac{1}{2}(500)\left(\frac{45}{60}\right)^2$

$S = 796.875\ m$

840. $V_0 = \frac{30\,km}{hr} x \frac{1\,hr}{3600\,sec} x \frac{1000\,m}{1\,km} = 8.333$ m/sec

$$S = V_0 t + \frac{1}{2} a t^2$$

$$21000 = 8.333[18(60)] - \frac{1}{2} a[18(60)]^2$$

$$a = 0.0206 \text{ m/s}^2$$

841. $S_1 = V_1 t_1$

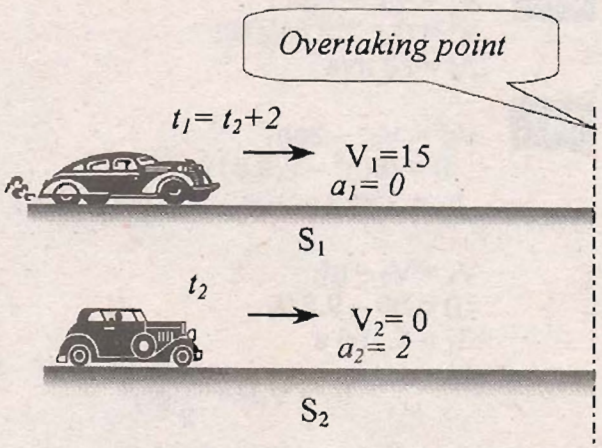
$= 15(t_2 + 2)$

$S_1 = 15t_2 + 30$ ☞ ❶

$S_2 = V_2 t_2 + \frac{1}{2} a t_2^2$

$= 0(t_2) + \frac{1}{2}(2)t_2^2$

$S_2 = t_2^2$ ☞ ❷



Equate (1) to (2):

$S_1 = S_2$

$15t_2 + 30 = t_2^2$

$t_2^2 - 15t_2 - 30 = 0$

By quadratic formula:

$$t_2 = \frac{15 \pm \sqrt{(15)^2 - 4(1)(-30)}}{2(1)} = \frac{15 \pm 18.574}{2}$$

Take + sign, $t_2 = 16.8$ seconds

842. $x = 3.4t^3 - 5.4t$

$$V = \frac{dx}{dt} = 3(3.4)t^2 - 5.4$$

$V = 10.2t^2 - 5.4$

$$a = \frac{dV}{dt} = 20.4t = 20.4(5)$$

$a = 102$ m/s^2

843. $x = 18t + 9t^2$

$$V = \frac{dx}{dt} = 18 + 18t = 18 + 18(2)$$

$V = 54$ m/s

844. $D = 20t + \frac{5}{t+1}$

$$V = \frac{dD}{dt} = 20 + \frac{-5}{(t+1)^2} = 20 - \frac{5}{(4+1)^2}$$

$V = 19.8$ m/s

845. Note: Since the ball was dropped, initial velocity of the ball is zero.

$$h = V_0t + \frac{1}{2}gt^2$$

$$100 = 0 + \frac{1}{2}(9.81)t^2$$

$t = 4.52$ seconds

846. $V^2 = V_0^2 + 2gh$
$V^2 = 0 + 2(9.81)(40)$
$V = 28$ m/s

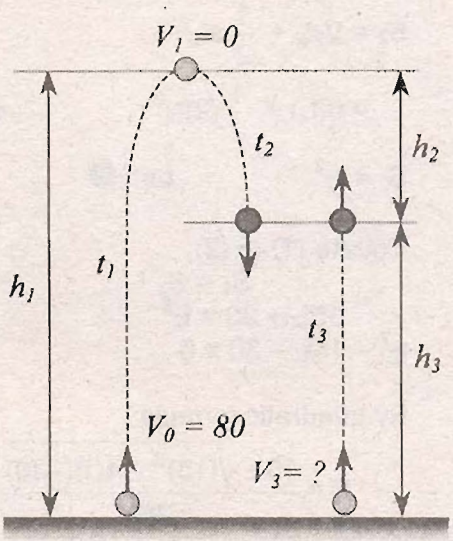
847. $V_1^2 = V_0^2 - 2gh_1$
$0 = (80)^2 - 2(9.81)h_1$
$h_1 = 326.2$ m

$V_1 = V_0 - gt_1$
$0 = 80 - 9.81t_1$
$t_1 = 8.155$ s

$$h_2 = V_1t_2 + \frac{1}{2}gt_2^2$$

$$326.2 - 150 = 0(t_2) + \frac{1}{2}(9.81)t_2^2$$

$t_2 = 6$ s



$t_3 = t_1 + t_2 - 5$
$= 8.155 + 6 - 5$
$t_3 = 9.155$ s

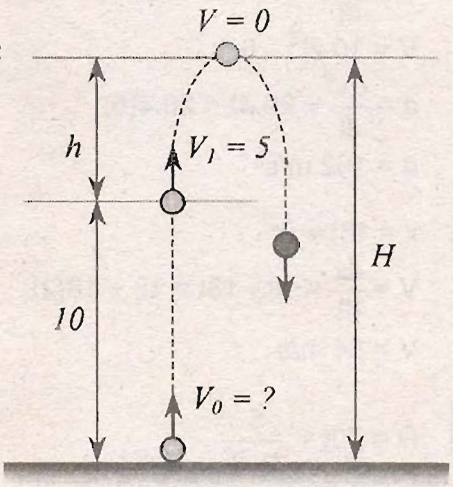
$$h_3 = V_3t_3 - \frac{1}{2}gt_3^2$$

$$150 = V_3(9.155) - \frac{1}{2}(9.81)(9.155)^2$$

$V_3 = 61.3$ m/s

848. $V^2 = V_1^2 - 2gh$
$0 = (5)^2 - 2(9.81)h$
$h = 1.3$ m.

$H = 10 + h$
$H = 10 + 1.3 = 11.3$ m.



849. $$h = V_0t + \frac{1}{2}gt^2$$

$$60 = (0)t + \frac{1}{2}(9.81)t^2$$

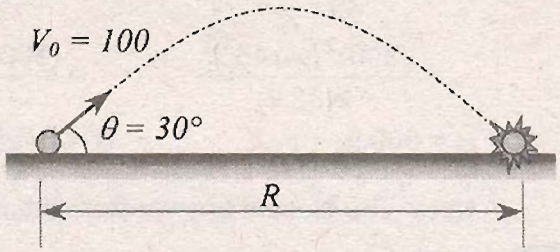
$t = 3.5$ s

850. Let: R = range of the projectile

$$R = \frac{V_0^2 \sin 2\theta}{g}$$

$$= \frac{(100)^2 \sin[2(30°)]}{9.81}$$

R = 883 m.

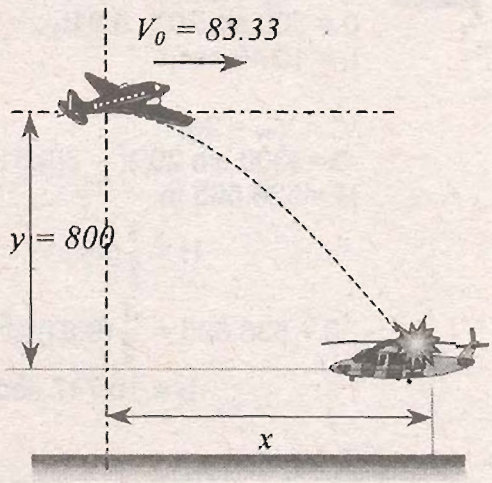


851. $V_0 = 300 \times \frac{1000}{3600} = 83.33$ m/s

$$y = x \tan\theta - \frac{gx^2}{2V_0^2 \cos^2\theta}$$

$$-800 = x \tan 0° - \frac{(9.81)x^2}{2(83.33)^2 \cos^2 0°}$$

x = 1064.2 m



852.

$$R = \frac{V_0^2 \sin 2\theta}{g}$$

$$10(5280) = \frac{(1500)^2 \sin 2\theta}{32.2}$$

$\theta = 24.54° = 24° (0.54)(60)'$

$\theta = 24°32.4'$

853.

$$R = \frac{V_0^2 \sin 2\theta}{g}$$

$$= \frac{(300)^2 \sin[2(45°)]}{32.2} = 2795 \text{ ft} \times \frac{1 \text{ yd}}{3 \text{ ft}} = 931.66 \text{ yd.}$$

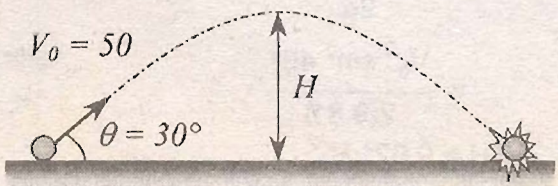
R = 932 yd.

854. Let: H = maximum height

$$H = \frac{V_0^2 \sin^2\theta}{2g}$$

$$= \frac{(50)^2 (\sin 30°)^2}{2(9.81)}$$

H = 31.86 m.



855. $H = \dfrac{V_0^2 \sin^2\theta}{2g}$ $\qquad R = \dfrac{V_0^2 \sin 2\theta}{g}$

$= \dfrac{(300)^2(\sin 45°)^2}{2(32.2)}$ $\qquad = \dfrac{(300)^2 \sin[2(45°)]}{32.2}$

$H = 699$ ft. $\qquad R = 2795$ ft.

Note: The closest answer from the choices is 700 ft and 2800 ft.

856. $V = V_{oy} - gt_1$

$0 = 300 \sin 20° - 9.81t_1$

$t_1 = 10.46$ sec.

$V^2 = V_{oy} - 2gH$

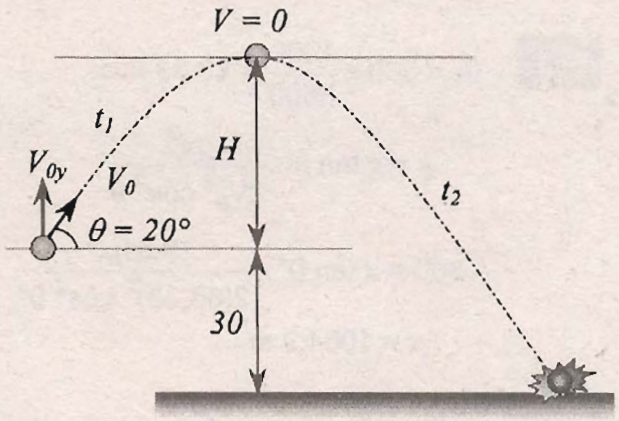
$0 = (300 \sin 20°)^2 - 2(9.81)H$

$H = 536.595$ m.

$$H = \frac{1}{2}gt_2^2$$

$$30 + 536.595 = \frac{1}{2}(9.81)t_2^2$$

$t_2 = 10.747$ sec.



Total time $= t_1 + t_2$

$= 10.46 + 10.747$

$= 21.2$ sec.

857. Let: t = total time (t) of the flight

$x = V_0 \cos\theta\, t$

$7 = V_0 \cos 40°\, t$

$$t = \frac{9.14}{V_0}$$ ☞ ❶

$V = V_{0y} - gt_1$

$V = V_0 \sin\theta - gt_1$

$0 = V_0 \sin 40° - 9.81t_1$

$t_1 = 0.066\, V_0$ ☞ ❷

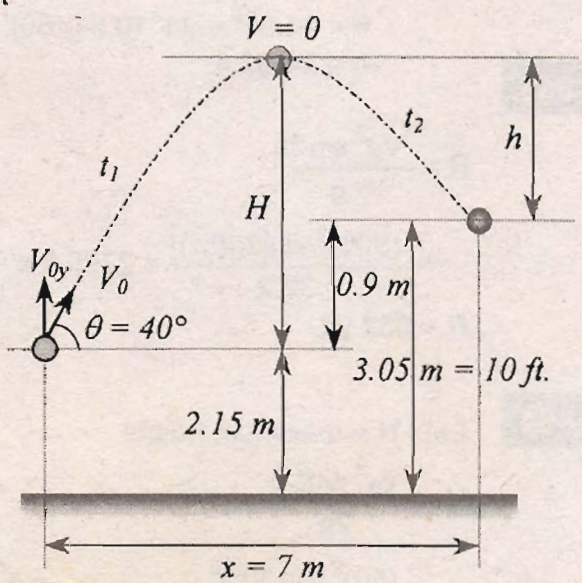
$$H = \frac{V_0^2 \sin^2\theta}{2g}$$

$$= \frac{V_0^2 \sin^2 40°}{2(9.81)}$$

$H = 0.021\, V_0^2$

$$h = \frac{1}{2}gt_2^2 = \frac{9.81}{2}t_2^2$$

$t_2 = 0.45\sqrt{h}$



$h = H - 0.9$

$h = 0.021\ V_0^2 - 0.9$

$t_2 = 0.45\ \sqrt{0.021V_0^2 - 0.9}$ ☞ ❸

$t = t_1 + t_2$

$$\frac{9.14}{V_0} = 0.066V_0 + 0.45\sqrt{0.021V_0^2 - 0.9}$$

$$9.14 = 0.0666\ V_0^2 + 0.45\ V_0\ \sqrt{0.021V_0^2 - 0.9}$$

$$9.14 - 0.0666\ V_0^2 = 0.45\ V_0\ \sqrt{0.021V_0^2 - 0.9}$$

Square both sides:

$$83.54 - 1.206\ V_0^2 + 0.004\ V_0^4 = 0.2025V_0^2\ (0.021V_0^2 - 0.9)$$
$$= 0.004V_0^4 - 0.18225V_0^2$$
$$83.54 - 1.206\ V_0^2 = -\ 0.18225V_0^2$$
$$1.02375\ V_0^2 = 83.54$$
$$V_0 = 9.03\ \text{m/s}$$

858. $V_1^2 = V_{0y}^2 - 2gH$

$0 = (300 \sin 20°)^2 - 2(9.81)h$

$H = 536.59$ m.

$V_{2y}^2 = V_1^2 + 2g(30 + h)$

$= 0 + 2(9.81)(30 + 536.59)$

$V_{2y} = 105.43$ m/s

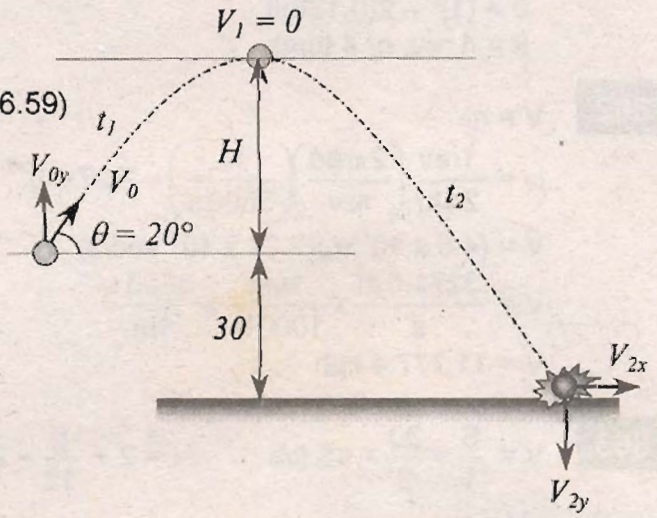
$V_{2x} = V_{ox} = 300 \cos 20°$

$V_{2x} = 281.9$ m/s

$$V_2 = \sqrt{(V_{2x})^2 + (V_{2y})^2}$$

$$= \sqrt{(281.9)^2 + (105.43)^2}$$

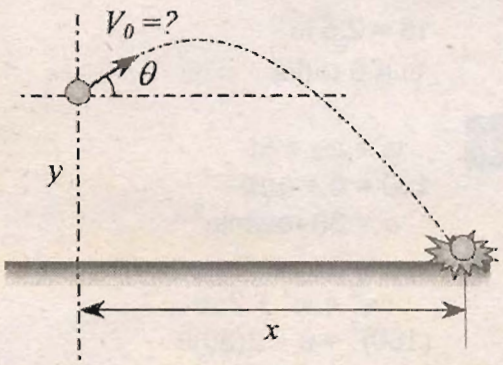
$V_2 = 301$ m/s



859. $$y = x \tan\theta - \frac{gx^2}{2V_0^2 \cos^2\theta}$$

$$-2 = 60 \tan 30° - \frac{(9.81)(60)^2}{2V_0^2 \cos^2 30°}$$

$V_0 = 25.35$ m/s



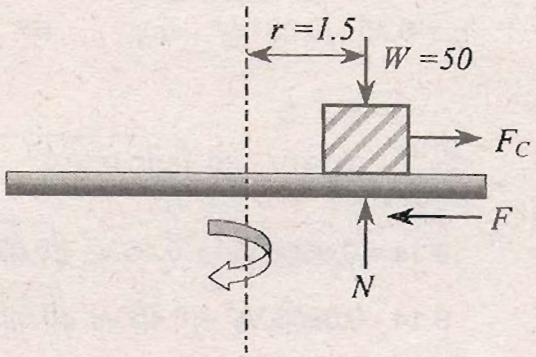
860. $V = r\omega$

$= 1.5(2)$

$V = 3$ m/s

$$F_C = \frac{WV^2}{gr} = \frac{50(3)^2}{9.81(1.5)}$$

$$F_C = 30.58$$



$$\sum F_H = 0$$

$$F - F_C = 0$$

$$F = 30.58$$

$$\mu = \frac{F}{N} = \frac{30.58}{50}$$

$$\mu = 0.61$$

861. $\omega_0 = \frac{60\text{rev}}{\text{min}} \times \frac{1\text{min}}{60\text{s}} = 1 \text{ rev/s}$

$$\omega = \omega_0 - \alpha t$$

$$0 = 1 - \alpha(8)$$

$$\alpha = 0.125 \text{ rev/s}^2$$

$$\omega^2 = \omega_0^2 - 2\alpha\theta$$

$$0 = (1)^2 - 2(0.125)\theta$$

$$\theta = 4 \text{ rev. or 4 turns}$$

862. $V = r\omega$

$$\omega = \frac{1\text{rev}}{24\text{hr}}\left(\frac{2\pi\text{rad}}{\text{rev}}\right)\left(\frac{\text{hr}}{3600\,\text{s}}\right) = 7.27\text{x}10^{-5} \text{ rad/s}$$

$$V = (4.5 \times 10^7 \text{ m})(7.27 \times 10^{-5} \text{ rad/s})$$

$$V = \frac{3271.5\,\text{m}}{\text{s}} \times \frac{1\text{km}}{1000\,\text{m}} \times \frac{3600\,\text{s}}{1\text{hr}}$$

$$V = 11{,}777.4 \text{ kph}$$

863. $V = \frac{S}{t} = \frac{30}{2} = 15 \text{ ft/s}$ $\qquad r = 2 + \frac{6}{12} = 2.5 \text{ feet}$

$$V = r\omega$$

$$15 = 2.5\,\omega$$

$$\omega = 6 \text{ rad/s}$$

864. $\omega = \omega_0 + \alpha t$

$$180 = 0 + \alpha(6)$$

$$\alpha = 30 \text{ rev/min}^2$$

$$\omega_0^2 = \omega^2 + 2\alpha\theta$$

$$(180)^2 = 0 + 2(30)\theta$$

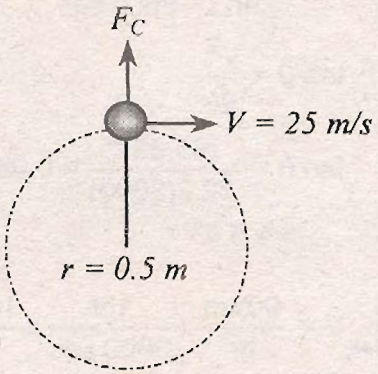
$$\theta = 540 \text{ rev.}$$

865. $\omega = \frac{500\,\text{rev}}{\text{min}} \times \frac{1\,\text{min}}{60\,\text{s}} \times \frac{2\pi\,\text{rad}}{\text{rev}} = 52.36\ \text{rad/s}$

$\omega = \omega_0 + \alpha t$

$52.36 = 0 + \alpha(20)$

$\alpha = 2.62\ \text{rad/sec}^2$



866. $F_C = \frac{WV^2}{gr} = \frac{mV^2}{r}$

$= \frac{0.08(25)^2}{0.5}$

$F_C = 100\ \text{N}$

867. Note: The force exerted by the string is equal to the centrifugal force

$F_C = \frac{WV^2}{gr} = \frac{mV^2}{r}$

$= \frac{1(10)^2}{1.5}$

$F_C = 66.67\ \text{N}$, approximately 67 N

868. $\sum F_H = 0$

$F_C = F = \mu N$

$\sum F_V = 0$

$N = W$

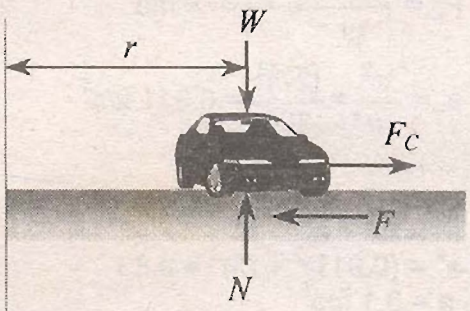
$F_C = \mu W$

$F_C = \frac{WV^2}{gr}$

$\mu W = \frac{WV^2}{gr}$

$0.3 = \frac{(15)^2}{9.81r}$

$r = 76.5\ \text{m}.$



869. $V = \frac{40\,\text{mi}}{\text{hr}} \times \frac{5280\,\text{ft}}{1\,\text{mi}} \times \frac{1\,\text{m}}{3.281\,\text{ft}} \times \frac{1\,\text{hr}}{3600\,\text{s}} = 17.88\ \text{m/s}$

$\tan\theta = \frac{V^2}{gr}$

$\tan 7° = \frac{(17.88)^2}{(9.81)r}$

$r = 265.41\ \text{m}.$

870. $V = \frac{65\,\text{mi}}{\text{hr}} \times \frac{1\,\text{hr}}{3600\,\text{sec}} \times \frac{5280\,\text{ft}}{\text{mi}} = 95.33\ \text{ft/s}$

$$\tan\theta = \frac{V^2}{gr}$$

$$\tan\theta = \frac{(95.33)^2}{32.2(3000)} = 0.094$$

$$\theta = 5.4°$$

871. $V = \frac{90\,\text{km}}{\text{hr}} \times \frac{1\,\text{hr}}{3600\,\text{sec}} \times \frac{1000\,\text{m}}{1\,\text{km}} = 25\ \text{m/s}$

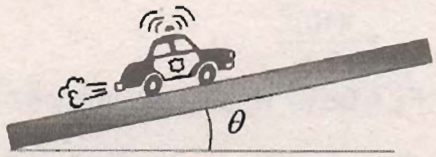
$$\tan\theta = \frac{V^2}{gr}$$

$$\tan\theta = \frac{(25)^2}{9.81(200)} = 0.3185$$

$$\theta = 17.67°$$

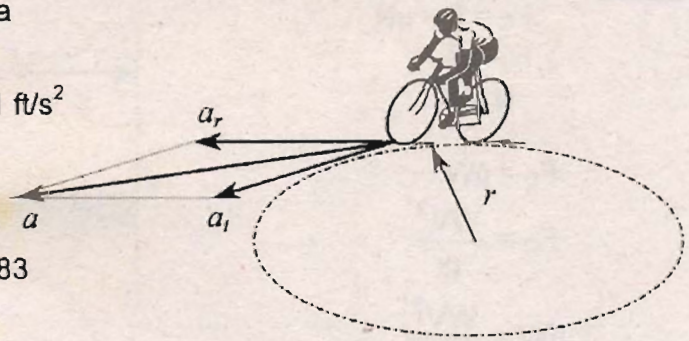
872. slope $= \tan\theta = 0.0825$

$\theta = 4.71°$



873. $F_C = \frac{WV^2}{gr} = \frac{mV^2}{r} = ma$

$$a_r = \frac{V^2}{r} = \frac{(27)^2}{800} = 0.911\ \text{ft/s}^2$$



$$a^2 = a_r^2 + a_t^2$$

$$a^2 = (0.911)^2 + (3)^2 = 9.83$$

$$a = 3.1\ \text{fps}^2$$

874. $V = \frac{60\,\text{mi}}{\text{hr}} \times \frac{5280\,\text{ft}}{1\,\text{mi}} \times \frac{1\,\text{hr}}{3600\,\text{s}} = 88\ \text{ft/s}$

$$\tan\theta = 0.25$$

$$\theta = 8.53°$$

$$\tan(\theta + \phi) = \frac{V^2}{gr}$$

$$8.53° + \phi = \tan^{-1}\frac{(88)^2}{32.2(500)}$$

$$\phi = 17.16°$$

$\mu = \tan\phi = \tan 17.16°$

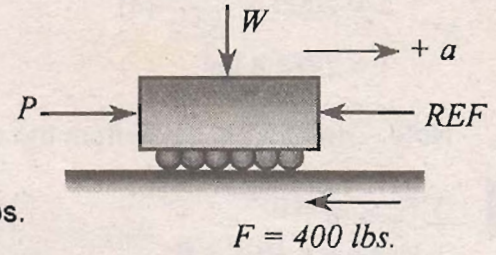
$\mu = 0.309$, thus the coefficient of friction is less than 0.310

875. $\sum F_H = 0$

$P = REF + F$

$P = \frac{W}{g}a + F$

$P = \frac{30{,}000}{32.2}(1.25) + 400 = 1556$ lbs.

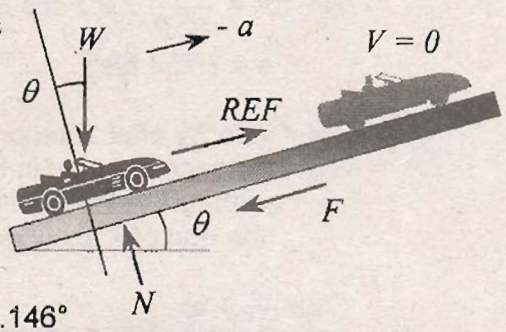


Note: REF means reverse effective force by d'Alemberts principle.

876. $\tan\theta = 0.02$

$\theta = 1.146°$

$V_0 = \frac{50\,mi}{hr} \times \frac{5280\,ft}{1mi} \times \frac{1hr}{3600\,s} = 44$ ft/s



$\sum F_{inclined} = 0$

$REF = F + W \sin\theta$

$\frac{W}{g}a = F + W \sin\theta$

$\frac{40(2000)}{32.2}a = 10(40) + 40(2000)\sin 1.146°$

$a = 0.8049$ ft/s^2

$V^2 = V_0^2 - 2aS$

$0 = (44)^2 - 2(0.8049)S$

$S = 1{,}203$ ft.

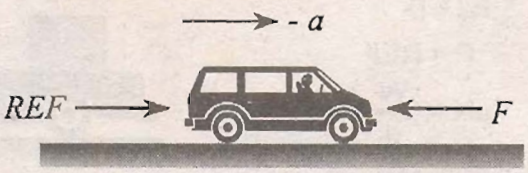
Note: The nearest value from the choices is 1,204 ft.

877. $\sum F_H = 0$

$F = REF$

$F = \frac{W}{g}a = ma$

$F = 1700\,(0.4) = 680$ N



878. $V = V_0 + at$

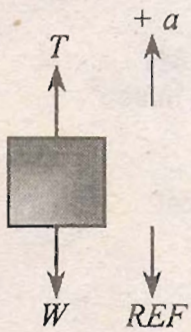
$16 = 0 + a(4)$

$a = 4$ ft/s^2

$\sum F_V = 0$

$T = W + REF$

$T = W + \frac{W}{g}a$



$$T = 2000 + \frac{2000}{32.2}(4)$$

$$T = 2248.4 \text{ lbs.}$$

Note: The nearest value from the choices is 2250 lbs.

879. $\sum F_{inclined} = 0$

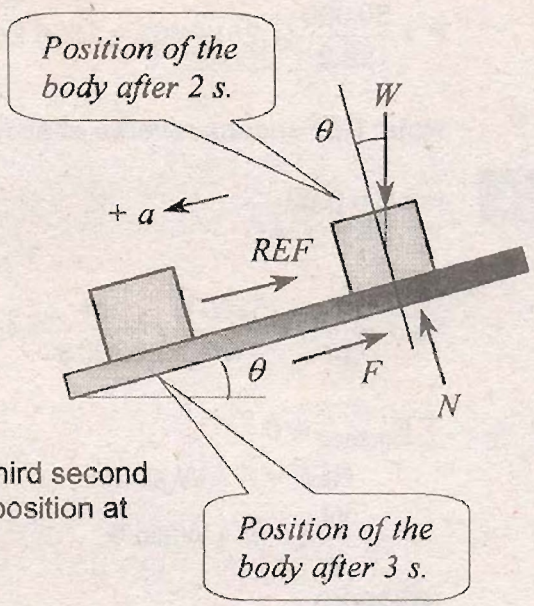
$$W \sin\theta = REF + F$$

$$W \sin\theta = \frac{W}{g}a + \mu N$$

$$W \sin\theta = \frac{W}{g}a + \mu (W \cos\theta)$$

$$\sin 30° = \frac{a}{32.2} + 0.3 \cos 30°$$

$$a = 7.734 \text{ ft/s}^2$$



Let: V = velocity after 2 seconds
S = distance traveled in the third second with reference from the position at the end of 2 seconds.

$$V = V_0 + at$$

$$V = 0 + 7.734(2) = 15.468 \text{ ft/s}$$

$$S = Vt + \frac{1}{2}at^2$$

$$S = 15.468(1) + \frac{1}{2}(7.734)(1)^2$$

$$S = 19.33 \text{ ft.}$$

880. $\sum F_H = 0$

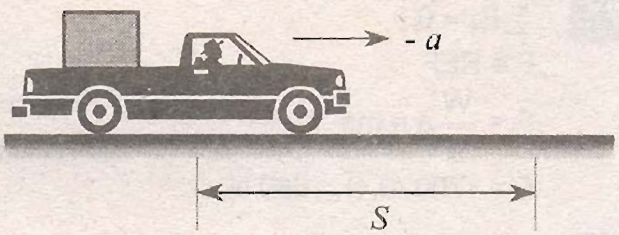
$$F = REF$$

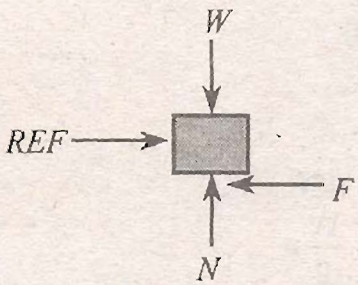
$$\mu N = \frac{W}{g}a$$

$$\mu W = \frac{W}{g}a$$

$$a = \mu g$$
$$= 0.4(9.81)$$
$$a = 3.924 \text{ m/sec}^2$$

$$V = V_0 - at$$
$$0 = 25 - 3.924t$$
$$t = 6.37 \text{ s.}$$



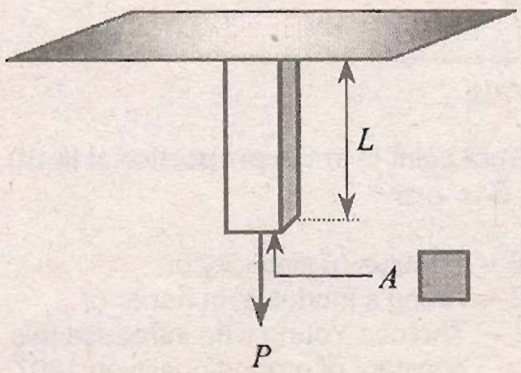


DAY 21

STRENGTH OF MATERIALS

SIMPLE STRESS

Simple stress is the force per unit area.



$$\sigma = \frac{P}{A}$$

where: P = force
A = cross-sectional area
σ = stress

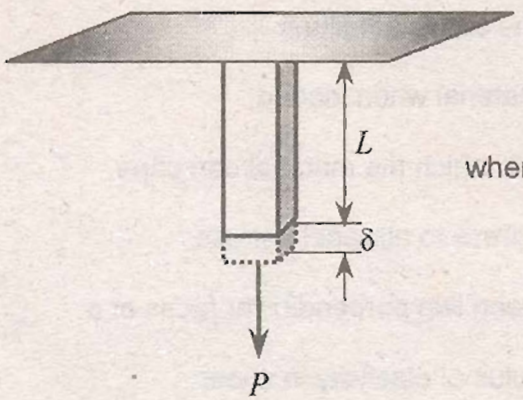
If force P is in newtons and area is in mm^2, then the resulting unit for stress is MPa.

Shearing stress (or tangential stress) is a stress caused by forces acting along or parallel to the area resisting the forces.

Bearing stress is one which is caused by forces acting perpendicular to the area resisting the forces. Normal stresses, like **tensile stress and compressive stress** are examples of bearing stress.

SIMPLE STRAIN

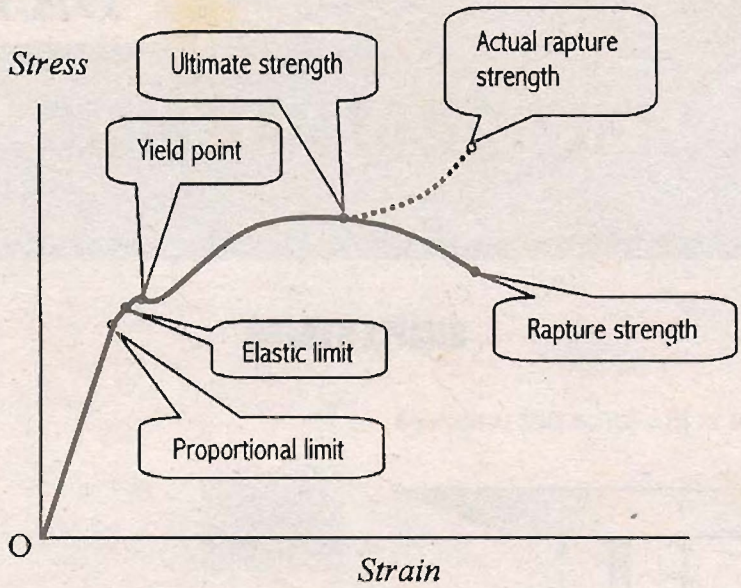
Simple strain is the ratio of the deformation or elongation to the original length.



$$\epsilon = \frac{\delta}{L}$$

where: δ = elongation
L = original length

Stress-strain diagram:



Hooke's Law states that within elastic limit (from point O to the **proportional limit**), the stress is proportional to strain. Thus, $S \propto \epsilon$ or

$$\frac{P}{A} = \frac{E\delta}{L}$$

where: E = modulus of elasticity or
E = Young's modulus, in honor of Thomas Young who introduce this constant of proportionality in 1807.

Hooke's law was formulated by **Robert Hooke** in 1678.

Elastic limit refers to the stress beyond which the material will not return to its original shape when the load is removed. The permanent deformation caused by excessive stress is called **permanent set.**

Yield point refers to the point where there is an appreciable elongation or yielding of the material even without any corresponding increase of load.

Ultimate stress (or ultimate strength) refers to the highest ordinate in the stress-strain diagram.

Rapture strength is sometimes known as the stress at failure.

Working stress is the actual stress of the material when loaded.

Allowable stress is the maximum safe stress which the material can carry.

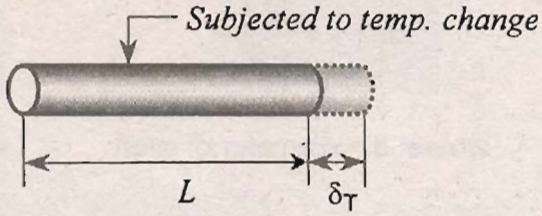
Factor of safety is the ratio of the ultimate stress to allowable stress.

Shearing strain is the angular change between two perpendicular faces of a differential element.

Modulus of rigidity (G) – refers to the modulus of elasticity in shear.

Thermal stress is the stress on the material caused by the internal forces due to change in temperature. The temperature deformation may be calculated using

$$\delta_T = \alpha L(\Delta T)$$



THIN WALLED CYLINDERS

A. Tangential Stress:

$$\sigma_t = \frac{pD}{2t}$$

B. Longitudinal Stress:

$$\sigma_l = \frac{pD}{4t}$$

where: p = pressure in N/m^3
D = inside diameter in mm
t = thickness in mm

Another term for tangential stress is **circumferential stress**, or **hoop stress** or **girth stress**.

Note that the longitudinal stress is one-half the value of the tangential stress.

TORSION

Torsion refers to the twisting of solid or hollow circular shafts.

A. Shearing stress:

$$\tau = \frac{T\rho}{J}$$

where: T = torque applied
ρ = radial distance from the center of cross-section
J = polar moment of inertia of the cross-section

B. Maximum shearing stress:

$$\text{Max. } \tau = \frac{Tr}{J}$$

where: r = radius of the cross-section

C. Maximum shearing stress of:

A. Solid shaft

$$\text{Max. } \tau = \frac{16T}{\pi d^3}$$

where: d = diameter of shaft

B. Hollow shaft

$$\text{Max. } \tau = \frac{16TD}{\pi\left(D^4 - d^4\right)}$$

where: d = inner diameter of shaft
D = outer diameter of shaft

D. Angular deformation, θ:

$$\theta = \frac{TL}{JG}$$

where: T = torque applied
L = length
J = polar moment of inertia of cross-section
G = modulus of rigidity

E. Transmit power, P

$$P = 2\pi f T$$

where: P = power in watts
T = torque in N-m
f = frequency or speed in revolutions per second

HELICAL SPRINGS

A. Maximum shearing stress:

$$\tau = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right) \quad \textit{or} \quad \tau = \frac{16PR}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right)$$

where: P = axial load
R = mean radius of helical spring
d = diameter of rod/wire of spring
m = ratio of the mean diameter of the spring to the mean diameter of the spring rod or wire

$$m = \frac{2R}{d} = \frac{D}{d}$$

B. Spring deformation:

$$\delta = \frac{64PR^3n}{Gd^4}$$

where: n = number of turns
G = modulus of rigidity



Tips: **Shear and Moment:**
For symmetrically loaded and simply supported beams:

Maximum shear occurs at the **support** and **equal to the reaction** while **maximum moment** occurs at the **midspan**.

Also, maximum moment occurs at a point where shear is equal to zero.

Did you know that... QED, the abbreviation of Quod Erat Demonstrandum, Latin for "which was to be demonstrated" was commonly used by mathematicians to indicated that a conclusion has been reached, was first introduced by Euclid using its Greek equivalent in the 3rd century B.C.!

Proceed to the next page for your 21st test. GOODLUCK ! ☞

Test 21

Time element: 2.0 hours & 30 minutes

Problem 881: ECE Board November 1998

An iron column of annular cross-section has an outer diameter of 200 mm and is subjected to a force of 74 kN. Find the thickness of the wall if the allowable compressive stress is 10 MPa.

A. 12.75 mm
B. 12.57 mm
C. 17.75 mm
D. 15.75 mm

Problem 882: ME Board April 1998

A force of 10 N is applied to one end of a 10 inches diameter circular rod. Calculate the stress.

A. 0.20 kPa
B. 0.15 kPa
C. 0.05 kPa
D. 0.10 kPa

Problem 883: ME Board April 1996

A steel tie rod on bridge must be made to withstand a pull of 5000 lbs. Find the diameter of the rod assuming a factor of safety of 5 and ultimate stress of 64,000 psi.

A. 0.75
B. 0.71
C. 0.84
D. 0.79

Problem 884: EE Board October 1996

Determine the outside diameter of a hollow steel tube that will carry a tensile load of 500 kN at a stress of 140 Mpa. Assume the wall thickness to be one-tenth of the outside diameter.

A. 111.3 mm
B. 109.7 mm
C. 113.7 mm
D. 112.4 mm

Problem 885: ME Board April 1996

If the ultimate shear strength of a steel plate is 42,000 psi, what force is necessary to punch a 0.75-inch diameter hole in a 0.625 inch thick plate?

A. 63,000
B. 68,080
C. 61,850
D. 66,800

Problem 886: ME Board October 1995

What force is required to punch a 1/2-inch hole on a 3/8 thick plate if the ultimate shearing strength of the plate is 42,000 psi?

A. 24,940
B. 24,620
C. 24,960
D. 24,740

Problem 887:

A single bolt is used to lap joint two steel bars together. Tensile force on the bar is 20,000 N. Determine the diameter of the bolt required if the allowable shearing stress is 70 MPa?

A. 17 mm
B. 18 mm
C. 19 mm
D. 20 mm

Problem 888: EE Board October 1996

What is the stress in a thin-walled spherical shell of diameter D and a wall thickness t when subjected to internal pressure ρ?

A. $S = D/\rho t$
B. $S = 4D/\rho t$
C. $S = \rho D/4t$
D. $S = \rho D/t$

Problem 889: ME Board April 1998

Compute the safe wall thickness of a 76.2 cm diameter steel tank. The tank is subjected to 7.33 MPa pressure and the steel material has a yield stress of 215.4 MPa. The factor of safety to use is 3.

A. 1 1/2 inches
B. 3.89 inches
C. 4.09 inches
D. 3.96 inches

Problem 890: EE Board April 1996

A cylindrical water tank is 8 m in diameter and 12 m high. If the tank is to be completely filled, determine the minimum thickness of the tank plating if the stress is limited to 40 MPa.

A. 11.77 mm
B. 13.18 mm
C. 10.25 mm
D. 12.6 mm

Problem 891: EE Board October 1990

A water reservoir of 24 m high and 12 m in diameter is to be completely filled with water. Find the minimum thickness of the reservoir plating if the stress is limited to 50 MPa.

A. 24.5 mm
B. 28 mm
C. 21 mm
D. 26 mm

Problem 892: EE Board April 1995

The stress in a 90-cm diameter pipe having a wall thickness of 9.5 cm and under a static head of 70 m of water is

A. 325 kPa
B. 32.5 kPa
C. 32.5 MPa
D. 3.25 MPa

Problem 893: ME Board October 1994

A cylindrical tank with 10 inches inside diameter contains oxygen gas at 2,500 psi. Calculate the required thickness in mm under a stress of 28,000 psi.

A. 11.44
B. 11.34
C. 10.60
D. 10.30

Problem 894: ME Board April 1995

A solid shaft 48.2 cm long is used for a transmission of mechanical power at a rate of 37 kW running at 1760 rpm. The stress is 8.13 MPa. Calculate the diameter.

A. 30 mm
B. 35 mm
C. 40 mm
D. 50 mm

Problem 895: ME Board October 1995

What is the modulus of elasticity if the stress is 44,000 psi and unit strain of 0.00105?

A. 41.905×10^6
B. 42.300×10^6
C. 41.202×10^6
D. 43.101×10^6

Problem 896: ME Board October 1995

A 2-inch solid shaft is driven by a 36-inch gear and transmits power at 120 rpm. If the allowable shearing stress is 12 ksi, what horsepower can be transmitted?

A. 29.89
B. 35.89
C. 38.89
D. 34.89

Problem 897: ME Board October 1995

A hollow shaft has an inner diameter of 0.035 m and an outer diameter of 0.06 m. Compute for the torque in N-m, if the stress is not to exceed 120 MPa.

A. 4500
B. 4100
C. 4300
D. 4150

Problem 898: ME Board October 1996

Compute the nominal shear stress at the surface in MPa for a 40-mm diameter shaft that transmits 750 kW at 1500 rpm. Axial and bending loads are assumed negligible.

A. 218
B. 312
C. 232
D. 380

Problem 899: ME Board October 1995

A hollow shaft has an inner diameter of 0.035 m and an outer diameter of 0.06 m. Determine the polar moment of inertia of the hollow shaft.

A. $1.512 \times 10^{-6}\ m^4$
B. $1.215 \times 10^{-6}\ m^4$
C. $1.152 \times 10^{-6}\ m^4$
D. $1.125 \times 10^{-6}\ m^4$

Problem 900: ME Board April 1997

What power would a spindle 55 mm in diameter transmit at 480 rpm. Stress allowed for short shaft is 59 N/mm^2

A. 42.12 kW
B. 50.61 kW
C. 96.88 kW
D. 39.21 kW

Problem 901:

A 30-m long aluminum bar is subjected to a tensile stress of 172 MPa. Find the elongation if E = 69,116 MPa?

A. 0.746 m
B. 0.007 m
C. 6.270 mm
D. 7.46 cm

Problem 902: EE Board October 1996

A steel wire is 4.0 m long and 2 mm in diameter. How much is it elongated by a suspended body of mass 20 kg? Young's modulus for steel is 196,000 MPa.

A. 1.123 mm
B. 1.385 mm
C. 1.374 mm
D. 1.274 mm

Problem 903:

A steel wire is 6 m long, hanging vertically supports a load of 2000 N. Neglecting the weight of the wire, determine the required diameter if the stress is not to exceed 140 MPa and the total elongation is not to exceed 4 mm. E = 200,000 MPa.

A. 3.4 mm
B. 4. 4 mm
C. 4.26 mm
D. 5.4 mm

Problem 904:

A copper rolled wire 10 m long and 1.5 mm diameter when supporting a weight of 350 N elongates 18.6 mm. Compute the value of the Young's modulus of this wire.

A. 200 GPa
B. 180.32 GPa
C. 148.9 GPa
D. 106.48 GPa

Problem 905: EE Board April 1996

A cylinder of diameter 1.0 cm at 30°C is to be slid into a hole on a steel plate. The hole has a diameter of 0.99970 cm at 30°C. To what temperature the plate must be heated? Coefficient of linear expansion for steel is 1.2×10^{-5} cm/°C.

A. 62°C
B. 65°C
C. 48°C
D. 55°C

Problem 906: EE Board April 1995

An iron steam pipe is 200 ft long at 0°C. What will its increase in length when heated to 100°C? Coefficient of linear expansion is 10×10^{-6} ft/°C.

A. 0.18 ft
B. 0.12 ft
C. 0.28 ft
D. 0.20 ft

Problem 907: ECE Board November 1996

A simple beam 10 m long carries a concentrated load of 200 kN at the midspan. What is the maximum moment of the beam?

A. 250 kN-m
B. 500 kN-m
C. 400 kN-m
D. 100 kN-m

Problem 908: ME Board October 1993

A beam supported at both ends and carrying a uniformly distributed load:

A. has its maximum bending moment at the supports
B. has its maximum shear at the center of the beam
C. has its maximum shear at the supports
D. has uniform shear throughout the length of the beam

Problem 909:

A simply supported beam, 10 m long carries a uniformly distributed load of 20 kN/m. What is the value of the maximum shear of the beam due to this load?

A. 250 kN
B. 100 kN
C. 1000 kN
D. 500 kN

Problem 910:

A simply supported beam, 10 m long carries a uniformly distributed load of 20 kN/m. What is the value of the maximum moment of the beam due to this load?

A. 10,000 kN-m
B. 5,000 kN-m
C. 2,000 kN-m
D. 250 kN-m

ANSWER KEY

881. A	891. B	901. D
882. A	892. D	902. D
883. B	893. B	903. B
884. D	894. D	904. D
885. C	895. A	905. D
886. D	896. B	906. D
887. C	897. A	907. B
888. C	898. D	908. C
889. B	899. D	909. B
890. A	900. C	910. D

RATING

- ☐ 25-30 Topnotcher
- ☐ 18-24 Passer
- ☐ 15-17 Conditional
- ☐ 0-14 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 21

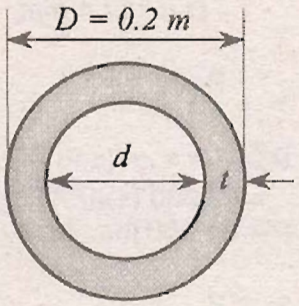
881. $\sigma = \frac{P}{A}$; $A = \frac{P}{\sigma}$

$$A = \frac{75{,}000}{10 \times 10^6} = 0.0075 \text{ m}^2$$

$$A = \frac{\pi}{4}D^2 - \frac{\pi}{4}d^2$$

$$0.0075 = \frac{\pi}{4}(0.2)^2 - \frac{\pi}{4}d^2$$

$$d = 0.1745 \text{ m.}$$



Solving for t:

$D = d + 2t$

$0.2 = 0.1745 + 2t$

$t = 0.01275$ m.

$t = 12.75$ mm.

882. $d = 10 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ m}}{3.281 \text{ ft}} = 0.254 \text{ m.}$

$$\sigma = \frac{P}{A} = \frac{P}{\frac{\pi}{4}d^2} = \frac{10}{\frac{\pi}{4}(0.254)^2} = 197.35 \text{ Pa} \cong 0.20 \text{ kPa}$$

883. $\sigma = k\frac{P}{A}$, k = factor of safety

$$A = \frac{kP}{\sigma} = \frac{5(5000)}{64000} = 0.3906 \text{ in}^2.$$

$$A = \frac{\pi}{4}d^2$$

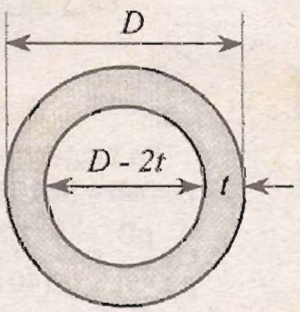
$$0.3906 = \frac{\pi}{4}d^2$$

$$d = 0.71 \text{ in.}$$

884. $\sigma = \frac{P}{A}$; $A = \frac{P}{\sigma}$

$$A = \frac{500{,}000}{140 \times 10^6} = 0.00357 \text{ m}^2$$

$$A = \frac{\pi}{4}D^2 - \frac{\pi}{4}(D - 2t)^2$$



Note: $t = 0.1D$

$$A = \frac{\pi}{4}D^2 - \frac{\pi}{4}[D - 2(0.1D)]^2$$

$A = 0.2827\ D^2$

$0.00357 = 0\ 2827\ D^2$

$D = 0.1124$ m.

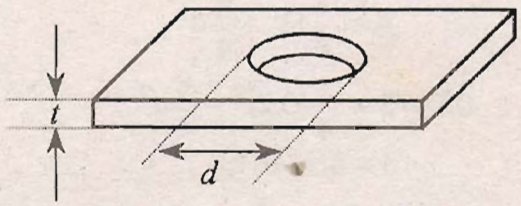
$D = 112.4$ mm.

885. $\sigma = \dfrac{P}{A}$

$P = \sigma A = \sigma\ (\pi dt)$

$= 42{,}000\ (\pi)(0.75)(0.625)$

$P = 61{,}850$ lbs.



886. $\sigma = \dfrac{P}{A}$

$P = \sigma A = \sigma\ (\pi dt)$

$= 42{,}000\ (\pi)(0.5)(0.375)$

$P = 24{,}740$ lbs.

887. $\sigma = \dfrac{P}{A} = \dfrac{P}{\frac{\pi}{4}d^2}$, d = diameter of the bolt

$70 \times 10^6 = \dfrac{20{,}000}{\frac{\pi}{4}d^2}$

$d = 0.019$ m.

$d = 19$ mm.

888. $\sigma_L = \dfrac{\rho D}{4t}$ ☞ *Formula!*

889. $\sigma_t = k\dfrac{\rho D}{2t}$, k = factor of safety

$215.4 \times 10^6 = \dfrac{3(7.33 \times 10^6)(0.762)}{2t}$

$t = 0.0389$ m.

$t = 3.89$ cm.

890. $\sigma_t = \dfrac{\rho D}{2t}$

Note: The biggest pressure occurs at the bottom of the tank

$p = \omega h = 9810\ N/m^3\ (12\ m) = 117.720$ Pa

$t = \dfrac{pD}{2\sigma_t} = \dfrac{117{,}720(8)}{2(40 \times 10^6)} = 0.01177$ m.

$t = 11.77$ mm.

891. $\sigma_T = \dfrac{\rho D}{2t}$

Note: The biggest pressure occurs at the bottom of the tank

$p = \omega h = 9810\ N/m^3\ (24\ m) = 235{,}440\ Pa$

$t = \dfrac{pD}{2S} = \dfrac{235{,}440(12)}{2(50 \times 10^6)} = 0.028\ m.$

$t = 28\ mm.$

892. $\sigma_T = \dfrac{pD}{2t}$

Note: The biggest pressure occurs at the bottom

$p = \omega h = 9810\ N/m^3\ (70\ m) = 686{,}700\ Pa$

$\sigma_T = \dfrac{pD}{2t} = \dfrac{686{,}700(0.9)}{2(0.095)} = 3{,}252{,}789.474\ Pa.$

$\sigma_T = 3.25\ MPa$

893. $\sigma_T = \dfrac{pD}{2t}$

$28{,}800 = \dfrac{2{,}500(10)}{2t}$

$t = 0.4464\ in \times \dfrac{2.45\ cm}{1\ in} \times \dfrac{10\ mm}{1\ cm}$

$t = 11.34\ mm$

894. $P = \dfrac{2\pi fT}{60}$

$37{,}000 = \dfrac{2\pi(1760)T}{60}$

$T = 200.75\ N\text{-}m$

$\tau = \dfrac{16T}{\pi d^3}$

$8.13 \times 10^6 = \dfrac{16(200.75)}{\pi d^3}$

$d = 0.050\ m.$

$d = 50\ mm.$

895. $E = \dfrac{\sigma}{\varepsilon} = \dfrac{44{,}000}{0.00105} = 41.905 \times 10^6\ Pa$

896. $$\tau = \frac{16T}{\pi d^3}$$

$$12{,}000 = \frac{16T}{\pi(2)^3}$$

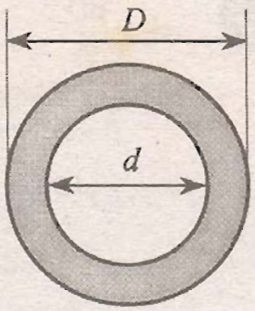
$$T = 18{,}849.55 \text{ lb-in}$$

$$P = \frac{2\pi fT}{33{,}000(12)} = \frac{2\pi(120)(18{,}849.55)}{33{,}000(12)} = 35.89 \text{ Hp}$$

897. $$\tau = \frac{16TD}{\pi(D^4 - d^4)}$$

$$120 \times 10^6 = \frac{16T(0.06)}{\pi[(0.06)^4 - (0.035)^4]}$$

$$T = 4{,}500 \text{ N-m}$$



898. $$P = \frac{2\pi fT}{60}$$

$$750{,}000 = \frac{2\pi(1500)T}{60}$$

$$T = 4{,}774.648 \text{ N-m}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(4{,}774.648)}{\pi(0.04)^3} = 380 \text{ MPa}$$

899. $$J = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}[(0.06)^4 - (0.035)^4]$$

$$J = 1.125 \times 10^{-6} \text{ m}^4$$

900. $$\tau = \frac{16T}{\pi d^3}$$

$$59 = \frac{16T}{\pi(55)^3}$$

$$T = 1{,}927{,}391.637 \text{ N-mm} \times \frac{1 \text{ m}}{1000 \text{ mm}} = 1{,}927.39 \text{ N-m}$$

$$P = \frac{2\pi fT}{60} = \frac{2\pi(480)(1927.39)}{60} = 96.88 \text{ kW}$$

901. $\delta = \frac{PL}{AE}$ ☞ ❶

$\sigma S = \frac{P}{A}$ ☞ ❷

Substitute (2) in (1):

$$\delta = \frac{\sigma L}{E} = \frac{(172 \times 10^6)(30)}{69{,}116 \times 10^6} = 0.0746 \text{ m.}$$

$\delta = 7.46$ cm.

902. $\delta = \dfrac{PL}{AE}$

where: P = weight of the body

P = mg = 20(9.81) = 196.2 N

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.002)^2 = 3.1416 \times 10^{-6} \text{ m}^2$$

Substitute:

$$\delta = \frac{196.2(4)}{3.1416 \times 10^{-6}(196{,}000 \times 10^6)} = 1.274 \times 10^{-3} \text{ m.}$$

$\delta = 1.274$ mm.

903. Considering limitations of stress:

$$\gamma = \frac{P}{A}$$

$$140 \times 10^6 = \frac{2000}{\frac{\pi}{4}d^2}$$

d = 0.00426 m. = 4.26 mm.

Considering limitations of elongation:

$$\delta = \frac{PL}{AE}$$

$$0.004 = \frac{2000(6)}{\frac{\pi}{4}d^2(200{,}000 \times 10^6)}$$

d = 0.0044 m. = 4.4 mm.

Note: To be safe for both stress and elongation, use d = 4.4 mm.

904. $\delta = \dfrac{PL}{AE}$

$$0.0186 = \frac{350(10)}{\frac{\pi}{4}(0.0015)^2 E}$$

$E = 1.0648 \times 10^{11}$ Pa

E = 106.48 GPa

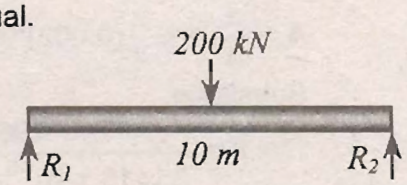
905. $\delta_T = 1 - 0.99970 = 0.0003$ cm.

$$\delta_T = \alpha L (t_2 - t_1)$$
$$0.0003 = 1.2 \times 10^{-5} (1) (t_2 - 30°)$$
$$t_2 = 55\ °C$$

906. $\delta_T = \alpha L (t_2 - t_1)$
$= 10 \times 10^{-6} (200) (100° - 0°)$
$\delta_T = 0.20$ ft.

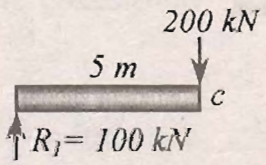
907. Note: Since the load is at midspan, then the reaction at both supports must be equal.

$\sum F_V = 0$
$R_1 + R_2 = 200$
$2R_1 = 200$
$R_1 = 100$ kN



Note: Maximum moment of a symmetrically loaded system is at midspan.

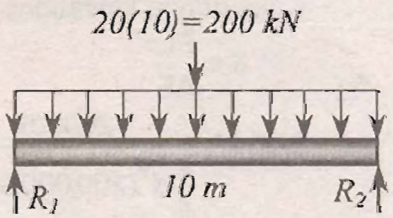
Consider the half of the beam:
Maximum moment $= \sum M_C$
$= R_1(5)$
Maximum moment = 100(5) = 500 kN-m.



908. Maximum shear of a symmetrically loaded system is equal to the reaction at the supports.

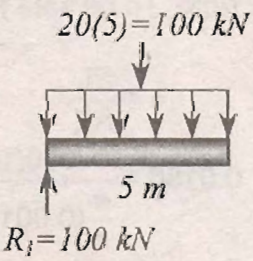
909. $\sum F_V = 0$
$R_1 + R_2 = 200$
$2R_1 = 200$
$R_1 = 100$ kN

Maximum shear = 100 kN



910. Refer to solution in Problem # 909:

Consider the half of the beam:
Maximum moment $= \sum M_C$
$= R_1(5) - 100(2.5)$
$= 100(5) - 250$
Maximum moment = 250 kN-m.



DAY 22

ENGINEERING ECONOMY (SIMPLE & COMPOUND INTEREST)

BASIC TERMS

Economics is a science which deals with the attainment of the maximum fulfilment of society's unlimited demands for goods and service.

Engineering Economy is the branch of economics which deals with the application of economics laws and theories involving engineering and technical projects or equipments.

Consumer goods and services refer to the products or services that are directly used by people to satisfy their wants. Examples are food, clothing, shelter or home, etc.

Producer goods and services are those that are used to produce the consumer goods and services. Examples are buildings, machines, factories, etc.

Utility refers to the satisfaction or pleasure derived from the consumer goods and services. This also means the power to satisfy human wants and needs.

Luxury products are those products that have an income-elasticity of demand greater than one. This implies that as income increases, more income will be spent on these products. Examples are appliances, entertainment, vacations, etc.

Supply the amount of goods or products that are available for sale by the suppliers.

Demand the want or desire or need for a product using money to purchase it.

Law of supply and demand: "When free competition exists, the price of the product will be that value where supply is equal to the demand."

Competition is a form of market structure where the number of suppliers is used to determine the type of the market.

Perfect competition a market situation wherein a given product is supplied by a very large number of vendors and there is no restriction of any additional vendor from entering the market.

Market is the place where the vendors or the sellers and vendees or the buyers come together.

The following are the different market situations:

Market situation	Sellers	Buyers
Perfect competition	many	many
Monopoly	one	many
Monopsony	many	one
Bilateral monopoly	one	one
Duopoly	two	many
Duopsony	many	two
Oligopoly	few	many
Oligopsony	many	few
Bilateral oligopoly	few	few

SIMPLE INTEREST

Interest is the amount of money or payment for the use of a borrowed money or capital.

Simple interest (I) is defined as the interest on a loan or principal that is based only on the original amount of the loan or principal. This means that the interest charges grow in a linear function over a period of time. It can be calculated using the formula

$$I = Pin$$

where: P = principal
i = interest per period
n = number of interest period

Ordinary simple interest is based on one banker's year. One banker's year is equivalent to 12 months of 30 days each. Also, 1 banker's year = 360 days.

$$n = \frac{d}{360} \qquad I = Pi\frac{d}{360}$$

Exact simple interest is based on the exact number of days in a given year. An ordinary year has 365 days while a leap year (which occurs once every 4 years) has 366 days.

$$n = \frac{d}{365}$$ For ordinary or normal year

$$n = \frac{d}{366}$$ For leap year

COMPOUND INTEREST

Compound interest is defined as the interest of loan or principal which is based not only on the original amount of the loan or principal but the amount of the loan or

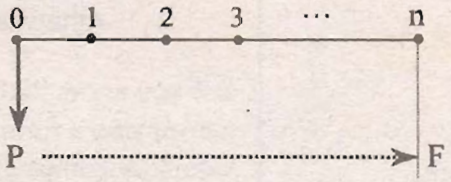
principal plus the previous accumulated interest. This means that the interest charges grow exponentially over a period of time.

Compound interest is used frequently in commercial practices than simple interest.

A. Total amount, F

$$F = P(1+i)^n$$

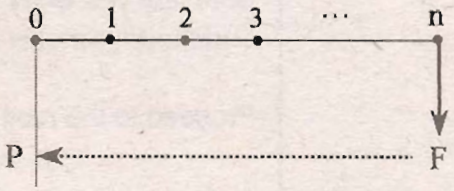
where : P = principal
i = interest per period
n = number of periods



Cash flow of P

B. Present worth, P

$$P = \frac{F}{(1+i)^n}$$



NOMINAL AND EFFECTIVE RATES OF INTEREST

Rate of interest is the cost of borrowing money. It also refers to the amount earned by a unit principal per unit time.

Nominal rate of interest is defined as the basic annual rate of interest while **effective rate of interest** is defined as the actual or the exact rate of interest earned on the principal during 1 year period.

For example: 5% compounded quarterly.

In this example, the nominal rate is 5% while the effective rate is greater than 5% because of the compounding that occurs four times during a year.

The effective rate of interest may be calculated using the following formula.

$$ER = (1+i)^m - 1$$

where: m = number of interest periods per year

Discount refers to the difference between the future worth of a negotiable paper and its present worth. It also refers to the sale of stock or share at reduced price. Discount may refer to the deduction from the published price of services or goods.

discount = future worth - present worth



Tip: **Determination of leap year:**
To determine a year whether a leap year or not, just divide the year by 4. If exactly divisible by 4, then it is a leap year. However years ending with two zeros or century years (i.e. 1900, 1800, etc.) must be divided by 400 not 4. If exactly divisible by 400 it is a leap year otherwise its not.

Did you know that... The Gregorean Calendar we are using now was named after a former teacher of law at the University of Bologna, Ugo Buoncompagni who became Pope Gregory XIII in 1572! In February 24, 1582, he issued a Papal edict directing the former Julian Calendar be allowed to catch up with the Lord's Time and that aside from leap year every four years, leap year be once in every four centennial years, i.e. every 400 years!

Proceed to the next page for your 22nd test. GOODLUCK ! ☞

Test 22

Time element: 3.0 hours

Problem 911: ME Board April 1995

P 4,000 is borrowed for 75 days at 16 % per annum simple interest. How much will be due at the end of 75 days?

A. P 4,133.33
B. P 4,333.33
C. P 4,166.67
D. P 4,150.00

Problem 912: CE Board May 1997

A deposit of P 110,000 was made for 31 days. The net interest after deducting 20% withholding tax is P 890.36. Find the rate of return annually.

A. 11.95 %
B. 12.75 %
C. 11.75 %
D. 12.25 %

Problem 913: ME Board April 1993

Agnes Abanilla was granted a loan of P 20,000 by her employer CPM Industrial Fabricator and Construction Corporation with an interest of 6 % for 180 days on the principal collected in advance. The corporation would accept a promissory note for P 20,000 non-interest for 180 days. If discounted at once, find the proceeds of the note.

A. P 18,600
B. P 18,800
C. P 19,000
D. P 19,200

Problem 914: ECE Board November 1998

What will be the future worth of money after 12 months, if the sum of P 25,000 is invested today at simple interest rate of 1% per month?

A. P 30,000
B. P 29,000
C. P 28,000
D. P 27,859

Problem 915: ECE Board November 1999

If you borrowed money from your friend with simple interest of 12%, find the present worth of P 50,000, which is due at the end of 7 months.

A. P 46,200
B. P 44,893
C. P 46,730
D. P 45,789

Problem 916:

Annie buys a television set from a merchant who ask P 1,250 at the end of 60 days. Annie wishes to pay immediately and the merchant offers to compute the cash price on the assumption that money is worth 8% simple interest. What is the cash price?

A. P 1,233.55
B. P 1,244.66
C. P 1,323.66
D. P 1,392.67

Problem 917: ME Board April 1998

It is the practice of almost all banks in the Philippines that when they grant a loan, the interest for one year is automatically deducted from the principal amount upon release of money to a borrower. Let us therefore assume that you applied for a loan with a bank and the P 80,000 was approved at an interest rate of 14 % of which P 11,200 was deducted and you were given a check of P 68,800. Since you have to pay the amount of P 80,000 one year after, what then will be the effective interest rate?

A. 15.90 %
B. 16.28 %
C. 16.30 %
D. 16.20 %

Problem 918: EE Board April 1996

A man borrowed P 20,000 from a local commercial bank which has a simple interest of 16% but the interest is to be deducted from the loan at the time that the money was borrowed and the loan is payable at the end of one year. How much is the actual rate of interest.

A. 12%
B. 14%
C. 10%
D. 19%

Problem 919: ME Board April 1998

A bank charges 12 % simple interest on a P 300.00 loan. How much will be repaid if the loan is paid back in one lump sum after three years?

A. P 408.00

B. P 415.00
C. P 551.00
D. P 450.00

Problem 920: EE Board October 1997

A man borrowed P 100,000 at the interest rate of 12% per annum, compounded quarterly. What is the effective rate?

A. 3 %
B. 13.2 %
C. 12 %
D. 12.55 %

Problem 921: ECE Board April 1999

What is the corresponding effective rate of 18% compounded semi-quarterly?

A. 19.25 %
B. 19.48 %
C. 18.46 %
D. 18.95 %

Problem 922: ME Board October 1995, EE Board October 1997

Mandarin Bank advertises 9.5 % account that yields 9.84 % annually. Find how often the interest is compounded.

A. Daily
B. Monthly
C. Bi-monthly
D. Quarterly

Problem 923: EE Board October 1993

A bank pays one percent interest on savings accounts four times a year. The effective annual interest rate is

A. 4.06 %
B. 1.00 %
C. 2.04 %
D. 3.36 %

Problem 924: ECE Board November 1998

The effective rate of 14% compounded semi-annually is

A. 14.49 %
B. 12.36 %
C. 14.94 %
D. 14.88 %

Problem 925: ME Board October 1996

An interest rate is quoted as being 7.5% compounded quarterly. What is the effective annual interest rate?

A. 7.71 %
B. 7.22 %
C. 15.78 %
D. 21.81 %

Problem 926: ECE Board April 1998

The amount of P 12,800 in 4 years at 5 % compounded quarterly is

A. P 14,785.34
B. P 15,614.59
C. P 16,311.26
D. P 15,847.33

Problem 927: ECE Board April 1999

Find the present worth of a future payment of P 100,000 to be made in 10 years with an interest of 12% compounded quarterly.

A. P 30,444.44
B. P 33,000.00
C. P 30,655.68
D. P 30,546.01

Problem 928: EE Board June 1990

On his 6th birthday a boy is left an inheritance. The inheritance will be paid in a lump sum of P 10,000 on his 21st birthday. What is the present value of the inheritance as of the boy's 6th birthday, if the interest is compounded annually? Assume i = 4%.

A. P 6,500
B. P 8,600
C. P 5,500
D. P 7,500

Problem 929: ECE Board April 1999

The amount of P 50,000 was deposited in the bank earning at interest of 7.5% per annum. Determine the total amount at the end of 5 years, if the principal and interest were not withdrawn during the period?

A. P 71,781.47
B. P 72,475.23
C. P 70,374.90
D. P 78,536.34

Problem 930: ME Board April 1993

Alexander Michael owes P 25,000.00 due in 1 year and P 75,000 due in 4 years. He agrees to pay P 50,000.00 today and the balance in 2 years. How much must he pay at the end of two years if money is worth 5% compounded semi-annually?

A. P 38,025.28
B. P 35,021.25
C. P 30,500.55
D. P 39,021.28

Problem 931: ECE Board November 1998

At an interest rate of 10% compounded annually, how much will a deposit of P 1,500 be in 15 years?

A. P 6,100.00
B. P 6,234.09
C. P 6,265.87
D. P 6,437.90

Problem 932: CE Board May 1995

How long (in years) will it take money to quadruple if it earns 7 % compounded semi-annually?

A. 20.15
B. 26.30
C. 33.15
D. 40.30

Problem 933: ECE Board April 1999

In how many years is required for P 2,000 to increase by P 3,000 if interest at 12% compounded semi-annually?

A. 8
B. 9
C. 10
D. 7

Problem 934: ME Board April 1996

Consider a deposit of P 600.00 to be paid back in one year by P 700.00. What are the conditions on the rate of interest, i% per year compounded annually such that the net present worth of the investment is positive? Assume $i \geq 0$.

A. $0 \leq i < 14.3\ \%$
B. $0 \leq i < 16.7\ \%$
C. $12.5\ \% \leq i < 14.3\ \%$
D. $16.7\ \% \leq i < 100\%$

Problem 935: ME Board October 1995

A company invests P 10,000 today to be repaid in 5 years in one lump sum at 12 % compounded annually. How much profit in present day pesos is realized?

A. P 7,632
B. P 7,236
C. P 7,326
D. P 7,362

Problem 936: ME Board April 1996

A firm borrows P 2,000 for 6 years at 8 %. At the end of 6 years, it renews the loan for the amount due plus P 2,000 more for 2 years at 8 %. What is the lump sum due?

A. P 5,355.00
B. P 5,892.00
C. P 6,035.00
D. P 6,135.00

Problem 937: ME Board October 1996

A deposit of P 1,000 is made in a bank account that pays 8 % interest compounded annually. Approximately how much money will be in the account after 10 years?

A. P 1,925.00
B. P 1,860.00
C. P 2,345.00
D. P 2,160.00

Problem 938: CE Board May 1996

P 200,000 was deposited on January 1, 1988 at an interest rate of 24 % compounded semi-annually. How much would the sum be on January 1, 1993?

A. P 401,170
B. P 421,170
C. P 521,170
D. P 621,170

Problem 939: CE Board November 1996

If P 500,000 is deposited at a rate of 11.25 % compounded monthly, determine the compounded interest after 7 years and 9 months.

A. P 660,550
B. P 670,650
C. P 680,750
D. P 690,850

Problem 940: ME Board October 1996

Fifteen years ago P 1,000.00 was deposited in a bank account, and today it is worth P 2,370.00. The bank pays interest semi-annually. What was the interest rate paid in this account?

A. 3.8 %
B. 4.9 %
C. 5.0 %
D. 5.8 %

Problem 941: ME Board April 1998

If P 5,000.00 shall accumulate for 10 years at 8 % compounded quarterly, find the compounded interest at the end of 10 years.

A. P 6,005.30
B. P 6,000.00
C. P 6,040.20
D. P 6,010.20

Problem 942: ME Board April 1998

A sum of P 1,000 is invested now and left for eight years, at which time the principal is withdrawn. The interest has accrued is left for another eight years. If the effective annual interest rate is 5 %, what will be the withdrawal amount at the end of the 16th year?

A. P 706.00
B. P 500.00
C. P 774.00
D. P 799.00

Problem 943: ME Board April 1998

P 1,500.00 was deposited in a bank account, 20 years ago. Today it is worth P 3,000.00. Interest is paid semi-annually. Determine the interest rate paid on this account.

A. 3 %
B. 2.9 %
C. 3.5 %
D. 4 %

Problem 944: ME Board April 1998

A merchant puts in his P 2,000.00 to a small business for a period of six years. With a given interest rate on the investment of 15 % per year, compounded annually, how much will he collect at the end of the sixth year?

A. P 4,400.00
B. P 4,390.15
C. P 4,200.00
D. P 4,626.00

Problem 945: ECE Board November 1998

A man expects to receive P 25,000 in 8 years. How much is that money worth now considering interest at 8% compounded quarterly?

A. P 13,859.12
B. P 13,958.33
C. P 13,675.23
D. P 13,265.83

Problem 946: CE Board November 1994

P 500,000 was deposited 20.15 years ago at an interest rate of 7% compounded semi-annually. How much is the sum now?

A. P 2,000,000
B. P 2,000,150
C. P 2,000,300
D. P 2,000,500

Problem 947: ME Board October 1995

In year zero, you invest P 10,000.00 in a 15% security for 5 years. During that time, the average annual inflation is 6 %. How much, in terms of year zero pesos will be in the account at maturity?

A. P 12,020
B. P 13,030
C. P 14,040
D. P 15,030

Problem 948: ECE Board April 1998

By the condition of a will, the sum of P 20,000 is left to a girl to be held in trust fund by her guardian until it amounts to P 50,000. When will the girl receive the money if the fund is invested at 8 % compounded quarterly?

A. 7.98 years
B. 10.34 years
C. 11.57 years
D. 10.45 years

Problem 949: ME Board October 1996

You borrow P 3,500.00 for one year from a friend at an interest rate of 1.5 % per month instead of taking a loan from a bank at a rate of 18% per year. Compare how much money you will save or lose on the transaction.

A. You will pay P 155.00 more if you borrowed from the bank.
B. You will save P 55.00 by borrowing from your friend.
C. You will pay P 85.00 more if you borrowed from the bank.
D. You will pay P 55.00 less if you borrowed from the bank

Problem 950: ME Board April 1996

What is the present worth of two P 100 payments at the end of the third year and fourth year? The annual interest rate is 8%.

A. P 153
B. P 160
C. P 162
D. P 127

ANSWER KEY

911. A	921. B	931. C	941. C
912. C	922. D	932. A	942. A
913. B	923. A	933. A	943. C
914. C	924. A	934. B	944. D
915. C	925. A	935. A	945. D
916. A	926. B	936. C	946. B
917. B	927. C	937. D	947. D
918. D	928. C	938. D	948. C
919. A	929. A	939. D	949. D
920. D	930. D	940. D	950. A

RATING

☐ 34-40 Topnotcher
☐ 26-33 Passer
☐ 20-25 Conditional
☐ 0-19 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 22

911. $F = P(1 + in)$

$$= 4000\left[1 + 0.16\left(\frac{75}{360}\right)\right]$$

$F = 4{,}133.33$

912. $890.36 = 0.8\ I$

$I = 1112.95$

$I = Pin$

Substitute:

$$1112.95 = 110{,}000\ (i)\left(\frac{31}{360}\right)$$

$i = 11.75\ \%$

913. Interest $= 0.06\ (20{,}000) = 1{,}200$

Proceeds on the note $= 20{,}000 - 1200 = 18{,}800$

914. $F = P(1 + in)$

$= 25{,}000\ [1 + 0.01(12)]$

$F = 28{,}000$

915. $F = P(1 + in)$

$$50{,}000 = P\left[1 + 0.12\left(\frac{7}{12}\right)\right]$$

$P = 46{,}728.97$

Note: From the choices, the nearest answer is 46,730

916. $F = P(1 + in)$

$$1250 = P\left[1 + 0.08\left(\frac{60}{360}\right)\right]$$

$P = P1233.55$

917. $I = Pin$

$11{,}200 = 68{,}800(i)(1)$

$i = 16.28\%$

918. $I = 0.16(20{,}00) = 3200$

$I = Pin$

$3200 = (20{,}000 - 3200)(i)(1)$

$i = 19\%$

919. $F = P(1 + in)$

$F = 300[1 + 0.12(3)]$

$F = 408$

920. $ER = \left(1+\frac{i}{4}\right)^4 - 1 = \left(1+\frac{0.12}{4}\right)^4 - 1$

$ER = 12.55\%$

921. $ER = \left(1+\frac{i}{8}\right)^8 - 1 = \left(1+\frac{0.18}{8}\right)^8 - 1$

$ER = 19.48\%$

922. $ER = \left(1+\frac{1}{n}\right)^n - 1$

$0.0984 = \left(1+\frac{0.095}{n}\right)^n - 1$

$1.0984 = \left(1+\frac{0.095}{n}\right)^n$

By trial and error, n = 4.
Thus, the mode of interest is quarterly.

923. $ER = \left(1+\frac{i}{4}\right)^4 - 1 = (1+0.01)^4 - 1$

$ER = 4.06\%$

924. $ER = \left(1+\frac{i}{6}\right)^2 - 1 = \left(1+\frac{0.14}{2}\right)^2 - 1$

$ER = 14.49\%$

925. $ER = \left(1+\frac{i}{4}\right)^4 - 1 = \left(1+\frac{0.075}{4}\right)^4 - 1$

$ER = 7.71\ \%$

926. $F = P(1 + i)^n$
where: $i = 0.05/4 = 0.0125$
$n = 4(4) = 16$

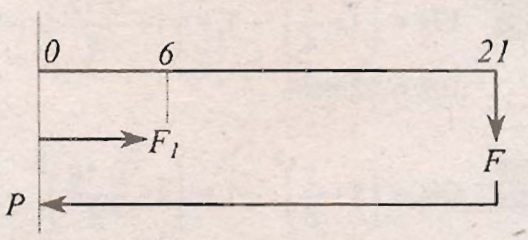
$F = 12{,}800(1 + 0.0125)^{16}$
$F = 15{,}614.59$

927. $F = P(1 + i)^n$
where: $i = 0.12/4 = 0.03$
$n = 4(10) = 40$

$100{,}000 = P(1 + 0.03)^{40}$
$P = 30{,}655.68$

928. $F = P(1 + i)^n$
where: $i = 0.04$
$n = 21$

$10{,}000 = P(1 + 0.04)^{21}$
$P = 4388.336$

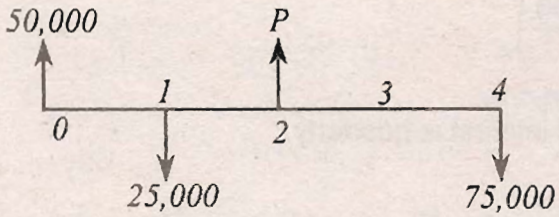


$F_1 = P(1+ i)^6$
$F_1 = 4388.336(1 + 0.04)^6 = 5552.645$

Note: From the choices, the nearest answer is 5,500

929. $F = P(1 + i)^5$
$= 50{,}000(1 + 0.075)^5$
$F = 71{,}781.47$

930.



Solving for the effective rate per year:

$$ER = \left(1+\frac{i}{2}\right)^2 - 1 = \left(1+\frac{0.05}{2}\right)^2 - 1 = 0.050625$$

$$50{,}000 + \frac{P}{(1+i)^2} = \frac{25{,}000}{(1+i)^1} + \frac{75{,}000}{(1+i)^4}$$

$$50{,}000 + \frac{P}{(1+0.050625)^2} = \frac{25{,}000}{(1+0.050625)^1} + \frac{75{,}000}{(1+0.050625)^4}$$

$$P = 39{,}021.28$$

931. $F = P(1 + i)^n$
$= 3{,}500(1 + 0.1)^{15}$
$F = 6{,}265.87$

932. $i = 0.07/2 = 0.035$

$F = P(1 + i)^{2n}$
$4P = P(1 + 0.035)^{2n}$
$4 = (1.035)^{2n}$

Take log on both sides:
$\log 4 = \log(1.035)^{2n}$
$\log 4 = 2n \log 1.035$
$n = 20.15$ years

933. $i = 0.12/2 = 0.06$

$$F = P(1 + i)^{2n}$$
$$2000 + 3000 = 2000\ (1 + 0.06)^{2n}$$
$$2.5 = (1.06)^{2n}$$

Take log on both sides:
$\log 2.5 = \log(1.06)^{2n}$
$\log 2.5 = 2n \log 1.06$
$n = 7.86$ years, approximately 8 years

934. $F = P(1 + i)^n$
$700 = 600(1 + i)^1$
$i = 16.67\%$

935. $F = P(1 + i)^n$
$= 10{,}000(1 + 0.12)^5$
$F = 17{,}623.42$

Profit = F – P = 17,623.42 – 10,000
Profit = 7,623.42

936. $F = P(1 + i)^8 + P(1 + i)^2$
$= 2000(1 + 0.08)^8 + 2000(1 + 0.08)^2$
$F = 6{,}034.66$

937. $F = P(1 + i)^n$
$= 1{,}000(1 + 0.08)^{10}$
$F = 2{,}158.92$

Note: From the choices, the nearest answer is 2,160.

938. $n = 2(1993 – 1988) = 10$
$i = 0.24/2 = 0.12$

$F = P\ (1 + i)^n$
$= 200{,}000(1 + 0.12)^{10}$
$F = 621{,}170$

939. $n = 12(7) + 9 = 93$
$i = 0.1125/12 = 0.009375$

$F = P(1 + i)^n$
$= 500{,}000\ (1 + 0.009375)^{93}$
$F = 1{,}190{,}848.73$

Interest = F – P = 1,190,848.73 – 500,000
Interest = 690,848.73

Note: From the choices, the nearest answer is 690,850

940. $n = 15(2) = 30$

$$F = P(1 + i)^n$$
$$2{,}370 = 1{,}000(1 + i/2)^{30}$$
$$2.37 = (1 + 0.5i)^{30}$$
$$i = 5.8\%$$

941. $n = 10(4) = 40$
$i = 0.08/4 = 0.02$

$$F = P(1 + i)^n$$
$$= 5{,}000(1 + 0.02)^{40}$$
$$F = 11{,}040.20$$

Interest = F – P = 11,040.2 – 5,000
Interest = 6,040.20

942. $F = P(1 + i)^n$
$= 1{,}000(1 + 0.05)^8$
$F = 1{,}477.455$

Money left after the principal is withdrawn = 1,477.455 – 1000 = 477.455

Let: F_{16} = total amount after the end of 16th year

$$F_{16} = P(1 + i)^8$$
$$= 477.455(1 + 0.05)^8$$
$$F_{16} = 705.42$$

Note: From the choices, the nearest answer is 706

943. $n = 20(2) = 40$

$$F = P(1 + i)^n$$
$$3{,}000 = 1{,}500(1 + i/2)^{40}$$
$$2 = (1 + 0.5i)^{40}$$
$$i = 3.5\%$$

944. $F = P(1 + i)^n$
$= 2{,}000(1 + 0.15)^6$
$F = 4{,}626$

945. $n = 8(4) = 32$
$i = 0.08/4 = 0.02$

$$F = P(1 + i)^n$$
$$25{,}000 = P(1 + 0.02)^{32}$$
$$P = 13{,}265.83$$

946. $n = 20.15(2) = 40.3$
$i = 0.07/2 = 0.035$

$F = P(1 + i)^n$
$= 500{,}000(1 + 0.035)^{43}$
$F = 200{,}166$

Note: From the choices, the nearest answer is 200,150

947. Let: F = value of the account after 5 years considering there was no inflation.
P' = value of the account in today's peso due to inflation

$F = P(1 + i)^n$
$= 10{,}000(1 + 0.15)^5$
$F = 20{,}113.57$

$F = P'(1 + i)^n$
$20{,}113.57 = P'(1 + 0.06)^5$
$P' = 15{,}030$

948. $i = 0.08/4 = 0.02$

$F = P(1 + i)^{4n}$
$50{,}000 = 20{,}000(1 + 0.02)^{4n}$
$2.5 = (1.02)^{4n}$

Take log on both sides:
$\log 2.5 = \log(1.02)^{4n}$
$\log 2.5 = 4n \log 1.02$
$n = 11.57$ years

949. Compute for the amount due after one year.

a. Borrow money from a friend
$F = P(1 + i)^n$
$F = 3{,}500(1 + 0.015)^{12} = 4{,}185$

b. Borrow money from a bank
$F = P(1 + i)^n$
$= 3{,}500(1 + 0.018)^1 = 4{,}130$

Thus, you will pay P55 less by borrowing the money from the bank.

950. $P = P_1 + P_2$

$$P = \frac{F_1}{(1+i)^3} + \frac{F_2}{(1+i)^4}$$

$$P = \frac{100}{(1+0.08)^3} + \frac{100}{(1+0.08)^4} = 153$$

DAY 23

ENGINEERING ECONOMY (ANNUITY, DEPRECIATION, BONDS, BREAKEVEN ANALYSIS, ETC.)

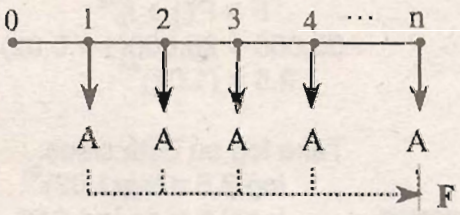
ANNUITY

Annuity is defined as a series of equal payments occurring at equal interval of time. When an annuity has a fixed time span, it is known as **annuity certain**. The following are annuity certain:

1. **Ordinary annuity** is a type of annuity where the payments are made the end of each period beginning on the first period.

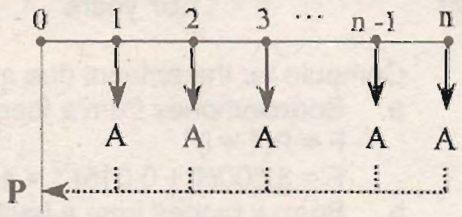
a) Sum of ordinary annuity

$$F = \frac{A\left[(1+i)^n - 1\right]}{i}$$



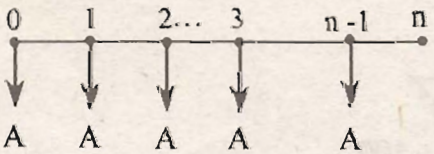
b) Present worth of ordinary annuity

$$P = \frac{A\left[(1+i)^n - 1\right]}{i(1+i)^n}$$



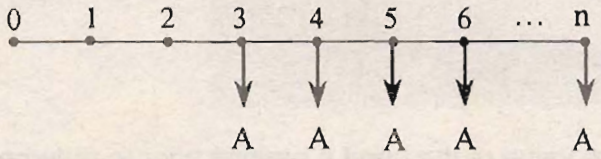
where: P = principal
i = interest per period
n = number of periods
A = uniform payment

2. **Annuity due** is the type of annuity where the payments are made at the beginning of each period starting from the first period.



cash flow of annuity due

3. **Deferred annuity** is the one where the first payment does not begin until some later date in the cash flow.



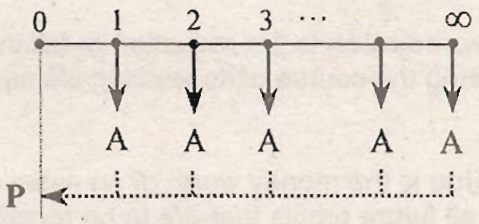
cash flow of deferred annuity

When an annuity does not have a fixed time span but continues indefinitely, then it is referred to as a **perpetuity**. The sum of a perpetuity is an infinite value.

Present worth of a perpetuity:

$$P = \frac{A}{i}$$

where: A = uniform payment
i = interest per period



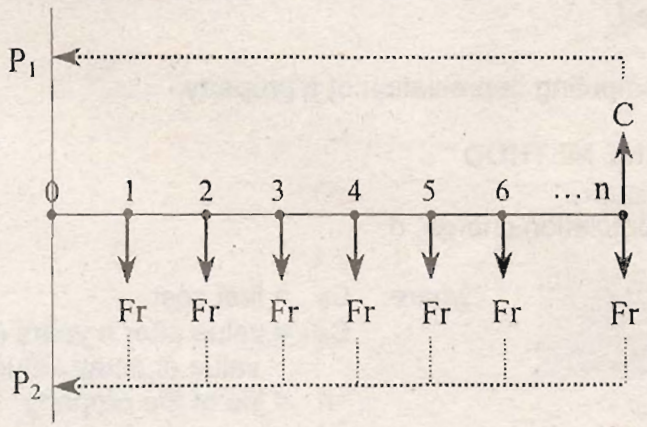
Capitalized cost of a property refers to the sum of its first cost and cost of perpetual maintenance. Thus,

Capitalized cost = first cost + cost of perpetual maintenance

BONDS

Bond is a long-term note or a financial security issued by businesses or corporation and guaranteed on certain assets of the corporation or its subsidiaries. Bonds are repayable on maturity and bear a fixed nominal rate of interest.

Bond rate refers to the rate of interest that is quoted in the bond.
Bond value is the present worth of the future payments that will be received.



$$V_n = P_1 + P_2$$

$$V_n = \frac{C}{(1+i)^n} + \frac{Fr\left[(1+i)^n - 1\right]}{i(1+i)^n}$$

Where : V_n = value of the bond n periods prior to redemption
C = redemption or amount at maturity (usually equal to F)
F = par value of the bond
n = number of periods prior to redemption
i = bond yield interest per period
r = bond rate per interest period

DEPRECIATION

Depreciation is the reduction or fall in the value of an asset or physical property during the course of its working life and due to passage of time.

Value is the money worth of an asset or product. It also refers to the present worth of all future profits that are to be received through ownership of a particular property.

Market value is the amount a willing buyer will pay to a willing seller for a property where each has equal advantage and neither one of them is under the compulsion to buy or sell.

Book value is the worth of the property as reflected in the book of records of the company.

Use value is the amount of the property which the owner believed to be its worth as an operating unit.

Fair value is the worth of the property determined by a disinterested person in order to establish an amount which is fair to both the buyer and the seller.

Salvage value the amount obtained from the sale of the property. This is also known as **resale value**. Salvage value implies that the property will still be use for the purpose it is intended.

Different methods of computing depreciation of a property:

1. **STRAIGHT LINE METHOD**

   a) Annual depreciation charge, d:

$$d = \frac{C_0 - C_n}{n}$$

where: C_0 = first cost
C_n = value after n years (salvage value or scrap value)
n = life of the property

b) Book value after "m" years, Cm:

$$C_m = C_0 - D_m$$

where: D_m = total depreciation after "m" years
$D_m = d(m)$

2. **SINKING FUND METHOD**

a) Annual depreciation charge

$$d = \frac{(C_0 - C_n)i}{[(1+i)^n - 1]}$$

b) Book value at the end of "m" years

$$D_m = C_0 - C_m$$

where: Dm = total depreciation after "m" years

$$D_m = \frac{d[(1+i)^n - 1]}{i}$$

3. **DECLINING BALANCE METHOD**. Also known as Diminishing Balance Method or Constant Percentage Method.

Use the Matheson Formula:

$$k = 1 - \sqrt[n]{\frac{Cn}{Co}} \quad \text{or} \quad k = 1 - \sqrt[m]{\frac{Cm}{Co}}$$

Note: This method is not applicable if the salvage or scrap value is zero.

4. **SUM-OF-YEARS' DIGITS METHOD**. Commonly known as SYD Method.

a) Sum of the years' digit, Σ_{years}:

$$\sum_{years} = \frac{(n)(n+1)}{2}$$

b) Respective depreciation charges:

First year: $$d_1 = (C_0 - C_n)\frac{n}{\sum_{years}}$$

Second year: $$d_2 = (C_0 - C_n)\frac{(n-1)}{\sum_{years}}$$

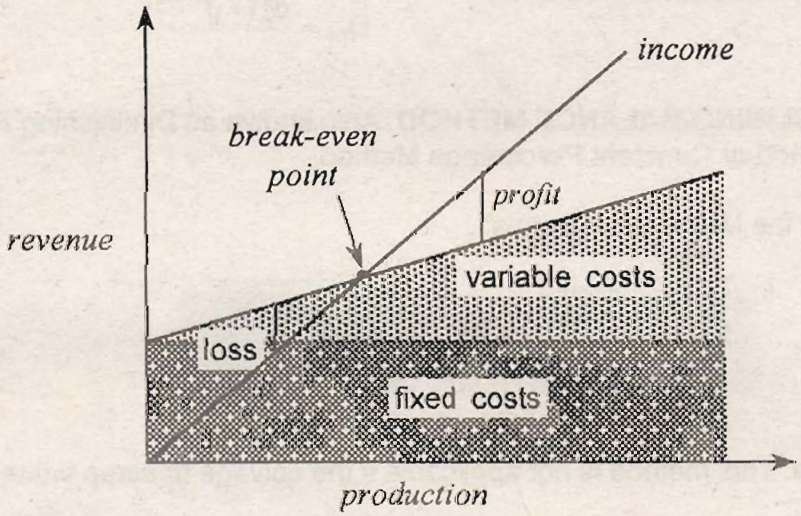
Third year: $$d_3 = (C_0 - C_n)\frac{(n-2)}{\sum_{\text{years}}}$$

And so on...

BREAK EVEN ANALYSIS

Break-even refers to the situation where the sales generated (income) is just enough to cover the fixed and variable cost (expenses). The level of production where the total income is equal to the total expenses is known as **break-even point.**

Break-even chart is a diagram which shows relationship between volume and fixed costs, variable costs, and income. The following is an example of a break-even chart.



LEGAL FORMS OF BUSINESS ORGANIZATIONS

The legal forms of business organizations are the following:

1. **Sole proprietorship** – considered as the simplest type of business organization wherein the firm is owned and controlled by a single person.

2. **Partnership** – is a firm owned and controlled by two or more persons who are bind to a partnership agreement.

3. **Corporation** – is a firm owned by a group of ordinary shareholders and the capital of which is divided up to the number of shares. It is also defined as a distinct legal entity separate from the individuals who owns it and can engage in any business transaction which a real person could do. This is sometimes known as joint-stock company or a cooperative.



Did you know that... There are three problems in Geometry, which attracted the interest of mathematicians in the ancient times, considered as "impossible problems". They are the following:

1. Duplication of a cube – to construct a cube whose volume shall be twice that of a given cube.
2. Squaring a circle – to construct a square whose area shall be equal to the area of a given circle.
3. Trisection of an arbitrary angle – to construct an angle that is exactly one-third of a given angle.

Proceed to the next page for your 23rd test. GOODLUCK ! ☞

Test 23

Time element: 4.0 hours

Problem 951: EE Board October 1997

A man purchased on monthly installment a P 100,000 worth of land. The interest rate is 12 % nominal and payable in 20 years. What is the monthly amortization?

A. P 1,101.08
B. P 1,121.01
C. P 1,152.15
D. P 1,128.12

Problem 952: ECE Board April 1998

Money borrowed today is to be paid in 6 equal payments at the end of 6 quarters. If the interest is 12 % compounded quarterly. How much was initially borrowed if quarterly payment is P 2000.00?

A. P 10,834.38
B. P 10,382.90
C. P 10,586.99
D. P 10,200.56

Problem 953: ME Board October 1996

You need P 4,000 per year for four years to go to college. Your father invested P 5,000 in 7 % account for your education when you were born. If you withdraw P 4,000 at the end of your 17th, 18th, 19th and 20th birthday, how much will be left in the account at the end of the 21st year?

A. P 1,700
B. P 2,500
C. P 3,400
D. P 4,000

Problem 954: ECE Board November 1998

What is the accumulated amount of five year annuity paying P 6,000 at the end of each year, with interest at 15 % compounded annually?

A. P 40,454.29
B. P 41,114.29
C. P 41,454.29
D. P 40,544.29

Problem 955: ME Board April 1998

How much must be deposited at 6% each year beginning on January 1, year 1, in order to accumulate P 5,000 on the date of the last deposit, January 1, year 6?

A. P 751.00
B. P 717.00
C. P 715.00
D. P 725.00

Problem 956: ECE Board November 1998

A debt of P 10,000 with 10 % interest compounded semi-annually is to be amortized by semi-annual payment over the next 5 years. The first due in 6 months. Determine the semi-annual payment.

A. P 1,200.00
B. P 1,295.05
C. P 1,193.90
D. P 1,400.45

Problem 957: EE Board October 1997

A young engineer borrowed P 10,000 at 12 % interest and paid P 2,000 per annum for the last 4 years. What does he have to pay at the end of the fifth year in order to pay off his loan?

A. P 6,919.28
B. P 5,674.00
C. P 6,074.00
D. P 3,296.00

Problem 958: EE Board April 1997

Mr. Cruz plans to deposit for the education of his 5 years old son, P 500 at the end of each month for 10 years at 12% annual interest compounded monthly. The amount that will be available in two years is

A. P 13,000
B. P 14,500
C. P 13,500
D. P 14,000

Problem 959: ME Board October 1994

If you obtain a loan of P 1M at the rate of 12% compounded annually in order to build a house, how much must you pay monthly to amortize the loan within a period of ten years?

A. P 13,994.17
B. P 12,955.21
C. P 15,855.45
D. P 12,900.25

Problem 960: ECE Board April 1998

How much must you invest today in order to withdraw P 2,000 annually for 10 years if the interest rate is 9%?

A. P 12,853.32
B. P 12,881.37
C. P 12,385.32
D. P 12,835.32

Problem 961: ECE Board April 1998

A person buys a piece of lot for P 100,000 downpayment and 10 deferred semi-annual payments of P 8,000 each, starting three years from now. What is the present value of the investment if the rate of interest is 12 % compounded semi-annually?

A. P 134,666.80
B. P 143,999.08
C. P 154,696.80
D. P 164,969.80

Problem 962: CE Board May 1998

A man loans P 187,400 from a bank with interest at 5% compounded annually. He agrees to pay his obligations by paying 8 equal annual payments, the first being due at the end of 10 years. Find the annual payments.

A. P 44,982.04
B. P 56,143.03
C. P 62,334.62
D. P 38,236.04

Problem 963:

A housewife bought a brand new washing machine costing P 12,000 if paid in cash. However, she can purchase it on installment basis to be paid within 5 years. If money is worth 8% compounded annually, what is her yearly amortization if all payments are to be made at the beginning of each year?

A. P 2,782.85
B. P 2,872.58
C. P 2,400.00
D. P 2,827.58

Problem 964: ME Board October 1996

Mr. Ayala borrows P 100,000 at 10% effective annual interest. He must pay back the loan over 30 years with uniform monthly payments due on the first day of each month. What does Mr. Ayala pay each month?

A. P 870.00
B. P 846.00
C. P 878.00
D. P 839.00

Problem 965: ME Board April 1998

A house and lot can be acquired by a downpayment of P 500,000 and a yearly payment of P 100,000 at the end of each year for a period of 10 years, starting at the end of 5 years from the date of purchase. If money is worth 14% compounded annually, what is the cash price of the property?

A. P 810,100
B. P 808,811
C. P 801,900
D. P 805,902

Problem 966: ME Board April 1998

A piece of machinery can be bought for P 10,000 cash or for P 2,000 down and payments of P 750 per year for 15 years. What is the annual interest rate for the time payments?

A. 4.61 %
B. 3.81 %
C. 5.71 %
D. 11.0 %

Problem 967: CE Board November 1996

A man inherited a regular endownment of P 100,000 every end of 3 months for 10 years. However, he may choose to get a single lump sum payment at the end of 4 years. How much is this lump sum if the cost of money is 14% compounded quarterly?

A. P 3,802,862
B. P 3,702,939
C. P 3,502,546
D. P 3,602,431

Problem 968: ME Board April 1998

A parent on the day the child is born wishes to determine what lump sum would have to be paid into an account bearing interest at 5 % compounded annually, in order to withdraw P 20,000 each on the child's 18th, 19th, 20th and 21st birthdays. How much is the lump sum amount?

A. P 35,941.73
B. P 33,941.73
C. P 30,941.73
D. P 25,941.73

Problem 969: ME Board April 1998

An instructor plans to retire in exactly one year and want an account that will pay him P 25,000 a year for the next 15 years. Assuming a 6 % annual effective interest rate, what is the amount he would need to deposit now? (The fund will be depleted after 15 years).

A. P 249,000

B. P 242,806
C. P 248,500
D. P 250,400

Problem 970: EE Board October 1997

An investment of P 350,000 is made to be followed by payments of P 200,000 each year for 3 years. What is the annual rate of return on investment for the project?

A. 41.7 %
B. 32.7 %
C. 51.1 %
D. 15 %

Problem 971: EE Board April 1997

A small machine has an initial cost of P 20,000, a salvage value of P 2,000 and a life of 10 years. If your cost of operation per year is P 3,500 and your revenues per year is P 9,000, what is the approximate rate of return (ROR) on the investment?

A. 25.0 %
B. 22.5 %
C. 23.9 %
D. 24.8 %

Problem 972: CE Board November 1996

A man paid 10% down payment of P 200,000 for a house and lot and agreed to pay the balance on monthly installments for "x" years at an interest rate of 15% compounded monthly. If the monthly installment was P 42,821.87, find the value of x?

A. 11
B. 9
C. 5
D. 7

Problem 973: ME Board April 1998

A manufacturing firm wishes to give each 80 employees a holiday bonus. How much is needed to invest monthly for a year at 12 % nominal interest rate compounded monthly, so that each employee will receive a P 2,000 bonus?

A. P 12,608
B. P 12,610
C. P 12,600
D. P 12,300

Problem 974: CE Board November 1995

Find the present value in pesos, of a perpetuity of P15,000 payable semi-annually if money is worth 8% compounded quarterly.

A. P 372,537

B. P 374,977
C. P 373,767
D. P 371,287

Problem 975:

A fund donated by a weathy person to IIEE to provide annual scholarships to deserving EE students. The fund will grant P 5,000 for each of the first five years, P 8,000 for the next 5 years and P 10,000 for each year thereafter. The scholarship will start one year after the fund is established. If the fund earns 8% interest, what is the amount of the donation?

A. P 101,605.71
B. P 101,505.21
C. P 100,506.21
D. P 99,601.71

Problem 976: ME Board April 1998

A company issued 50 bonds of P 1,000.00 face value each, redeemable at par at the end of 15 years to accumulate the funds required for redemption. The firm established a sinking fund consisting of annual deposits, the interest rate of the fund being 4 %. What was the principal in the fund at the end of the 12th year?

A. P 35,983.00
B. P 38,378.00
C. P 41,453.00
D. P 37,519.00

Problem 977: ME Board April 1992

A unit of welding machine cost P 45,000 with an estimated life of 5 years. Its salvage value is P 2,500. Find its depreciation rate by straight-line method.

A. 17.75 %
B. 19.88 %
C. 18.89 %
D. 15.56 %

Problem 978: EE Board April 1997

A machine has an initial cost of P 50,000 and a salvage value of P 10,000 after 10 years. Find the book value after 5 years using straight-line depreciation.

A. P 12,500
B. P 30,000
C. P 16,400
D. P 22,300

Problem 979: ME Board October 1992

The initial cost of a paint sand mill, including its installation, is P 800,000. The BIR approved life of this machine is 10 years for depreciation. The estimated salvage value of the mill is P 50,000 and the cost of dismantling is estimated to be P 15,000. Using straight-line depreciation, what is the annual depreciation charge and what is the book value of the machine at the end of six years?

A. P 74,500 ; P 340,250
B. P 76,500 ; P 341,000
C. P 76,500 ; P 342,500
D. P 77,500 ; P 343,250

Problem 980: CE Board November 1997

The cost of equipment is P 500,000 and the cost of installation is P 30,000. If the salvage value is 10% of the cost of equipment at the end of 5 years, determine the book value at the end of the fourth year. Use straight-line method.

A. P 155,000
B. P 140,000
C. P 146,000
D. P 132,600

Problem 981: ME Board April 1998

An asset is purchased for P 500,000.00. The salvage value in 25 years is P 100,000.00. What is the total depreciation in the first three years using straight-line method?

A. P 48,000
B. P 24,000
C. P 32,000
D. P 16,000

Problem 982: ME Board April 1998

A machine has an initial cost of P 50,000 and a salvage value of P10,000.00 after 10 years. What is the book value after 5 years using straight line depreciation?

A. P 35,000.00
B. P 25,000.00
C. P 15,500.00
D. P 30,000.00

Problem 983:

An equipment costs P 10,000 with a salvage value of P 500 at the end of 10 years. Calculate the annual depreciation cost by sinking fund method at 4 % interest.

A. P 791.26
B. P 950.00
C. P 971.12
D. P 845.32

Problem 984: CE Board November 1995

A machine costing P 720,000 is estimated to have a book value of P 40,545.73 when retired at the end of 10 years. Depreciation cost is computed using a constant percentage of the declining book value. What is the annual rate of depreciation in %?

A. 28
B. 25
C. 16
D. 30

Problem 985: CE Board May 1996

A machine costing P 45,000 is estimated to have a book value of P 4,350 when retired at the end of 6 years. Depreciation cost is computed using a constant percentage of the declining book value. What is the annual rate of depreciation in %?

A. 33.25%
B. 32.25%
C. 35.25%
D. 34.25%

Problem 986: ECE Board November 1998

ABC Corporation makes it a policy that for any new equipment purchased, the annual depreciation cost should not exceed 20% of the first cost at any time with no salvage value. Determine the length of service life necessary if the depreciation used is the SYD method.

A. 9 years
B. 10 years
C. 12 years
D. 19 years

Problem 987: ME Board April 1998

A company purchases an asset for P 10,000.00 and plans to keep it for 20 years. If the salvage value is zero at the end of 20th year, what is the depreciation in the third year? Use SYD method.

A. P 1,000.00
B. P 857.00
C. P 937.00
D. P 747.00

Problem 988: ECE Board April 1999

A Telephone company purchased a microwave radio equipment for P 6 million, freight and installation charges amounted to 4% of the purchased price. If the equipment will be depreciated over a period of 10 years with a salvage value of 8%, determine the depreciation cost during the 5th year using SYD.

A. P 626,269.10
B. P 622,786.07

C. P 638,272.08
D. P 627,989.90

Problem 989: ME Board April 1998

An asset is purchased for P 9,000.00. Its estimated life is 10 years after which it will be sold for P 1,000.00. Find the book value during the first year if sum-of-years' digit (SYD) depreciation is used.

A. P 8,000.00
B. P 6,500.00
C. P 7,545.00
D. P 6,000.00

Problem 990: EE Board April 1997

The maintenance cost for a sewing machine this year is expected to be P 500. The cost will increase P 50 each year for the subsequent 9 years. The interest is 8 % compounded annually. What is the approximate present worth of maintenance for the machine over the full 10-year period?

A. P 4,700
B. P 5,300
C. P 4,300
D. P 5,500

Problem 991: CE Board November 1996

At 6%, find the capitalized cost of a bridge whose cost is P 250M and life is 20 years, if the bridge must be partially rebuilt at a cost of P 100M at the end of each 20 years.

A. P 275.3M
B. P 265.5M
C. P 295.3M
D. P 282.1M

Problem 992: CE Board May 1997

A corporation uses a type of motor truck which costs P 5,000 with life of 2 years and final salvage value of P 800. How much could the corporation afford to pay for another type of truck of the same purpose whose life is 3 years with a final salvage value of P 1,000. Money is worth 4%.

A. P 8,450.66
B. P 7,164.37
C. P 6,398.24
D. P 9,034.56

Problem 993: ME Board October 1995

A company must relocate one of its factories in three years. Equipment for the loading dock is being considered for purchase. The original cost is P 20,000, the salvage value of the equipment after three years is P 8,000. The company's rate of return on the money is 10%. Determine the capital recovery rate per year.

A. P 5,115
B. P 4,946
C. P 5,625
D. P 4,805

Problem 994: EE Board October 1998

The annual maintenance cost of a machine shop is P 69,994. If the cost of making a forging is P 56 per unit and its selling price is P 135 per forged unit, find the number of units to be forged to break-even.

A. 886 units
B. 885 units
C. 688 units
D. 668 units

Problem 995: CE Board May 1998

A manufacturer produces certain items at a labor cost of P 115 each, material cost of P 76 each and variable cost of P2.32 each. If the item has a unit price of P 600, how many number of units must be manufactured each month for the manufacturer to break even if the monthly overhead is P 428,000.

A. 1053
B. 1138
C. 946
D. 1232

Problem 996: ME Board April 1996

Steel drum manufacturer incurs a yearly fixed operating cost of $ 200,000. Each drum manufactured cost $ 160 to produce and sells $ 200. What is the manufacturer's break-even sales volume in drums per year?

A. 1250
B. 2500
C. 5000
D. 1000

Problem 997:

JRT Industries manufactures automatic voltage regulators at a labor cost of P 85.00 per unit and material cost of P 350.00 per unit. The fixed charges on the business are P 15,000 per month and the variable costs are P 20.00 per unit. If the automatic voltage regulators are sold to retailers at P 580.00 each, how many units must be produced and sold per month to breakeven?

A. 104

B. 200
C. 120
D. 150

Problem 998: ME Board October 1990

Compute for the number of locks that an ice plant must be able to sell per month to break even based on the following data:

Cost of electricity per block	- P 20.00
Tax to be paid per block	- P 2.00
Real Estate Tax	- P 3,500.00 per month
Salaries and Wages	- P 25,000.00 per month
Others	- P 12,000.00 per month
Selling price of ice	- P 55.00 per block

A. 1228
B. 1285
C. 1373
D. 1312

Problem 999: EE Board October 1997

The annual maintenance cost of a machine is P 70,000. If the cost of making a forging is P 56 and its selling price is P 125 per forged unit. Find the number of units to be forged to break even.

A. 1015 units
B. 985 units
C. 1100 units
D. 1000 units

Problem 1000: ME Board April 1998

XYZ Corporation manufactures bookcases that sells for P 65.00 each. It costs XYZ Corporation P 35,000 per year to operate its plant. This sum includes rent, depreciation charges on equipment, and salary payments. If the cost to produce one bookcase is P 50.00, how many cases must be sold each year for XYZ to avoid taking a loss?

A. 2334
B. 539
C. 750
D. 2333

Problem 1001: ME Board April 1998

A company which manufactures electric motors has a production capacity of 200 motors a month. The variable costs are P 150.00 per motor. The average selling price of the motors is P 275.00. Fixed costs of the company amount to P 20,000 per month which includes taxes. The number of motors that must be sold each month to break even is closest to:

A. 40
B. 150
C. 80
D. 160

ANSWER KEY

951. A	964. D	977. C	990. A
952. A	965. B	978. B	991. C
953. A	966. A	979. B	992. B
954. A	967. B	980. C	993. C
955. B	968. C	981. A	994. A
956. B	969. B	982. D	995. A
957. A	970. D	983. A	996. C
958. C	971. B	984. B	997. C
959. A	972. C	985. B	998. A
960. D	973. D	986. A	999. A
961. B	974. B	987. B	1000. A
962. A	975. D	988. A	1001. D
963. C	976. D	989. C	

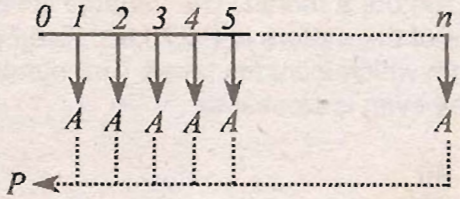
RATING

- ☐ 43-51 Topnotcher
- ☐ 33-42 Passer
- ☐ 25-32 Conditional
- ☐ 0-24 Failed

If FAILED, repeat the test.

SOLUTIONS TO TEST 23

951. $i = 0.12/12 = 0.01$
$n = 12(20) = 240$



$$P = \frac{A\left[(1+i)^n - 1\right]}{(1+i)^n i}$$

$$100{,}000 = \frac{A\left[(1+0.01)^{240} - 1\right]}{(1+0.01)^{240}(0.01)}$$

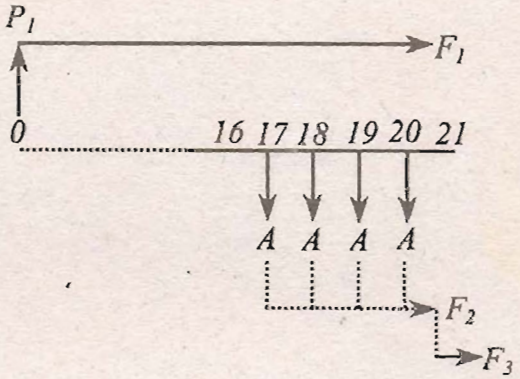
$$A = 1{,}101.08$$

952. $i = 0.12/4 = 0.03$
$n = 6$

$$P = \frac{A\left[(1+i)^n - 1\right]}{(1+i)^n i} = \frac{2000\left[(1+0.03)^6 - 1\right]}{(1+0.03)^6(0.03)}$$

$$P = 10{,}834.38$$

953. $F_1 = P_1(1+i)^n$
$= 500(1+0.07)^{21}$
$F_1 = 20{,}702.81$



$$F_2 = \frac{A\left[(1+i)^n - 1\right]}{i}$$

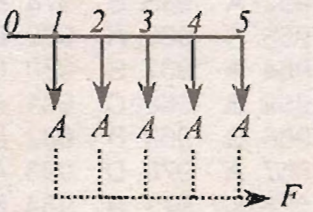
$$= \frac{4000\left[(1+0.07)^4 - 1\right]}{0.07}$$

$$F_2 = 17{,}759.772$$

$F_3 = F_2(1+i)$
$= 17{,}759{,}772\ (1+0.07)$
$F_3 = 19{,}002.95$

Money left $= F_1 - F_3$
$= 20{,}702.81 - 19.002.95$
Money left = 1,699.86, approximately 1,700.

954.



$$F = \frac{A\left[(1+i)^n - 1\right]}{i} = \frac{6000\left[(1+0.15)^5 - 1\right]}{0.15}$$

$$F = 40{,}454.29$$

955.

$$F = \frac{A\left[(1+i)^n - 1\right]}{i}$$

$$5{,}000 = \frac{A\left[(1+0.06)^6 - 1\right]}{0.06}$$

$$A = 717$$

956. $i = 0.10/2 = 0.05$
$n = 2(5) = 10$

$$P = \frac{A\left[(1+i)^n - 1\right]}{(1+i)^n i}$$

$$10{,}000 = \frac{A\left[(1+0.05)^{10} - 1\right]}{(1+0.05)^{10}(0.05)}$$

$$A = 1{,}295.05$$

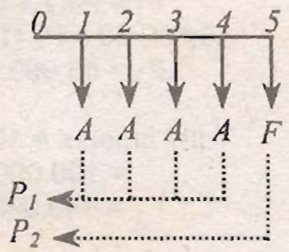
957.

$$P_1 + P_2 = 10{,}000$$

$$\frac{A\left[(1+i)^n - 1\right]}{(1+i)^n i} + \frac{F}{(1+i)^n} = 10{,}000$$

$$\frac{2000\left[(1+0.12)^4 - 1\right]}{(1+0.12)^4(0.12)} + \frac{F}{(1+0.12)^5} = 10{,}000$$

$$F = 6{,}917.72$$



Note: From the choices, the nearest answer is 6,919.28

958. $i = 0.12/12 = 0.01$
$n = 12(2) = 24$

$$F = \frac{A\left[(1+i)^n - 1\right]}{i} = \frac{500\left[(1+0.01)^{24} - 1\right]}{0.01}$$

$$F = 13{,}486.7$$

Note: From the choices, the nearest answer is 13,500

959. Solving for the interest rate per month:

$$i = \left(1 + \frac{i}{12}\right)^{12} - 1$$

$$0.12 = \left(1 + \frac{i}{12}\right)^{12} - 1$$

$$\frac{i}{12} = 0.009488$$

$$n = 12(10) = 120$$

$$P = \frac{A\left[(1+i)^n - 1\right]}{(1+i)^n i}$$

$$1{,}000{,}000 = \frac{A\left[(1+0.009488)^{120} - 1\right]}{(1+0.009488)^{120}(0.009488)}$$

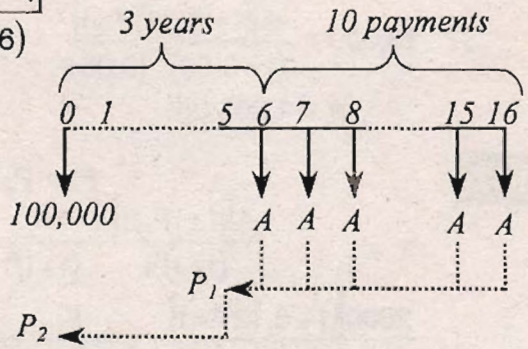
$$A = 13{,}994.17$$

960. $P = \dfrac{A\left[(1+i)^n - 1\right]}{(1+i)^n i} = \dfrac{2000\left[(1+0.09)^{10} - 1\right]}{(1+0.09)^{10}(0.09)}$

$P = 12{,}835.32$

961. $P_1 = \dfrac{A\left[(1+i)^n - 1\right]}{(1+i)^n i} = \dfrac{8000\left[(1+0.06)^{10} - 1\right]}{(1+0.06)^{10}(0.06)}$

$P_1 = 58{,}880.69$



$P_1 = P_2(1 + i)^n$

$58{,}880.69 = P_2\,(1 + 0.06)^5$

$P_2 = 43{,}999.078$

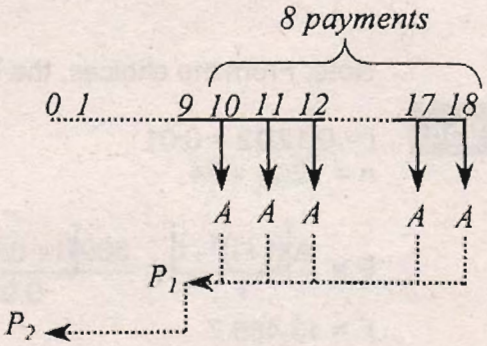
Total amount = 100,000 + P_2

= 100,000 + 43,999.078

= 143,999.078

962. $P_1 = \dfrac{A\left[(1+i)^n - 1\right]}{(1+i)^n i} = \dfrac{A\left[(1+0.05)^8 - 1\right]}{(1+0.05)^8(0.05)}$

$P_1 = 6.643\ A$



$P_1 = P_2(1 + i)^n$

$6.643\ A = 187{,}400(1 + 0.5)^9$

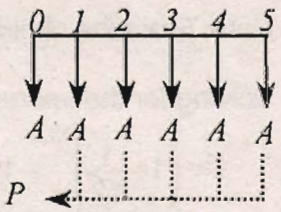
$A = 44{,}982.04$

963. Cash price = A + P

$12{,}000 = A + \dfrac{A\left[(1+i)^n - 1\right]}{(1+i)^n i}$

$12{,}000 = A + \dfrac{A\left[(1+0.08)^5 - 1\right]}{(1+0.08)^5(0.0.08)}$

$A = 2{,}400$

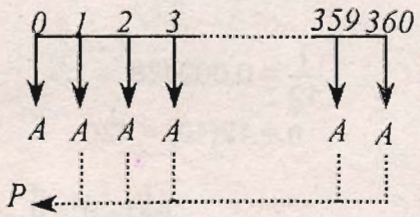


964. Solving for the interest rate per month:

$i + 1 = \left(1 + \dfrac{i}{12}\right)^{12}$

$1.10 = \left(1 + \dfrac{i}{12}\right)^{12}$

$\dfrac{i}{12} = 0.007974$



Total number of payments = 12(30) = 360

Borrowed money = A + P

$$100{,}000 = A + \frac{A\left[(1+0.007974)^{359}-1\right]}{(1+0.007974)^{359}(0.0.007974)}$$

$$A = 839$$

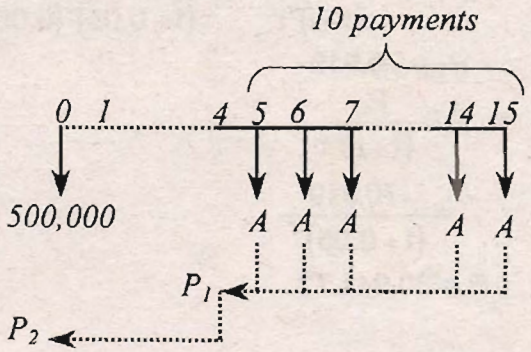
965. $P_1 = \dfrac{A\left[(1+i)^n-1\right]}{(1+i)^n i}$

$$= \frac{100{,}000\left[(1+0.14)^{10}-1\right]}{(1+0.14)^{10}(0.14)}$$

$P_1 = 521{,}611.56$

$$P_2 = \frac{P_1}{(1+i)^n} = \frac{521.611.56}{(1+0.14)^4}$$

$P_2 = 308{,}835.9$



Cash price = 500,000 + P_2

= 500,000 + 308,835.9

Cash price = 808,835.9

Note: From the choices, the nearest answer is 808,811

966. Cash price = Down payment + Present worth of the annuity

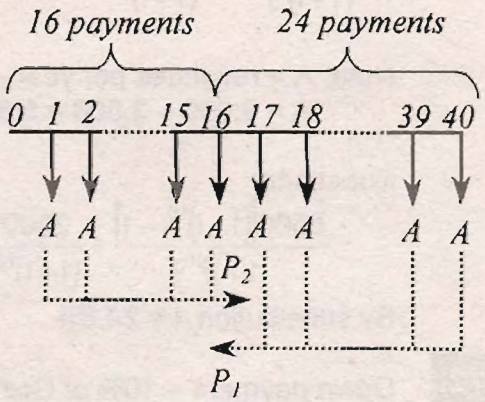
$$10{,}000 = 2000 + \frac{A\left[(1+i)^n-1\right]}{(1+i)^n i}$$

$$10{,}000 = 2000 + \frac{750\left[(1+i)^{15}-1\right]}{(1+i)^{15} i}$$

$$10{,}6667 = \frac{(1+i)^{15}-1}{(1+i)^{15} i}$$

By substitution, i = 4.61%

967. i = 0.14/4 = 0.035



$$P = \frac{A\left[(1+i)^n-1\right]}{(1+i)^n i}$$

$$= \frac{100{,}000\left[(1+0.035)^{24}-1\right]}{(1+0.035)^{24}(0.035)}$$

$P = 1{,}605{,}836.76$

$$F = \frac{A\left[(1+i)^n-1\right]}{i}$$

$$F = \frac{100{,}000\left[(1+0.035)^{16}-1\right]}{0.035} = 2{,}097{,}102.97$$

Lump sum price = P + F

= 1605836.76 + 2097102.97

Lump sum price = 3,702,939.73

968. Note: From the choices, the nearest answer is 12,610

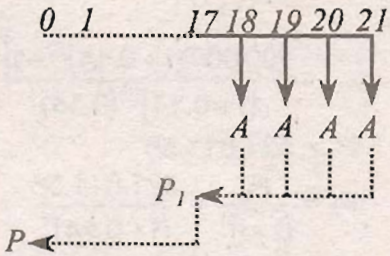
$$P_1 = \frac{A\left[(1+i)^n - 1\right]}{(1+i)^n i} = \frac{2{,}000\left[(1+0.05)^4 - 1\right]}{(1+0.05)^4(0.05)}$$

$P_1 = 70{,}919$

$$P = \frac{P_1}{(1+i)^n}$$

$$= \frac{70{,}919}{(1+0.05)^{17}}$$

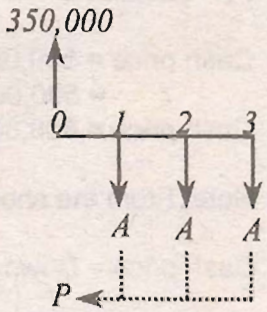
$P = 30{,}941.72$



969.

$$P = \frac{A\left[(1+i)^n - 1\right]}{(1+i)^n i} = \frac{25{,}000\left[(1+0.06)^{15} - 1\right]}{(1+0.06)^{15}(0.06)}$$

$P = 242{,}806$

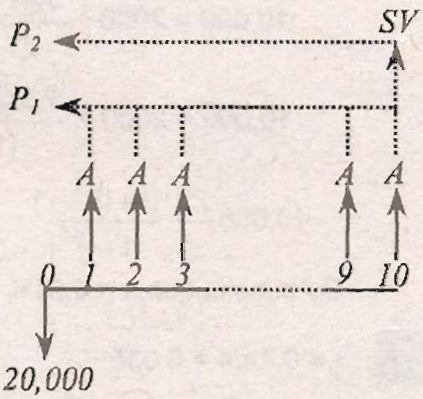


970.

$$P = \frac{A\left[(1+i)^n - 1\right]}{(1+i)^n i}$$

$$350{,}000 = \frac{200{,}000\left[(1+i)^3 - 1\right]}{(1+i)^3(i)}$$

By substitution, i = 32.7%



971.

$P_1 + P_2 = 20{,}000$

$$\frac{A\left[(1+i)^n - 1\right]}{(1+i)^n i} + \frac{SV}{(1+i)^n} = 20{,}000$$

Note: A = revenues per year -- operating cost per year

A = 9,000 – 3,500 = 5,500

Substitute:

$$\frac{5500\left[(1+i)^{10} - 1\right]}{(1+i)^{10} i} + \frac{2000}{(1+i)^{10}} = 20{,}000$$

By substitution, i = 24.8%

972. Down payment = 10% of Cost of house and lot

200,000 = 0.10(Cost)

Cost = 2,000,000

Balance = Cost – Down payment
= 2,000,000 – 200,000
= 1,800,000

i = 0.15/12 = 0.0125

$$P = \frac{A\left[(1+i)^n - 1\right]}{(1+i)^n i}$$

$$1{,}800{,}000 = \frac{42{,}821.87\left[(1+0.0125)^{12x} - 1\right]}{(1+0.0125)^{12x}(0.0125)}$$

$$(1.0125)^{12x} - 1 = 0.52543(1.0125)^{12x}$$

$$(1.0125)^{12x} = 2.1072$$

Take log on both sides:

$$12x \log 1.0125 = \log 2.1072$$

$$x = 5 \text{ years}$$

973. i = 0.12/12 = 0.01
n = 12
F = 80(2000) = 160,000

$$F = \frac{A\left[(1+i)^n - 1\right]}{i}$$

$$160{,}000 = \frac{A\left[(1+0.01)^{12} - 1\right]}{0.01}$$

$$A = 12{,}615.80$$

974.

$$\left(1+\frac{i}{4}\right)^4 = \left(1+\frac{i}{2}\right)^2$$

$$\left(1+\frac{0.08}{4}\right)^4 = \left(1+\frac{i}{2}\right)^2$$

$$\frac{i}{2} = 0.404$$

$$P = \frac{A}{i} = \frac{15{,}000}{0.404}$$

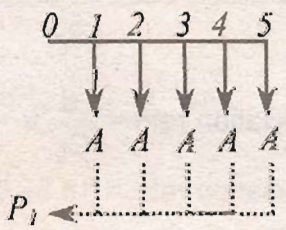
P = 371,287.128

975.

$$P_1 = \frac{A_1\left[(1+i)^n - 1\right]}{(1+i)^n i}$$

$$= \frac{5000\left[(1+0.08)^5 - 1\right]}{(1+0.08)^5(0.08)}$$

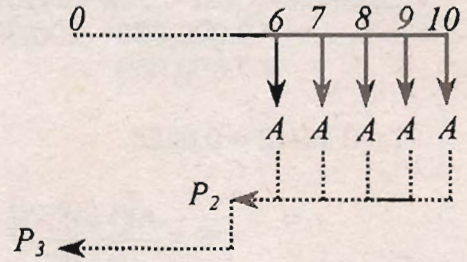
P_1 = 19,963.55



$$P_2 = \frac{A_2\left[(1+i)^n - 1\right]}{(1+i)^n i}$$

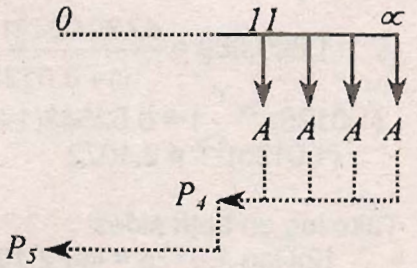
$$= \frac{8000\left[(1+0.08)^5 - 1\right]}{(1+0.08)^5(0.08)}$$

$$P_2 = 31941.68$$



$$P_3 = \frac{P_2}{(1+i)^5} = \frac{31941.68}{(1+0.08)^5}$$

$$P_3 = 21{,}738.97$$



$$P_4 = \frac{A}{i} = \frac{10{,}000}{0.08} = 125{,}000$$

$$P_5 = \frac{P_4}{(1+i)^{10}} = \frac{125{,}000}{(1+0.08)^{10}}$$

$$P_5 = 57899.186$$

$$\text{Total} = P_2 + P_4 + P_5$$

$$= 57899.186 + 21{,}738.97 + 19{,}963.55$$

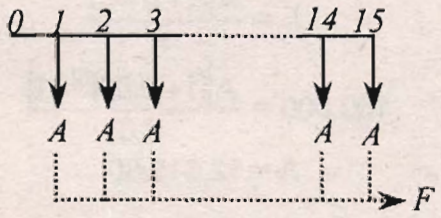
$$\text{Total} = 99{,}601.71$$

976.

$$F = \frac{A\left[(1+i)^n - 1\right]}{i}$$

$$50(1{,}000) = \frac{A\left[(1+0.04)^{15} - 1\right]}{0.04}$$

$$A = 2{,}497$$



Let: F_1 = value of the given annuity when n = 12 years

$$F_1 = \frac{A\left[(1+i)^n - 1\right]}{i}$$

$$= \frac{2{,}497\left[(1+0.04)^{12} - 1\right]}{0.04}$$

$$F_1 = 37{,}519$$

977.

$$d = \frac{C_0 - C_n}{n} = \frac{45{,}000 - 2{,}500}{5} = 8{,}500$$

$$\text{Depreciation rate} = \frac{d}{C_0} = \frac{8{,}500}{45{,}000} \times 100\%$$

$$\text{Depreciation rate} = 18.89\ \%$$

978. $d = \frac{C_0 - C_n}{n} = \frac{50{,}000 - 10{,}000}{10} = 4{,}000$

$C_m = C_0 - d(m) = 50{,}000 - 4000(5)$
$C_m = 30{,}000$

979. $d = \frac{C_0 - (C_n - \text{cost to dismantle})}{n} = \frac{80{,}000 - (50{,}000 - 15{,}000)}{10}$

$d = P\ 76{,}500$

$C_m = C_n - d(m) = 800{,}000 - 76{,}500(6)$
$C_m = 341{,}000$

980. $C_0 = 500{,}000 + 30{,}000 = 530{,}000$
$C_n = 0.10(500{,}000) = 50{,}000$

$d = \frac{C_0 - C_n}{n} = \frac{530{,}000 - 50{,}000}{5} = 96{,}000$

$C_m = C_0 - d(m) = 530{,}000 - 96{,}000(4)$
$C_m = 146{,}000$

981. $d = \frac{C_0 - C_n}{n} = \frac{500{,}000 - 100{,}000}{25} = P16{,}000$

Let: D = total depreciation after "m" years

$D = dm = 16{,}000(3)$
$D = 48{,}000$

982. $d = \frac{C_0 - C_n}{n} = \frac{50{,}000 - 10{,}000}{10} = 4000$

$C_m = C_0 - d(m) = 50{,}000 - 4000(5)$
$C_m = 30{,}000$

983. $d = \frac{(C_0 - C_n)i}{(1+i)^n - 1} = \frac{(10{,}000 - 500)(0.04)}{(1+0.04)^{10} - 1}$

$d = 791.26$

984. $C_m = C_0(1-k)^m$
$40{,}545.73 = 720{,}000(1-k)^{10}$
$(1-k)^{10} = 0.0563$
$k = 0.25$ or 25%

985. $C_m = C_0(1-k)^m$
$4{,}350 = 45{,}000(1-k)^6$
$(1-k)^6 = 0.09666$
$k = 0.3225$ or 32.25%

986. Note: Using SYD, the largest charge of depreciation is the first year.

$$d_1 = (C_0 - C_n)\left(\frac{n}{\sum \text{years}}\right)$$

$$0.2C_0 = C_0\left(\frac{n}{\sum \text{years}}\right)$$

$$\sum \text{years} = 5n$$

Using formula for sum of an A.P.

$$\sum \text{years} = \frac{n(n+1)}{2}$$

$$5n = \frac{n(n+1)}{2}$$

$$10n = 2n + n^2 - n$$

$$9n = n^2$$

$$n = 9 \text{ years}$$

987. $$d_1 = (C_0 - C_n)\left(\frac{n}{\sum \text{years}}\right)$$

Using formula for sum of an A.P.

$$\sum \text{years} = \frac{n(n+1)}{2} = \frac{20(20+1)}{2} = 210$$

Substitute:

$$d_3 = (10{,}000 - 0)\left(\frac{18}{210}\right)$$

$$d_3 = 857$$

988. $$d_1 = (C_0 - C_n)\left(\frac{n}{\sum \text{years}}\right)$$

Using formula for sum of an A.P.

$$\sum \text{years} = \frac{n(n+1)}{2} = \frac{10(10+1)}{2} = 55$$

$C_0 = 6{,}000{,}000 + 0.04(6{,}000{,}000) = 6{,}240{,}000$
$C_n = 0.08(6{,}240{,}000) = 499{,}200$

Substitute:

$$d_5 = (6{,}240{,}000 - 499{,}200)\left(\frac{6}{55}\right)$$

$$d_5 = 626{,}269.10$$

989. $C_{m1} = C_0 - d_1$

$$d_1 = (C_0 - C_n)\left(\frac{n}{\sum \text{years}}\right)$$

$$\sum \text{years} = \frac{n(n+1)}{2} = \frac{10(10+1)}{2} = 55$$

$$d_1 = (9{,}000 - 1{,}000)\left(\frac{10}{55}\right) = 1{,}454.54$$

Substitute:

$C_{m1} = 9{,}000 - 1{,}454.54$

$C_{m1} = 7{,}545.46$

990. Using uniform gradient formula:

$$P = \frac{A\left[(1+i)^n - 1\right]}{(1+i)^n i} + G\left[\frac{(1+i)^n - 1}{i^2(1+i)^n} - \frac{n}{i(1+i)^n}\right]$$

$$= \frac{500\left[(1+0.08)^{10} - 1\right]}{(1+0.08)^{10}(0.08)} + 50\left[\frac{(1+0.08)^{10} - 1}{(0.08)^2(1+0.08)^{10}} - \frac{10}{(0.08)(1+0.08)^{10}}\right]$$

$P = 4{,}653.88$

Note: From the choices, the nearest answer is 4,700

991. $$\text{Capitalized cost} = C_0 + \frac{C_0 - C_n}{(1+i)^n - 1}$$

$$= 250 + \frac{100}{(1+0.06)^{20} - 1}$$

Capitalized cost = 295.3 million

992. Let: AC_1 = annual cost of the old motor truck

AC_2 = annual cost of the new motor truck

$$AC_1 = (C_{01})i + \frac{(C_{01} - C_{n1})i}{(1+i)^n - 1}$$

$$= (5{,}000)(0.04) + \frac{(5{,}000 - 800)(0.04)}{(1+0.04)^2 - 1}$$

$AC_1 = 2{,}258.82$

$$AC_2 = (C_{02})i + \frac{(C_{02} - C_{n2})i}{(1+i)^n - 1}$$

$$AC_2 = (C_{02})(0.04) + \frac{(C_{02} - 1000)(0.04)}{(1+0.04)^3 - 1}$$

$$AC_1 = AC_2$$

$$2{,}258.82 = (C_{02})(0.04) + \frac{(C_{02} - 1000)(0.04)}{(1+0.04)^3 - 1}$$

$$2{,}258.82 = 0.04C_{02} + 0.32C_{02} - 320.35$$

$$C_{02} = 7{,}164.36$$

993. Capital recovery rate per year = Annual cost

$$= (C_0)i + \frac{(C_0 - C_n)i}{(1+i)^n - 1}$$

$$= 20{,}000(0.10) + \frac{(20{,}000 - 8{,}000)(0.10)}{(1+0.10)^3 - 1}$$

Capital recovery rate per year = 5,625

994. Let: x = number of units to be forged

Income = Expenses

$135x = 56x + 69{,}994$

$79x = 69{,}994$

$x = 886$ units

995. Let: x = number of units to be manufactured per month

Income = Expenses

$600x = 115x + 76x + 2.32x + 428{,}000$

$406.68x = 428{,}000$

$x = 1{,}052.42$, approximately 1,053 units

996. Let: x = number of units to be sold out per year

Income = Expenses

$200x = 200{,}000 + 160x$

$40x = 200{,}000$

$x = 5{,}000$ units

997. Let: x = number of units to be produced per month

Income = Expenses

$580x = 85x + 350x + 20x + 15{,}000$

$125x = 15{,}000$

$x = 120$ units

998. Let: x = number of blocks to be sold per month

Income = Expenses

$55x = 20x + 2x + 3{,}500 + 25{,}000 + 12{,}000$

$33x = 40{,}500$

$x = 1{,}227.3$ blocks, approximately 1,228 blocks

999. Let: x = number of units to be forged

Income = Expenses
125x = 56x + 70,000
69x = 70,000
x = 1,014.49 units, approximately 1,015 units

1000. Let: x = number of cases to be sold each year

Income = Expenses
65x = 50x + 35,00
15x = 35,000
x = 2,333.33 cases, approximately 2,334 cases

1001. Let: x = number of motors to be sold each month

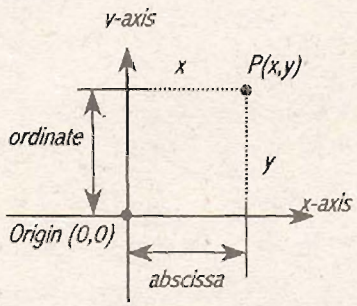
Income = Expenses
275x = 150x + 20,000
125x = 20,000
x = 160 motors

Appendix A: Glossary

A

A the number 10 in the hexadecimal number system

abscissa the position of any point on a plane from the y-axis. Also known as the x-coordinate. The plural form of abscissa is abscissae.



Rectangular coordinate system showing abscissa and ordinate. The abscissa of point P is x.

absolute error the difference between the approximate value and the exact value.

absolute term (syn. constant term) a term in an expression which does not have a variable.

absolute value the number written arithmetically omitting the sign that prefixed it. The absolute value of the number x is denoted as $|x|$.

absolute value of complex number the distance of the complex number from the origin when the complex number is represented as the point with rectangular coordinates (a,b). Absolute value of a + bi is $\sqrt{a^2+b^2}$

absolute value of a vector the numerical value of the length of a directed line segment representing the vector. Absolute value of the vector ai + bj + ck is $\sqrt{a^2+b^2+c^2}$.

abstract algebra the part of algebra that deals with study of groups, semi-groups, rings, modules, fields and similar structures.

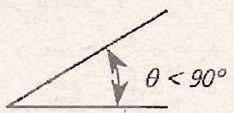
abundant number a natural number where the sum of its distinct factors exceeds the number. For example, the factors of 12 are 1,2,3,4 and 6. The sum of the factors is 16. Since 16 is greater than 12, hence 12 is an abundant number.

acceleration the rate of change of velocity per unit time. Acceleration is a vector quantity. The standard unit is meter per second per second (m/s^2). The gravitational acceleration on earth is denoted as g and is equal to 32.2 feet per $second^2$ or 9.81 meters per $second^2$.

accuracy a measure of precision of a numerical value of some quantity.

acre British unit of area which is equal to 4840 square yards. In metric equivalent, 1 acre = 4046.8 square meters or 0.40468 hectares.

acute angle an angle less than a right angle or 90 degrees.



The angle θ is an acute angle.

acute triangle a triangle having all angles acute angles.

add to combine numbers of quantities by getting the total number of units contained in them.

addend the number added to a certain number to produce a sum. Example: In the equation 5 + 2 = 7, the number 2 is the addend while the number 5 is the augend and 7 is called the sum.

addition the process of combining numbers or quantities. The result of addition is called sum.

addition formulae formulae which express a trigonometric functions of the sum or difference of two angles in terms of the function of the individual angle. The following are the addition formulae:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$
$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

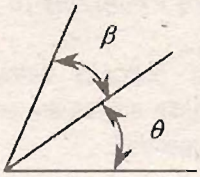
additive identity an identity element of an additive operation. The additive identity is the integer zero.

additive inverse the negative of the number. The additive inverse of 5 is -5.

ad infinitum (latin term) continuing with no end.

adjacent lying next to. An adjacent angle (side) is an angle (side) that lies next to another angle (side).

adjacent angles two angles that have the same point or vertex and a common side between them.



θ and β are adjacent angles

agonic another term for *skew as in agonic line or skew line.(see skew)*

algebra the study of operation and relation among numbers through the use of variables or literal symbols instead of just constants. The Latin term "algebra" comes from the Arabic "al-jabr" which means "to set or consolidate".

algebraic curve a curve that describes an algebraic equation

algebraic equation an equation of the form f(x) = 0 where f is a polynomial of degree n with coefficients in a given base field, usually rationals, n is the degree of the algebraic equation. For example, $x^2 + 3x + 4 = 0$ is a second degree (n = 2) algebraic equation.

algebraic expression any combination of symbols and numbers related to the fundamental operation of algebra.

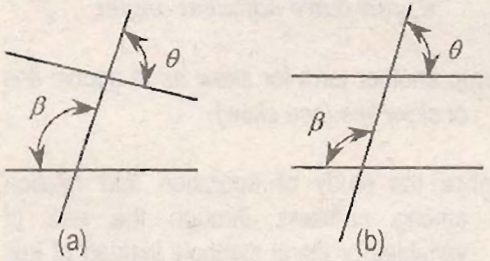
algorithm a step by step procedure by which an operation can be carried out.

Al-jabr wa'l muqabalah Rough Translations, Balancing Equations, an Arab texbook written in the early 800s by Al-Khowzrizmi (from whose name the word algorithm was coined). This is where the name algebra came from and from this text Europe came out of the dark ages and learned algebra.

alphanumeric (syn. alphameric character) combination of alphabet, numerical symbols, punctuation marks and other symbols used in computer works.

alternate angles either pair of angles contained between two given lines and a transversal

and lying on opposite sides of the transversal. These angles are equal if the given lines are parallel.



θ and β are alternate angles.
Figure 5b shows equal alternate angles since the lines are parallel.

alternating series an infinite series whose successive terms are alternately positive and negative, such as
1 - 1/2 + 1/3 - 1/4 + 1/5 - ...

altitude a line segment drawn from a vertex perpendicular to the opposite side (called base).

altitude of a trapezoid a perpendicular distance between the bases or parallel sides of a trapezoid.

altitude of a triangle the perpendicular segment from a vertex of the triangle to the line containing the opposite side. It also refers to the perpendicular distance from the apex of a triangle to the base.

ambiguous case the case of a solution of a plane triangle where the given data lead to two solutions.

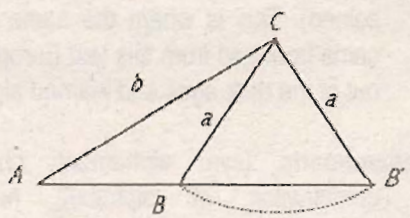


Figure shows that if sides a, b and angle A are given, each of the two triangles ABC and AB'C satisfies the given conditions.

amicable numbers two integers such that each is equal to the sum of the distinct proper factors of the other. An example of amicable numbers are the numbers 220 and 284. The number 220 has a proper factors of 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110 which have the sum of 284, while the number 284 have proper factors of 1, 2, 4, 71, and 142 which sums up to 220.

ampere an SI unit of electric current

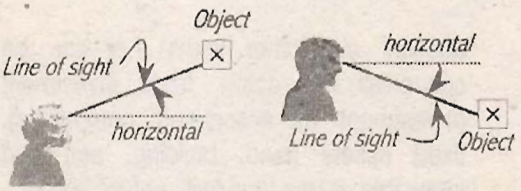
amplitude the maximum displacement between in either positive or negative direction from a reference level.

amortization as applied to the capitalized cost, the distribution of the initial cost by periodic charges to operation as in depreciation or the reduction of a debt by either periodic or irregular prearranged program.

Analytic Geometry the branch of Mathematics which deals with the properties, behaviors and solutions of points, lines, curves, angles, surfaces and solids by means of algebraic method in relation to the coordinate system. This was invented by Rene Descartes.

angle the basic figure formed by two line segments or rays with a common end point. The symbol for angle is "∠".
Angles are measured in different units, namely degrees, radians, grads and mils.
1 revolution = 360 degrees
= 2π radian
= 400 grads
= 6400 mils

angle of depression or elevation the angle between the horizontal and the line of sight to the observed point. If the observed point is below the horizontal from the observer, it is an angle of depression. It is an angle of elevation if the observed point is above the observer.



Angle of elevation Angle of depression

angle of inclination the smallest positive angle that the straight line makes with the positive x-axis.

angular relative to or in terms of angles such as angular acceleration, angular velocity etc.

annulus (*syn. circular ring*) a plane figure which contains an area of a ring-shaped region lying between two concentric circles. Another term for annulus is ring. Area of annulus is $\pi(R^2 - r^2)$.



Annulus or Circular ring

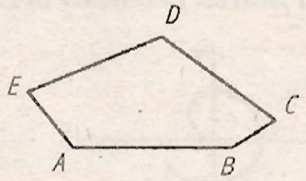
annuity a series of equal payments occurring at equal interval of time. Type of annuity are, ordinary annuity, annuity due, deferred annuity and perpetuity.

annuity due a type of annuity where the payment is made at the beginning of each period starting from the first period.

antecedent the first term of the ratio. In the ratio 2:3, 2 is the antecedent and 3 is the consequent. (See consequent). The antecedent is equal to the product of the ratio and the consequence.

antilogarithm the inverse function of a logarithm.

apex the highest point of a figure with respect to the base or plane of the base.



D is the apex of the polygon

apothem the radius of the circle inscribed in a regular polygon. A line from the center of a polygon perpendicular to one of its sides.

appreciation *(ant. depreciation)* increase in the value of an asset.

approximation a number which is a close estimate of another number. The symbol of approximation is ≈.

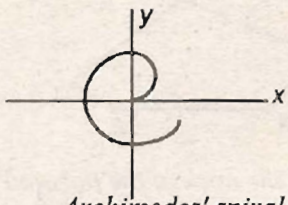
arabic numerals the symbols 0,1,2,3,4,5,6,... that represent the counting system in the decimal number system (base 10). (see numerals)

arbitrary constant a non-numerical symbol holding a place for an unspecified constant. For example, in the general linear equation $y = mx + b$, m and b are arbitrary constants while x and y are variables.

arc a part of a circle between two points on the circle. A portion of a curve between two of its points A and B.

Archimedes Principle Any body immersed in a fluid is subjected to a buoyant force which is equal to the weight of the fluid displaced. This was discovered by Greek Mathematician, Archimedes(287 – 212 B.C).

Archimedes' spiral *(syn. Archimedean spiral)* spiral with the polar equation $r = a\theta$, with its graph as shown in the figure. The radius vector, increases with polar angle, θ, a is the constant of proportionality.



Archimedes' spiral

are *a larger unit of area in the metric system. It is equivalent to 100 square meters.*

area a numerical measure of a two dimensional geometric figure enclosed within a specific boundary

Argand diagram *(syn. Gaussian Plane)* the rectangular coordinate system used for the representation of a complex number. The x-axis and the y-axis are known as the axis of reals and axis of imaginaries, respectively. Named after the Parisian bookkeeper, Jean Robert Argand (1768 – 1822).

arithmetic the branch of mathematics which deals with calculation of integers using the fundamental operations of addition, subtraction, multiplication, division and the extraction of roots.

arithmetic mean *(syn. mean, average)* the sum of "n" numbers divided by n. For example, the mean of 2,9 and 7 is 6.

arithmetic progression is a sequence in which the difference between any two successive terms is a constant and is called the common difference.

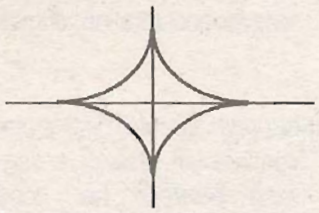
arithmetic series the sum of the terms of an arithmetic progression.

arm one of the lines forming the angle.

array an arrangement of numbers in rows and columns (see matrix)

assets refers to everything a company or corporation owns and has a money value. Assets are as current assets (cash, bank account and other items that can be converted into cash), trade investment (investment in associated companies), fixed assets (land, building, etc) and intangible assets (goodwill, patent, etc.)

astroid *(syn. star curve)* a hypocycloid with four cusps and with parametric equation of $x = \cos^3 t$, $y = \sin^3 t$. The rolling circle of this hypocycloid has a diameter one-fourth that of the fixed circle.



Astroid

astronomical unit (AU) the mean distance between the earth and the sun. It is about 1.495×10^8 km.

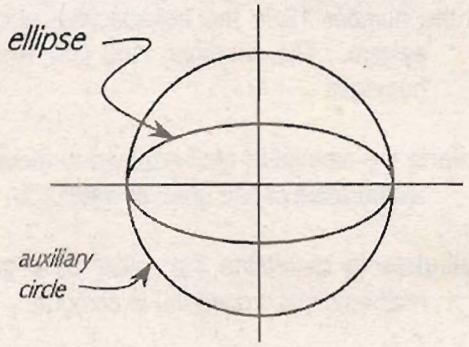
asymmetric not symmetric

asymptote a straight line that approaches the curve more and more closely but never really touches it except as a limiting position at infinity. The word asymptote was coined by Thomas Hobbes (1588 - 1679), using various latin stems meaning roughly "to fall together but not touch".

atmospheric pressure the pressure caused by the weight of air at a given point. Standard value is 14.7 pounds per square inch or 760 mm of mercury. It is also equal to the weight of a column of water about 30 feet high and in metric system, it is equal to 100 kPa.

augend the number or quantity to which the addend is added. Example: 5 + 2 = 7. The number 5 is the augend while 2 is the addend.

auxiliary circle a circle with radius equal to half the major (transverse) axis and its center is at the center of the ellipse (hyperbola).



Auxiliary circle

average the usual term used for arithmetic mean.

average acceleration the change in velocity of a body divided by the time interval during which the change occurs.

average velocity the displacement of a body divided by the time interval during which the displacement occurs.

axiom a statement of truth which is admitted without proof.

axiom of induction "Any property that belongs to zero, and also to the immediate successor of any natural number to which it belongs, belongs to all natural numbers."

axis the fixed reference line used in a coordinate system.

axis of symmetry a line around which a geometric figure is symmetrical.

axis of the conic the line through the focus and perpendicular to the directrix.

B

B the number 11 in the hexadecimal number system.

bar graph a chart or diagram consisting of horizontal or vertical rectangles or bars, each of which represents an interval of values of a variable and has height proportional to the quantities.

barycenter (see center of gravity)

base a side of a polygon which is at the bottom of the orientation.

billion refers to one million million (10^{12}) in the United Kingdom and Germany and one thousand million (10^9) in the United States and France. In US and France, 1 million = 1,000,000.

binary relating to the binary notation or binary code.

binary logarithm logarithm of a number to the base 2

binary number system *(syn. Dyadic number system)* a system of notation for real numbers that uses the place value method with 2 as the base. Only two digits are considered, 0 and 1 sometimes called as "bit" (abbreviation of binary digits).

binomial a mathematical expression of two terms.
Example: $5x + 4y$

binomial distribution *(syn. binomial probability)* the distribution of probabilities where there are two possible outcomes of an event (i.e. success and failure) and the possibilities if the outcomes are independent and constant.

binomial expansion expansion of a binomial in the form of $(x + y)^n$ in accordance with the binomial theorem.

binomial theorem the theorem that gives the form of the expansion of any positive integral power of a binomial $(x + y)^n$. Its general equation is

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \ldots$$
$$\ldots + n\,x\,y^{n-1} + y^n$$

bisect to divide a geometric figure into two equal parts.

bisector *(syn.bisectrix)* a line or plane that bisects a given angle or line or any geometric figure.

bit (abbreviation of binary digit) the digits 0 and 1 in the binary number system.

book value the recorded current value of an asset. The value of an asset that is recorded in the book of records of the corporation.

Boolean Algebra an algebra which deals with the operation of complementation, union and intersection. It is devised by the British mathematician George Boole (1815 - 1864) who is best known for his innovatory work in formal logic.

Boyle's Law At constant temperature, the pressure is inversely proportional to the volume.

Briggsian logarithm (see common logarithm)

British thermal unit (BTU) the amount of heat required to raise the temperature of one pound of water 1° F. 1 BTU = 252 calories

bulk modulus the ratio of the volume stress to the volume stress.

bundle a family of lines or planes which all passes through a single point.

byte a sequence of bits; a unit of information equivalent to a single character; a unit capacity of a computer.

C

C the number 12 in the hexadecimal number system. The number 100 in Roman numerals

calorie the amount of heat required to raise the temperature of one gram of water 1°C.

calculate to determine the value of a given mathematical procedure; to compute.

Calculus the branch of mathematics created in the seventeenth century by Isaac Newton(1642-1727) and German mathematician, Gottfreid Wilhelm von Leibniz (1646-1716) which rest on the basic principles of limits.

Calculus is divided into two subjects, namely, Differential Calculus and Integral Calculus.

cardinal numbers numbers used to count objects. Example: one, two, three,... twenty...

cardioid a heart-shaped curve generated by a fixed point on a circle as it rolls round another circle of equal radius. Its equation is $r = a(1 - \cos\phi)$.

Cartesian coordinates (syn. rectangular coordinates) a method of locating a point by pair of numbers denoting the distances from two fixed reference intersecting lines. The first number is called abscissa which is the distance from the y-axis while the second is called ordinate, which is the distance from the x-axis. The two intersecting lines are called coordinates axes.

cash flow the flowback of profit plus depreciation from a given project.

catenary a plane curve described by a heavy uniform, flexible cable hanging freely between two points. The term "catenary" comes from the latin word which means chain. Its equation is $y = a \cos h\ (x/a)$.

catenoid the surface generated when a catenary is rotated about its directrix.

Cavalieri's Principle Given two solids and a plane. Suppose that every plane parallel to the given plane, intersecting one of the two solids, also intersects the other and gives a cross-sections with the same area, then the two solids have the same volume. Named after Bonaventura Cavalieri (1598 – 1647).

center of gravity *(syn. centroid, center of mass, barycenter)* the point through which the resultant of the distributed gravity force passes regardless of the orientation of the body in space.

center of mass (see center of gravity)

centesimal degree (see gon or grad)

centillion a number 10 raised to a power of 600 or 10^{600}.

central angle an angle whose vertex is at the center of a given circle.

central conic a conic with a center. Examples are ellipse and hyperbola. The only non-central conic is the parabola.

central tendency a central value between the upper and lower limits of a distribution around which the scores are distributed.

centrifugal force a force acting outward on a body as it traverses a curvilinear path

centroid (see center of gravity)

chain rule a rule of differentiating a function of a function , $f\ [u(x)]: df/dx = (df/du) \cdot (du/dx)$

characteristic the integer part of the logarithm.

chord a segment whose end points lie on the circle.

chord of contrast the chord joining the points of tangency of the two tangent lines from a point P outside the circle.

cipher an old name for zero. It is derived from Arabic "sifr", Latin "cephirum" and Italian "zevero".

circle a close plane figure every point of which is equidistant from a fixed point called the center. Its equation is $(x - h)^2 + (y - k)^2 = r^2$ where center is at (h,k) and radius is r. The parametric equation is
$x = r \cos \theta$ or $y = r \sin \theta$.

circular cone a cone whose base is a circle.

circular cylinder a cylinder with a circular right section.

circular ring (syn. annulus) see annulus

circumcenter the point of concurrency of the perpendicular bisectors of the sides of a triangle.

circumcircle a circle that circumscribed a given polygon.

circumference the boundary of geometric figure, especially a circle

circumscribe to draw a geometric figure around another geometric figure in such a way that they are in contact but does not intersect.

coefficient In algebra, it refers to the numerical factor of a term. For example: In the term 5x, 5 is the coefficient of the term.

coefficient of kinetic friction the ratio of the frictional force to the perpendicular force. It is denoted as mu (μ).

coefficient of restitution the ratio of the total momentum after collision to that of before collision. It is denoted as "e".

coefficient of static friction the ratio of the limiting frictional force (maximum) to the normal force. The coefficient of static friction force. The coefficient of the static friction is always greater than the coefficient of kinetic friction.

coefficient of volume expansion (see bulk modulus)

cofunction (syn. complementary function) In trigonometry, the function of a pair have equal values for complementary angles. Example: Sin 30° = Cos (90° - 30°) or Cos (60°). Hence, sine and cosine are complementary functions.

coincident having all points in common.

collinear points points that lie on the same line.

cologarithm the logarithm of the reciprocal of a number.

columns the numbers in order which appears vertically in a matrix.

combination an arrangement of a set of objects in no specific order.

The combination of "n" different things taken "r" at a time is given as

$$C_{(n,r)} = \frac{n!}{(n-r)!\,r!}$$

If taken all, $C_{(n,n)} = 1$

common denominator an integer or polynomial that is exactly divisible by each denominator.

common difference the difference of two successive terms of an arithmetic progression or sequence.

common factor (syn. common divisor) Of two or more integers or polynomials, an integer of polynomial which is a factor of each. For example: 5 is a common factor of 20 and 25.

common logarithm (syn. Briggs logarithm) logarithm to the base 10. This was created by the geometry professor of Gresham College in London, Henry Briggs (1561-1630) as an improvement of the natural logarithm.

common multiple an integer or polynomial that is multiple of each in a given set. For example: 90 is a multiple of 15 and 18.

common tangent a line that is tangent to two or more curves.

commutative law law stating that the sum or product is unaffected by the order of the terms. In addition, a + b = b + a. In multiplication, a times b = b times a.

complementary angles two angles whose sum is 90° or right angle

completing the square the process of modifying a quadratic polynomial to obtain a perfect square (trinomial).

complex fraction any fraction which contains one or more fractions in either numerator or denominator.

complex number a number of the form a + bi with a and b real constants and i = the square root of -1.

composite numbers an integer which is the product of two integers, both different from 1 and -1. The integer 15 is a composite number since 15 = 5 times 3.

compound interest the interest charges under the condition that interest is charged on any previous interest earned in any period of time, as well as the principal. Fomula for

future amount of a principle in compound interest is $F = P(1 + i)^n$, with n as the number of periods and i is the interest per period.

compound number a quantity expressed in different but related units. Example. 5 hours and 32 minutes.

concentric circles circles having the same center with unequal radii.

conclusion a part of the theorem which is to be proved.

concrete number a number that counts a physical quantity. For example: 5 apples, 8 cats

concurrent having a common point.

conditional equation an equation which is satisfied by some, but not all, of the values of the variables for which the members of the equation are defined.

cone a solid bounded by a conical surface (lateral surface) whose directrix is a closed curve and a plane (base) which cuts all the elements.

confocal conics two conics which share the same focus. A confocal ellipse and hyperbola intersect at right angle.

congruent geometric figures having identical shape and size.

conjecture a mathematical statement which has neither been proved nor denied by counterexamples. Some of the famous conjectures are Fermat's Last Theorem and Goldbach Conjecture.

conjugate angles (see explementary angles)

conjugate arcs two circular arcs together make a full circle

conic (see conic section)

conic section the locus of a point which moves such that its distance from a fixed point (focus) is in constant ratio, e (eccentricity) to its distance to a fixed straight line (directrix)

conical surface a surface generated by a moving straight line (generator) which always intersects a fixed plane curve (directrix) and which always passes through a fixed point (vertex) not in the plane of the curve

conoid a surface or solid formed by rotating a conic section about one of its axes.

consequent the second term of a ratio. See antecedent

constant of integration an arbitrary constant term in the expression of indefinite integral of a function.

constant term a term in a polynomial that does not contain a variable

convergent A convergent sequence has a finite limit. A convergent series has a finite sum. Opposite of convergent is divergent.

converse of a theorem another theorem wherein the hypothesis and conclusion of the first are reversed; that is, the hypothesis becomes the conclusion and the conclusion becomes the hypothesis

convex polygon a polygon with no side extended will pass through the center of the polygon. Also it contains no interior angle greater than 180°. Regular polygons are said to be convex.

coordinates the abscissa and ordinate together. Also known as rectangular coordinates or Cartesian coordinates.

coplanar points set of points that lie on the same plane

corollary a statement of truth which follows with little or no proof from the theorem

corporation (type of business organization) a distinct legal entity separate from the individuals who own it and which can engage in any business transaction which a real person can do.

cosine law a law which relates the sides and angle of an oblique triangle. It is used in solving the parts of an oblique triangle.

coterminal angles angles of rotation which the same initial side and terminal side. The coterminal angle of 75° is 435 and -285°. The difference of coterminal angles is always 360°.

counting numbers (see natural numbers)

couple a pair of forces equal in magnitude, opposite in direction and not in the same line

Cramer's rule a method of solving linear equations of several unknowns simultaneously using determinants or matrices. It is named after the Swiss mathematician and physicist Gabriel Cramer (1704 - 1752).

critical point *(syn. Stationary point)* a point at which a function has a first derivative of infinity, thus having a slope which is vertical.

cross product (see vector product)

cube a polyhedron whose six faces are all squares. A cube is a regular hexahedron.

curve (see locus)

cyclic quadrilateral a quadrilateral with all vertices lie on a circle

cycloid the plane curve traces out by a fixed point P on the circle as the circle rolls along a line, the base of the cycloid

cylinder a solid bounded by a closed cylindrical surface and two parallel planes (see circular cylinder)

cylindrical surface a surface generated by a moving straight line (generator) which is always parallel to a fixed line, and which always intersects a fixed plane curve (directrix) not in the plane with the fixed line

cylindroid a cylinder with elliptical cross-section.

D

D the number 13 in the hexadecimal number system; the number 500 in the roman numerals.

D' Alemberts principle When a body is subjected to an acceleration, it is reacted upon be a reverse effective force opposite in the direction of the acceleration to balance. The value of the reverse effective force is equal to the mass time acceleration. Named after the French mathematician and physicist, Jean Le Rond d' Alember (1717 – 1783).

decagon a polygon of ten sides

decahedron a polygon with ten faces. There is no such thing as a regular decahedron.

decimal fraction number that consists of an integer part (which may be zero) and a decimal part (less the one) that follows the decimal marker, which may be a point or a comma.

decimal number system a system of notation for real number that uses the place value method with 10 as the base.

decimal point a dot place between the integral and fractional parts of a number.

declining balance method a method of computing depreciation in which the annual charge is a fixed percentage of the depreciated book value at the beginning of the year to which the depreciation applies. This method is also known as percent on diminishing value.

decrement the negative of an increment.

defective equation any equation which, because of some mathematical process, has fewer roots than its original

defective number (see deficient number)

deferred annuity a type of annuity where the payment of the first amount is deferred a certain number of periods after the first.

deflection angle (syn. exterior angle)

deficient number (syn. defective number) any natural number the sum of its proper divisors. All prime numbers are deficient numbers.

definite integral an expression of integrating an integrand between two limits of integration. Integrals with limits are definite integrals.

degenerate conic conic obtained when the cutting plane passes through the vertex of the cone. Example of degenerate conics are the point-ellipse, two coincident lines and two intersecting lines

degree 1. A measure of an angle which is equal to 1/360 of a revolution. 1degree = 60 minutes and 1 minute = 60 seconds **2.** The highest exponent or sum of exponents in any given term of a polynomial. The polynomial $7x^5y^2 + 8x^4y^5-2x^2y$ is 9th degrees polynomial.

deltoid a non-convex quadrilateral with two pairs of adjacent equal sides.

denominator the number written below in a given fraction. The number written above is called the numerator. For example, 3/4 has a denominator of 4.

density (syn. specific weight) the mass per unit volume of a substance.

depletion the lessening of the value of an asset due to the decrease in the quantity available. This refers to the natural resources such as coal, oil and timber in the forest

depreciated book value the first cost of the capitalized asset minus the accumulation of annual depreciation cost charges

depreciation the decline in the value of an asset due to the passage of time and constant use.

determinant a number which is expressed as a square matrix (with the number of rows equals the number of columns)

deviation in statistics, it refers to the difference between any one of the sequence of observed value of a variable to some value such as the mean.

diagonal a segment joining two non consecutive vertices. It may be calculated using the formula, n/2 (n-3), where n is the number of sides of the polygon

digit any specific symbol use to denote a number whether singly or in combination.

diameter a chord containing the center of the circle

difference the result of subtraction.

differential calculus a branch of calculus that deals with the evaluation and use of derivatives and differentials.

differential equation an equation to be solved for an unknown function which involves the first or higher derivatives of the function ; an equation that contains one or more terms involving derivatives of one variable with respect to another variable.

Ordinary differential equation - one that involves function of a single variable and some of its derivatives.

Partial differential equation - one that involves functions to two or more variables and some of their partial derivatives

Order of a differential equation - refers to the order of the highest derivatives that is present in the equation.

Degree of a differential equation - the highest power of the highest-order derivatives.

differentiation the operation of finding the derivative

dihedral angle the angle between two planes intercepted by a plane perpendicular to the common edge

dimensions of the matrix the number of rows and columns of a matrix

Diophantine equations If there exist more unknowns than the number of equation but still can be solve because the values of the unknown are integers, these equations are called Diophantine equations (named after Diophantus of Alexandria, a Greek mathematician in the 3rd century BC). Diophantus in his book "Arithmetica" carried out his extensive study to the solution to indeterminate equations.

directrix a fixed line opposite the focus of a conic section which the eccentricity of the conic section is defined.

discriminant the quantity that discriminates among the possibilities of a quadratic equation. It is expressed as $b^2 - 4ac$. The discriminant determines the nature of the roots.

If $b^2 - 4ac = 0$, the roots are real and equal.
If $b^2 - 4ac > 0$, the roots are real and unequal
If $b^2 - 4ac < 0$, the roots are imaginary and unequal

displacement a vector quantity which represent the charge in position of a point. It is equal to the product of the velocity and time.

distance the length of the shortest line segment between two points.

distance formula the formula used to measure the distance between two points. This formula was derived by the use of the Pythagorean theorem.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

divergent not having a finite limit

dividend In the expression a / b, a is the dividend and b is the divisor

division the process of obtaining the quotient; the inverse of multiplication

divisor *(see dividend)* the number that divides the dividend

dodecagon a polygon with 12 sides

dodecahedron a polyhedron of 12 faces. Each face of a regular dodecahedron is a regular pentagon.

domain the set of all first elements of a relation

dot product (see scalar product)

dozen a term use to denote the number 12. This is derived from French "douze" which means twelve. 1 dozen = 12 items.

duodecimal number system a number system using 12 as a place value. This number system still survive today as in 1 year = 12 months, a clock dial has 12 hours, 1 foot = 12 inches, 1 dozen = 12 items and 1 gross = 12 dozens.

dyadic number system (see binary number system)

dynamic (syn. kinetic) branch of mechanics that deals with the forces that produced a motion.

dyne a unit of force in the cgs metric system and is equivalent to 1 gram-cm/s^2. 1 dynes = 10^{-5} N.

E

E the number 14 in the hexadecimal notation.

e a transcendental number which is approximately equal to 2.71828...It is commonly called as the Euler Number because it was introduced by Leonhard Euler(1707 – 1783) in the 18th century. It has the equation

eccentric geometric figures not having the same center.

eccentricity the ratio of the distance from point to the focus (focal distance) to the distance from the point to the directrix. Eccentricity of parabola, ellipse and hyperbola are 1.0, <1.0 and >1.0, respectively.

ecenter (see. excenter)

ecircle (see escribed circle)

economic return the profit derived from a project or business enterprise without consideration of obligations to financial contributors and claims of others based on profit

effective interest the true value of interest rate computed by the equations for composed interest for a 1 year period.

Effective interest can be computed using the formula,

$$ER = (1 + i)^m - 1$$

where : m = number of interest period per year

i = interest per period

Egyptian triangle a right triangle with sides 3,4 and 5 units

element **1.**member of the set **2.** part of a geometric figure such as point, line or plane).

eliminant (see resultant)

ellipse a locus of a point which moves such that the sum of distance from two fixed points (foci) is constant and is equal to the length of the major axis (2a).

Eccentricity of ellipse is always less than 1. Standard equation of the ellipse is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

ellipsoid (syn. spheroid) solid of revolution of an ellipse when rotated about one of its axes. When rotated about its longer axis, it is called as prolate ellipsoid. When rotated about its shorter axis, it is called oblate ellipsoid.

empirical based only on observations and experimental evidences

empty set a set the contains no element at all

Engineering Economy the application of engineering or mathematical analysis and synthesis to economic decisions

ennea - (syn. nona) a prefix which means nine. A polygon of nine sides is called a nonagon or enneagon.

envelope an envelope of a family of plane curves is a curve that is tangent to every member of the family

epicycloid the plane curve traced by a fixed point on a circle as it rolls along the outside of a fixed circle.

equal having the same value. The symbol of equal (=) was introduced in 1557 by Robert Recorde in his algebra textbook "The Whetstone of Witte".

equally likely a term used to describe equiprobable events.

equation a mathematical statement showing that two expressions have the same value.

equiangular having all angles equal. An equilateral triangle is also equiangular.

equidistant being of equal distance to any given points or lines.

equilateral hyperbola (see hyperbola)

equilateral triangle a triangle having all sides equal in length. An equilateral triangle is also equiangular. Each interior angle of an equilateral triangle is equal to 60°.

equilibrium the condition when a body is acted on by no forces of several forces such that their vector sum (resultant) is zero. Forces are said to be in equilibrium if they are concurrent (having a common point for the forces' line of action).

equiprobable having the same probability.

equivalent having some properties (geometric) in common.

eradius *(see exradius)*

error the difference between an approximate value and the true value which it approximates.

escribed circle (syn. excircle, ecircle) a circle tangent to one side and to the extension of the two other sides of a triangle.

Euler number the number e which is equal to 2.71828...To obtain this number, get the anti-natural logarithm of 1. Or ln e = 1. The formula for e is,

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

The symbol "e" was introduced by the Swiss Mathematician, Leonard Euler (1707-1783).

even number numbers which are exactly divisible by two. All even numbers has for its last digit an even number.

event in probability, it refers to the possible outcome of a trial.

evolution the operation of root extraction. The opposite of involution.

exact accurate, precise

excenter (syn. ecircle) the center of the escribed circle.

excircle (see escribed circle)

explementary angles (syn. conjugate angles) two angle whose sum is 360°

exponent (syn. index) number (usually written superscript to another number) that is used to simplify repeated product. Example $x^3 = x \cdot x \cdot x$

exradius *(syn. eradius)* the radius of the escribed circle.

exterior angle (syn deflection angle) is the angle formed by the prolongation of one side and the adjacent side of the polygon. The sum of all exterior angles in any polygon is always equal to 360°.

extract to find the value of a root.

extrapolate to estimate the value of a quantity or measurement beyond the values which are already known. Opposite of extrapolation is interpolation.

extremes the first and the fourth terms in the proportion of the four quantities



F the number 15 in hexadecimal notation.

face a plane surface of a geometric figure.

factor (syn. divisor) each of two or more numbers which is multiplied together to form a product.

factorial the product of the n consecutive positive integers from 1 to n. Example: n! = 1x 2 x 3 x ... x (n - 1) x n. By definition, 0! = 1. The symbol factorial (!) was introduce by Christian Kramp (1760 – 1826) in 1808.

Recursion formula states that:

$$(n!)(n + 1) = (n + 1)!$$

Substituting 0 for n, results to 0! = 1

factoring the process of converting expressions as sums into equivalent expressions as products.

fallacy a contradictory or false proposition; an invalid argument of form of argument.

family (syn. family of curves) a collection of related geometric configurations; a set of related curves or surfaces whose equations vary only in the values of the constants.

Fermat's Last Theorem (syn. Fermats' Great Theorem) a famous conjecture of mathematics which states that $x^n + y^n = z^n$, where n is 3 or greater x, y and z are all positive integers. This is regarded as the the least proved theorem in Mathematics. Fiere de Fermat(1601-1665) claimed that he founded a marvellous root to this theorem, but the margin is too narrow to contain it. This is also known as the hardest problem in Mathematics or the "Mount Everest of Mathematics".

Fibonacci numbers (syn. Fibonacci sequence) the unending sequence of integers formed according to the rule that each integers is the sum of the preceding two. The Fibonacci sequence is 1,1,2,3,5,8,13,34,55,89.....
Name after italian merchant and mathematician, Leonardo Fibonacci (1170 – c. 1250).

figure 1. any arrangement of points, lines, curves forming a geometric shape **2.** The symbol for an integer, such as "8" or "69".

figurate numbers numbers that are represented by arrangements of dots as geometric figures. In plane figures, the following are examples of figurate numbers: Triangular numbers, Square numbers, Oblong numbers, Pentagonal numbers, Gnomon numbers. In solid figues are the Cubic numbers, Tetrahedral numbers and Square Pyramidal numbers.

Triangular numbers are numbers which can be drawn as dots and arranged in

triangular figure. Example: 1, 3, 6, 10, 15, 20, ...

Square numbers are numbers which can be drawn as dots and arranged in square figure.
Example: 1, 4, 9, 16, 25,...

Oblong numbers are numbers which can be drawn as dots and arranged in rectangular figure.
Example: 2, 6, 12, 20,...

Pentagonal numbers are numbers which can be drawn as dots and arranged in pentagonal figure.
Example: 1, 5, 12, 22, 35,...

Gnomon numbers are numbers which can be drawn as dots on equally long legs of a right angle. Example: 1, 3, 5, 7, 9,...

Cubic numbers are numbers which can be drawn as dots and arranged as a cube.
Example: 1, 8, 27, 64, ...

Tetrahedral numbers are numbers which can be drawn as dots and arranged as a tetrahedron.
Example: 1, 4, 10, 20, ...

Square pyramidal numbers are numbers which can be drawn as dots and arranged as a pyramid with square base. Example: 1, 5, 14, 30,...

finite can completely be counted off from 1 to the last whole number.

first cost the initial cost of a capitalized property, including transportation, installation, preparation for service, taxes, and other related initial expenditure in order to make the property functional

first derivative the derivative of a function. Normally the first derivative of the function is the slope of the function.

First Proposition of Pappus the area of a surface generated by rotating any plane curve about a non intersecting axis in its plane is equal to the product of the length L of the curve and the distance traveled by its centroid. Or expressed as

Area = length of arc x circumference described by the centroid of arc

This was formulated by Pappus (c.300-c.350) of Alexandria.

flat angle *(syn. straight angle)* an angle whose measure is 180°.

flow chart a sequence of logical computations often represented with rectangles, parallelograms and arrows.

focal measurement from the focus to a certain point.

focal chord a line segment joining two of its points and passing through a focus of a conic

focal radius is a line segment from a focus to one point of the conic.

focus a fixed point on the concave side of a conic section.

Folium of Descartes a plane curve which forms a loop on one side and intersect itself at a node. Its standard equation is
$x^3 + y^3 = 3axy$ where $x + y + a = 0$ is the equation of the line.

formula a symbolic statement of mathematical expression which is syntactically correct.

fourth proportion In the proportion a:b = c:d, d is the fourth proportion.

fraction a ratio of two integers such as a/b , with a not a multiple of b and b is not zero or one. The value "a" is called the numerator and the value "b" is called the

denominator. When the numerator is less than the denominator, it is a common or vulgar or proper fraction otherwise it will be called improper fraction. An improper fraction is always written as a mixed number, that is an integer and a proper fraction such as 3½.

frame a structure with at least one of its individual member is a multiforce member.

free body diagram the diagram of an isolated body with the representation of all external forces acting on it

frequency the number of times an event occurs within a given period.

frequency polygon a graph on which the frequencies of classes are plotted at the class mark and the class marks are connected by straight lines

friction the limited amount of resistance to sliding between the surfaces of two bodies in contact

frustum a part of cone or pyramid lying between the base and a plane parallel to the base.

frustum of a regular pyramid the portion of a regular pyramid included between the base and a section parallel to the base

frustum of a right circular cone the portion of a right circular cone include between the base and a section parallel to the base.

function a relation in which every ordered pair (x, y) has one and only one value of y corresponding to the value of x

fundamental operations of arithmetic referring to the four operations - addition, subtraction, multiplication and division. In algebra, fundamental operation follows the sequence of "My Dear Aunt Sally" which means that Multiplication and Division must be calculated first before Addition and Subtraction.

future worth the equivalent value at a designated future based on time value of money.

G

g notation for gravitational constant. g = 32.2 feet per second per second or 9.81 meters per second per second.

G notation for gravitational constant. G = 6.67 x 10^{-11} $N \cdot m^2/kg^2$.

generator (syn. generatrix) a point, line or plane whose motion forms a geometric figure.

geometric mean (syn. geometric average) The geometric mean of n numbers is the nth root of the product of the numbers. For example, the geometric mean of a and b is $\sqrt{a \cdot b}$. The geometric mean is always less than the arithmetic mean except if all numbers are equal.

geometric progression a sequence of numbers such that the ratio of any term to the preceding term is constant.

geometric series a series whose terms form a geometric progression.

Geometry the branch of mathematics which deals with the properties and relations of constructible plane and solid figures.

gnomon a geometric figure which is formed by cutting a parallelogram from one corner of another but larger parallelogram.

gnomon numbers (see figurate numbers)

Goldbach conjecture the conjecture that every even number (except 2) equals the sum of

two prime numbers. This was named after the Prussian-born number theorist and analyst, Christian Goldbach (1690-1764).

gon (syn. centesimal degree, grad) 1/400 of the full angle. 1 revolution = 400 gons and 1 right angle = 100 gons. 1 gon is divided into 100 centesimal minutes or centigon and 1 centesimal minute is divided into 100 centesimal seconds or milligon.

googol the number represented by 1 followed by a hundred zeros or 10^{100}. The name was coined by a 9 year old nephew of the American mathematician Edward Kasner in 1930s.

googolplex the number represented by 1 followed by a google of zeros. 1 googolplex = $10^{\text{google}} = 10^{10^{100}}$

grad (syn. gon or centesimal degree) unit of angle measurement with one revolution equivalent to 400 grads.

gradient slope of line or the ratio of vertical change to horizontal change.

gravitational acceleration the constant acceleration of a free falling body. Denoted as "g" and is equal to 9.81 meter per second per second or 32.2 feet per second per second on earth's surface. On moon, g = 1.62 m/s^2 and on sun, g = 274 m/s^2.

gravity the force of attracting between two bodies due to their mass.

great circle the intersection of the sphere and the plane through the center of the sphere.

gross twelve dozen. This is equivalent to 144 items.

$\mathcal{H}$

half-angle formulas formulas that express a trigonometric function of half an angle.

harmonic mean a term in between two harmonic terms of a harmonic progression. It is the reciprocal of arithmetic mean.

harmonic progression a sequence of numbers whose reciprocals form an arithmetic progression.

height the measure of an altitude of any polygon.

helix a curve in space which lies on a cylinder and crosses its elements at a constant angle.

hemisphere a portion of sphere cut off by a plane through its center. A hemisphere is one half of a sphere.

heptagon a polygon with seven sides

Heron's formula the formula for the area of a triangle with all sides given. This was named after the first century Greek mathematician Heron (c.A.D.75). The formula is

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where a,b and c are the length of the sides and $s = \dfrac{(a+b+c)}{2}$

hexadecimal number system a number system which uses a place value of 16. This number system uses 16 symbols (10 basic numbers and the letters A, B , C, D, E, and F).

hexagon a polygon of six sides.

hexahedron a polyhedron with six faces. A cube is a regular hexahedron.

histogram a vertical bar graph that shows the frequencies of scores or classes of scores by the height of the bars

homogeneous For polynomials, it is the one having all terms the same degree. For example, $x^3 + 3x^2y + y^3$.

Hooke's Law Within the elastic limit, the stress is proportional to strain. The constant of proportionality is called the modulus of elasticity, E or sometimes known as the Young's modulus. Named after the English mathematician and scientist, Robert Hooke (1635-1703).

Hydraulics the branch of Physics that deals with the properties and characteristics of fluids.

hydrodynamics is the branch of hydraulics that deals with fluids in motion.

hydrostatics is the branch of hydraulics that deals with the fluids at rest.

hyperbola the locus of a point which moves such that the difference of the distances from two fixed points (foci) is constant and is equal to the length of the transverse axis (2a). Eccentricity of hyperbola is always greater than 1.

hyperbolic functions functions of angle expressed in exponential functions such as

$$\sinh x = \frac{\left(e^x - e^{-x}\right)}{2}$$

$$\cosh x = \frac{\left(e^x + e^{-x}\right)}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

hyperboloid a surface of revolution produced by rotating a hyperbola in space about one of its axes, called its axis of revolution

hypocycloid the plane curve traced by a fixed point on a circle as it roll along the inside of a fixed circle

hypotenuse the longest side of a right triangle or the side opposite to the right angle.

hypothesis the part of the theorem which is assumed to be true.

I

i notation for imaginary number, $\sqrt{-1}$

I roman numeral for 1.

icosahedron a polyhedron with twenty faces. A regular icosahedron has faces equilateral triangles.

identity referring to the property of being the same.

imaginary number number in the form of $\sqrt{-1}$ and denoted as i

improper fraction a ratio of positive integer in which the value of the numerator is greater than that of the denominator.

impulse the product of the force and the time during which it acts

incenter the point of concurrency of the angle bisectors of a triangle

incircle the center of the circle inscribed in a triangle.

inclination (see angle of inclination)

increment small change in the value of the variable. The increment of x from x = a to x = b is the difference, b - a.

indeterminate an expression with no direct meaning as a number. Example are 0/0, ∞/∞, $0/\infty$, 0^0...

index *(syn. exponent)* refers to the number n of a radical $\sqrt[n]{}$

inelastic not elastic; having elasticity less than one.

inequality a statement that one mathematical expression is greater than or less than another

inertia property of a body resisting any change in its state of rest or of uniform motion in a straight line.

inferential statistics the methods used to describe a population (universe) by studying a random sample of that population

infinite not finite, having value which are extremely large amount.

infinity the concept of being beyond the finite value. The symbol of infinity (∞) was introduced by John Wallis (1616-1703) in his *Arithmetia in fini torum* in 1655.

inscribe to draw a geometric figure inside another geometric figure in such a way that the two figures having common but not intersecting points.

inscribed angle an angle whose vertex is a point on the circle and whose sides are chords

instantaneous power the limit of the rate of work done as time approaches zero.

instantaneous velocity the limit of average velocity of the body as the time interval approaches zero

integers are counting numbers (natural numbers) and the negative of the counting numbers and the number 0.

Integrand the function to be integrated

Integral the result of an integration. The integral sign $\int$ by Leibneiz. The word *integral* comes from a Latin origin which means "making up a whole".

Indefinite integral (syn. Primitive integral or antidiverivative) an integral with no restrictions imposed on its independent variable.

Definite integral an integral defined by the limit values of the independent variable.

Double integral an integral in which the integrand is integrated twice.

Triple integral an integral in which the integrand is integrated thrice.

Integral Calculus the branch of calculus which deals with evaluation of integrals and their applications.

integral part the biggest integer not greater than the given number. In the number 5.12, the integral part is 5. In the number -5.12, the integral part is -6.

integration the operation of transforming a function to its definite or indefinite integrals.

intercept refers to the intersection of two geometric figures. X and y intercepts are the intersection of the curve with the x and y axes respectively.

interest the periodic compensation for the use of money. Sometimes referred to as the *time value of money.*

interest rate the ratio of the interest payment to the principal for a given unit of time and is usually expressed as a percentage of the principal.

International System (SI) the metric system of unit defined by the General Conference of Weights and Measurements in 1960. SI stands for the French equivalent, *Systeme International*

interpolation the procedure for estimating intermediate value that are not listed in a table of numerical values. The simplest form of interpolation is ***linear interpolation***, which has for its variation, which has for its variation of the functional described by a straight line. If the function does not satisfy the condition of linearity of variation, ***graphical interpolation*** is used.

intersection a point where the curve crosses the coordinate axis. This also refers to the set consisting of the elements that are common to the original set.

intersection of two sets the set of all objects common to both sets

involute of a circle a curve which is the path of the end of a taut string as it is unwound from a circle.

involution the operation of raising to an integral power, x^n. This is opposite to evolution.

irrational equation an equation in which a variable appears under the radical sign

irrational number any number which cannot be expressed as a quotient of two integers. Examples are π, e, $\sqrt{2}$, etc.

isodiametric having all diameters of equal length.

isogon an equilateral polygon.

isoperimetric figures are figures that have the same perimeters

isosceles having two sides of equal length. An isosceles triangle has two sides and two angles equal.

isosceles trapezoid a trapezoid having non parallel sides equal in length

isosceles triangle a triangle having two side equal in length

J

j notation for imaginary number , $\sqrt{-1}$ for engineering and physics applications.

joule (pronounced "jewel") a unit of work in the SI units, named in honor of English physicist, James Prescott Joule (1835 – 1889). 1 joule = (1 Newton)(1 meter).

K

Kepler's Laws the laws which described the motion of stars, planets and comets, formulated by Johannes Kepler (1571 – 1630).

Kepler's three laws of planetary motion:

1. All the planets of the solar system describe elliptical orbits, having the sun as one of the foci.
2. A radius vector joining any planet to the sun sweeps out equal areas in equal periods of time.
3. The squares of the periods of revolution of the planets about the sun are directly proportional to the

cubes of their mean distances from the sun (the major semi-axes of the elliptical orbits).

kilogram (kg) mass of a particular cylinder of platinum-iridium alloy which is now kept at the International Bureau of Weights and Measures at Sevres, near Paris. 1kg = 1000 grams.

kilowatt a unit of power equivalent to 1000 Watts; equivalent to $\frac{1\ kN-m}{s}$

kilowatt-hour (kWh) the usual commercial unit of electrical energy. Kilowatt-hour is a unit of work or energy, not power. 1 kWh = 3.6 MJ

kinematics the study of motion without reference to the forces which causes the motion

kinematic viscosity the ratio of viscosity to the density of the body.

kinetic another name for dynamic.

kinetic energy In Physics, the quantity equivalent to 1/2 mv^2, where m is mass and v is the velocity.

L

L the number 50 in the roman numerals.

lamina a thin sheet of uniform thickness and density

lateral area area of the surface exclusive of bases.

lateral edge the intersection of the lateral faces.

lateral face any side of the polyhedron other than the base.

lateral surface the union of the lateral faces of a prism

latus rectum a line through the focus, parallel to the directrix and intersecting the curve

Law of Universal Gravitation "Every particle in the universe attracts every other particle with a force that is directly proportional to the masses of the two particles and inversely proportional to the square of the distance between the centers of masses".

leading coefficient the coefficient of the term of highest degree in a polynomial of one variable. In the polynomial, $4x^3 + 3x^2 - 8x - 10$, 4 is the leading coefficient.

leg Of a right triangle, any of the two sides other than the hypotenuse.

lemma a proved proposition which is used mainly as a preliminary to the proof of a theorem

lemniscate (*syn. Lemniscate of Bernoulli*) a curve whose equation is $r^2 = a^2 \cos 2\theta$, where (r,θ) are polar coordinates. This was conceived by Jakob Bernoulli (1654 – 1705) in 1694.

length measure of a line segment.

L'Hospital Rule (pronounced as Lopital's Rule) a method in calculus in evaluating indeterminate quantities such as 0/0 and ∞/∞. Name in honor of Johann Bernoulli's pupil, Guillaume Francois de L'Hospital, (1661 – 1704).

life the period of after which a machine or facility should be discarded or replaced because of its excessive costs or reduced profitability. Also refers to the period of time after which a machine or facility can no longer be repaired in order to perform its design function properly.

line segment a portion of a line bounded by two points.

linear equation an equation in which the variable or unknown appears only the first power and only in the numerator of any fractions

literal equation an equation in which some or all of the known quantities are represented by letters.

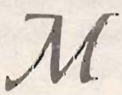
lituus a plane curve with equation $r^2 = \frac{a}{\theta}$ with (r,θ) as polar coordinates; It resembles a trumpet shaped spiral and originated with the English mathematician, Roger Cortes (1682 – 1716). The word lituus, is of Latin origin which means "a crooked staff". Plural form of lituus is litui.

locus the path of a point which moves according to a given law or equation. Plural form is loci.

logarithm an exponent when a number, N is represented as a power of a fixed number called based.

long radius the distance between a center and a vertex of a regular polygon.

lune the portion of a sphere lying between two semi-circles of great circles.



M the number 1000 in the roman numerals.

magnitude the absolute value of a vector quantity.

mantissa the decimal part of a logarithm .

mass a measure of inertia of a body, which is its resistance to a change in velocity.

mathematical expectation another term for expected value.

mathematics the group of subjects (Algebra, Trigonometry, Calculus, Geometry, etc.) used in investigation of numbers, space and the many generalization of these concepts.

Matheson Formula the formula used for *Declining Balance Method* depreciation, $k = \sqrt[n]{\frac{Co}{Cn}}$, where Co and Cn are the first and last cost, respectively. With this method of computing depreciation, the last cost, C_n should not be equal to zero.

matrix a rectangular arrays of numbers forming m rows and n columns.
Types of matrices:

1. *Square matrix* – a matrix where the number f columns equals the number of rows.
2. *Row matrix* – a matrix which only one row.
3. *Column matrix* – a matrix which has only one column.
4. *Lower triangular matrix* - a matrix where all entries above the main diagonal are zero
5. *Upper triangular matrix* – a matrix where all entries below the main diagonal are zero.
6. *Scalar matrix* – a diagonal matrix where $a_{11} = a_{22} = a_{33} = \ldots = k$, where k is a constant.
7. *Unit matrix or identify matrix* – a scalar matrix where k =1
8. *Null matrix* - a matrix in which all entries are zero
9. *Complex matrix* - a matrix with at least one of the entries a complex number.

mean (syn. average) **1.** The arithmetic average of all the scores in the distribution **2.** the average of two quantities.

mean proportional the second and the third terms of a proportion with the second term equals the third term.

means the second and the third terms in the proportional of four quantities

mechanics a branch of physical science which deals with state of rest or motion of bodies under the action of forces

median 1. a line drawn from the vertex of a triangle to the midpoint of the base **2.** (in statistic) a point in the distribution of scores at which 50 percent of the scores falls below and the 50 percent of the scores fall above

median of a trapezoid a line joining the midpoints of the non parallel sides

mediator the perpendicular bisector of a line segment.

Mersenne number a positive integer of the form $2^n - 1$, where n is a prime number. This was named after a French mathematician and Franciscan Priest, Marin Mersenne (1558-1648).

meter defined in 1960 as 1,650,763,73 wavelength of the orange-red light emitted by atoms of krypton 86(^{86}Kr) in a glow discharge tube. Redefined in 1983 as the distance that light travels (in vacuum) in 1/299,792,458 second. The latest definition is more accurate than the first.

mil a unit of angle measure with one revolution equivalent to 6400 mils

mile a unit of length.
Statue mile = 5280 feet
Nautical mile = 1 minute angle on a great circle
= 6280 feet.
The nautical mile is 800 feet longer than the statue mile.

milliard In United Kingdom, a thousand million. Billion (instead of milliard) is used in the United States of America.

minuend In an expression x - y, x is the minuend

Mirifici Logarithmorum Canonis Description a book published by John Napier in 1614 which means " A Description of the Marvelous Rule of Logarithm".

mixed number a number formed of an integer and a proper fraction. Example $5\frac{1}{2}$.

Mobius strip a twisted surface in space formed by turning one side of a rectangle through 180° (relative to the opposite side) and join it to the opposite side. This was named after a German mathematician Agustus Mobius (1790-1868).

mode the most frequently appearing score or group of scores appearing in the distribution

modulus of common logarithm in converting natural logarithms into common logarithms, the following formula is applied; log N = 0.43429 in N. The number 0.43429 is the modulus of common logarithm

modulus of elasticity the ratio of the tensile stress to tensile strain. This is sometimes called Youg's Modulus, in honor of Thomas Young.

moment of force another name for torque.

moment of inertia the quantity equivalent to the area times the square of the distance from the centroid to the axis considered. It has a unit to the 4th power.

momentum the product of the mass and velocity of the body

monomial an algebraic expression of only one term

motion any change in the position of a body.

multinomial the algebraic expression consisting of a sum of any number of terms.

multiple any number of polynomial that is a product of a given number or polynomial and a multiplier. For example; 15 is a multiple of 3.

multiplicand the number being multiplied by another. In an expression 5 x 2 = 10, 5 is the multiplicand and 2 is the multiplier. Both 5 and 2 are factors of 10.

multiplicative inverse the reciprocal of the number. The multiplicative inverse of 5 is 1/5.

multiplier (see multiplicand)

mutually exclusive a condition where two events cannot happen at the same time, or when one occur, the other one will not occur and vice versa.

N

nadir the point on the celestial sphere directly below the observer.

Napierian Logarithm logarithm to the base e = 2.718281828... This is also known as Natural logarithm. Denoted as ln or $\log_e$.

Napier's rules rules used in solving spherical right triangles.

Rule 1 (Tan-ad rule) *The sine of any middle part is equal to the product of tangent of the adjacent parts.*

Rule 2 (Co-op rule) *The sine of any middle part is equal to the product of the cosine of the opposite parts.*

natural logarithm (see Napierian Logarithm)

natural number are numbers considered as counting numbers. Example: 1, 2, 3,.... Zero and negative numbers are not considered as natural numbers.

negative less than zero.

negative angle angle measured in counterclockwise rotation.

newton the amount of force that gives an acceleration of one meter per second squared to a body with mass of one kilogram. Denoted as N. 1 N = 1 kg-m/s^2

Newton's First Law Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed on it. This is also known as the law of inertia.

Newton's Second Law The rate of change of linear momentum of a particle is equal to the total applied force.

Newton's Third Law For every action, there is always an equal and opposite reaction.

nominal interest the number employed loosely to described the annual interest rate

nonagon a polygon of 9 sides

normal another name of perpendicular or orthogonal.

null equal to zero; empty

null hypothesis a hypothesis that is being tested for rejection

numeral symbol or combination of symbols representing a number such as Arabic and

Roman numerals. Arabic numerals (e.g. 0, 1, 2, 3, 4,..) are the modification of the Hindu-Arabic Numbers. The Roman numerals are certain letters of the Latin alphabet.

Roman numeral	Arabic equivalent
I	1
V	5
X	10
L	50
C	100
D	500
M	1000

numerator In the fraction $\frac{x}{y}$, x is the numerator, y is the denominator.

numerical integration *(syn. approximate integration)* the process of finding an approximate value of a definite integral without carrying out the process of evaluating the indefinite integral.

O

oblate spheroid *(syn. oblate ellipsoid)* produced by rotating an ellipse through a complete revolution about its minor axis.

oblique angle consist of all angles except right and straight angles

oblique cartesian coordinate system a cartesian coordinate system in which the x and y axes are not perpendicular

oblique circular cone a circular cone whose axis is not perpendicular to the base of the cone.

oblique triangle a triangle with no right angle.

oblong numbers (see figurate numbers)

obtuse angle an angle that is greater than 90 degrees but less than 180 degrees.

obtuse triangle a triangle having one obtuse angle.

octagon a polygon of eight sides

octahedron a polyhedron with eight faces

octal number system a number notation which uses base 8 as a place value. It uses the digits 0, 1, 2, 3, 4, 5, 6, and 7.

octants referring to the eight compartments of the rectangular coordinate systems in space

ogive a graph of cumulative frequency distribution plotted at the class marks and connected by straight lines

ordinal numbers numbers which state their position in a sequence. Example: First, second, third,...

ordinary annuity an annuity where the payments are made the end of each period starting from the first period.

ordinate the position of any point on a plane from the x-axis. Also known as the y-coordinate.

origin the intersection of the x and y axes of the cartesian or rectangular coordinate systems. The origin has coordinates of (0,0).

orthocenter the point of intersection of all the altitudes of a triangle.

orthogonal normal or perpendicular. Its symbol is $\perp$.

P

Pappus theorems theorems that determine that surface area and volume of a figure

generated about an axis. This is named after Pappus of Alexandria.

First proposition of Pappus:
"If an arc is rotated about an axis, it generates as surface are which is equal to the length of arc times the circumference described by the centroid of the arc."

Second proposition of Pappus:
"If an area is rotated about an axis, it generates a solid of revolution, whose volume is equal to the area times the circumference described by the centroid of the area".

parabola a locus of a point which moves so that it is always equidistant from a fixed point (focus) and to a fixed line (directrix). Eccentricity of parabola = 1.

parabolic segment the plane region bounded by a chord a parabola perpendicular to the axis and the arc of the parabola cut off by the chord. The area of the parabolic segment is 2/3 of the circumscribing rectangle.

$$A = \frac{2}{3}bh$$

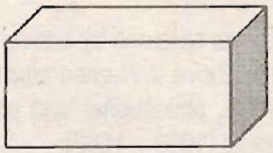
paraboloid a solid of a revolution of a parabola. Volume of the paraboloid is always equal to one-half of the volume of the circumscribing cylinder.



Paraboloid

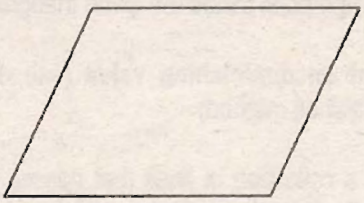
parallel line or plane that will never intersect no matter how far they are extended.

parallelepiped a prism whose base is a parallelogram region.



Rectangular parallelepiped

parallelogram a quadrilateral in which both pairs of opposite sides are parallel.



Parallelogram

parameter generally an arbitrary constant.

partial fraction the parts of an algebraic expression which contain a polynomial in a single variable in the denominator, or in the denominator and numerator, when split

partnership (type of business organization) an association of two or more persons for the purpose of engaging in a business for profit.

pascal (*Pa*) a unit of pressure. 1 pascal = 1 newton per square meter.

Pascal's Law "If any external pressure is applied to a confined fluid, the pressure will be increased at every point in the fluid by the amount of the external pressure."

Pascal's Triangle a triangular array of numbers which is made up of the binomial coefficient of the binomial expansions.

```
          1
        1   1
      1   2   1
    1   3   3   1
  1   4   6   4   1
```

The number can be found by adding the two numbers above it. For example, 6 in

the triangle was obtained by adding 3 an 3, the numbers above it. Named after French mathematician, philosopher and physicist, Blaised Pascal (1623 – 1662).

pedal triangle a triangle inscribed in a given triangle whose vertices are the feet of the three perpendiculars to the sides from some point inside the given triangle.

percent on diminishing value (see declining balance method)

pencil a collection of lines that passes through a fixed point or a given point.

pentagon a polygon of five sides

pentagonal numbers (see figurate numbers)

pentagram *(syn. pentangle, pentacle)* a star-shaped figure formed by extending the sides of a regular pentagon and meet at vertices.

pentedecagon a polygon of fifteen sides

per cent a word of latin origin which means every hundred. Its symbol is %.

percentage a ratio by which the denominator is 100.

perfect number a number the sum of whose factors including one but excluding itself is exactly equal to the number. Example: The factors of 6 are 3, 2 and 1. Adding the factors will yield the number itself such as

$1 + 2 + 3 = 6$. Hence, 6 is a perfect number.

perigon an angle equal to one revolution (360°)

perimeter the sum of the sides of a polygon. This is known as circumference for a circle.

permutation an arrangement of a set of objects in a particular order. The permutation of "n" different things taken "r" at a time is given as $P_{(n,r)} = \dfrac{n!}{(n-r)!}$

perpendicular (syn. normal, orthogonal) forming a right angle.

perpetuity an annuity where the payment periods extends forever or in which the periodic payments continue indefinitely.

Philosophiate Naturalis Principia Mathematica ("Mathematical Principle of Natural Science")a book publish by Sir Isaac Newton in 1686. This book clearly states the fundamental laws of nature which is now referred to as the Newton's Law, the cornerstone of mechanics.

pi an irrational number which the ratio of the circumference to the diameter of a circle. Its value is 3.14159... and has for its symbol π. This symbol for pi (π) was introduced in 1706 by William Jones (1675 – 1749).

pie chart a circular diagram divided into sectors of which the area are in proportion to the magnitude of the represented values.

plane a surface such that a straight line joining any two points in it lies wholly in the surface

planimetry the measurement of plane areas.

point of inflection the point on the graph where the curve changes from concave up to concave down and vice versa

Poisson ratio The ratio of the unit deformations or strains in a transverse directions is constant for stresses within the proportional limit.

point of tangency is the point of contact of the tangent and the curve

polar angle (syn. vectorial angle, the argument, the amplitude of the azimuth of the point) the angle the vector makes with the polar axis.

polar coordinates coordinates in the form of (r,θ) used to locate a point in the rectangular coordinates system.
To convert polar to rectangular, use the following relations: $x = r\cos\theta$, $y = r\sin\theta$

polygon a closed figure bounded by line segments.

polygonal region is the plane figure formed by fitting together a finite number of triangular regions

polyhedron a solid bounded by planes

polynomial an expression of several terms. It may include any number of terms.

population (syn. universe) in statistics, it refers to all the members of a particular group of items or individuals.

positive having values greater than zero.

postulate in Geometry, the construction or drawing of lines and figures the possibility of which is admitted without proof.

pound a unit of force in the English (British) system. If is equivalent to 1 slug-ft/s^2. Also 1 pound =4.448221615260 N.

power 1. the rate at which work is done or energy is transferred **2.** *(syn. exponent)* the number of times the number is multiplied by itself.

power series an infinite series in which successive terms are of the form of constants times successive integral power of the variable. It takes the form of $a_0+a_1x+a_2x^2+a_3x^3$ x .....

power set the set of all subsets of a given set, containing the original set as well as the empty set.

precision the accuracy in which a calculation is performed.

present worth the equivalent value at the present , base on time value of money

pressure the force per unit area. It has a unit of pascals in the metric system. Since pascal is a small unit, the unit bar or MPa are used instead. 1 pascal = 1 newton per square meter. 1 bar = 10^5 Pascals.

The gauge pressure can be calculated using the formula: $p = \omega h$, where ω = density (specific weight) of the fluid and h is the pressure head.

The absolute pressure is the sum of the gauge pressure and the atmospheric pressure.

Standard atmospheric pressure = 14.7 psi = 1.01 x 10^5 Pa = 760 mm of Hg

primitive integral (see integral)

prime number an integer which has no other factors except 1 and itself

principal In economics, it is the amount invested.

prism a polyhedron of which two faces are equal polygons in parallel planes, and the order faces are parallelograms

prismatoid a polyhedron having bases two polygons in parallel planes and for lateral faces triangles or trapezoids with one side lying one base and the opposite vertex or side lying on the other base of the polyhedron.

prismoid a prismatoid in which the two bases are polygons of equal number of sides and the lateral faces are quadrilaterals.

prismoidal formula formula used in finding the volume of a prismatoid such as follows,

$$V = \frac{L}{6}(A_1 + 4A_m + A_2).$$

where, L is the distance between end areas, A_1 and A_2 are end areas and Am is the area at the mid-section.

probability the ratio of the successful outcome of an event to the total possible outcome of an event. The value of the probability is always less than 1.

product the result of multiplication.

progression a sequence of numbers in which one is designated as first, another as second , another as third and so on. Types of progression are Arithmetic progression, Geometric progression, Harmonic progression, Infinite Geometric progression, etc.

projectile initial velocity of a body and then follows a path determined entirely by the effects of gravitational acceleration and air resistance.

prolate spheroid *(syn. prolate ellipsoid)* an ellipsoid produced by rotating an ellipse through a complete revolution about its major axis

proper fraction a ratio of positive integers in which the value of the numerator is less than that of the denominator

Ptolemy's Theorem In cyclic quadrilateral, the sum of the product of two opposite sides is equal to the product of the diagonals. Named after Ptolemy of Alexandria or Claudius Ptolemaeus (c.100 – c. 168).

pure quadratic a quadratic equation of the form $ax^2 + c = 0$, that is , the coefficient of the first degree term, b is equal to zero

pyramid a polyhedron of which one face, called the base, is a polygon of any number of sides and the other faces are triangles which have a common vertex

pyramidal numbers (see figurate numbers)

Pythagorean theorem The sum of the squares of the sides of a right triangle is equal to the square of the hypotenuse; in equation, $a^2 + b^2 = c^2$ with a and b are legs while c is the hypotenuse. This is named after the Greek philosopher and mathematician, Pythagoras (c. 580 – c. 500 B.C.) of Samos.

Q

QED latin word *quod erat demonstrandum*, which means that a proposition has been proven. The Greek equivalent was used by Euclid in the 3rd century B.C.

quadrantal spherical triangle a spherical triangle with at least one side a quarter of a great circle.

quadrants referring to the four divisions of the rectangular coordinates system.

quadratic equation an equation in which the maximum power of the unknown or variable is 2. Standard quadratic equation is in the form $ax^2 + bx + c = 0$.

quadrature formulas – refers to the formulas used in numerical intregration.

quadrilateral a polygon of four sides

quantity something with a magnitude or numerical value.

quarternary having four variables.

quinary number system which pertains to place value notation of base 5.

quotient the result of division.

$\mathcal{R}$

radian the angle between two radii with an intercepted arc equal to the radius of the circle. 1 revolution is equal to 2π radians.

radical the symbol $\sqrt{}$. This symbol was introduced by Christoff Rudolff in 1525.

radicand the quantity inside the radical (square root sign)

radius a segment from the center to a point of the circle

radius of gyration the distance from a given axis that a particle of the same mass as a rigid body must be placed in order to have the same moment of inertia.

radius vector the distance of any point P from the origin in the polar coordinate system

radix the base of a number system. For example, 2 is the radix of a binary number system and 10 is the radix of the decimal number system

random variable a numeric quantity which can be measured in a random experiment.

range the set of all second elements of a relation

rate of return the interest rate at which the present work of the cash flow on a project is zero, or the interest earned by an investment

ratio the quotient of two numerical measure of two magnitudes of the same kind. The word ratio comes from the Latin verb "*ratus*" which means "to estimate".

ratio of similitude the common ratio of the corresponding sides of two similar polygons

rational equation an equation which is satisfied by all value of the variables for which the members of the equation are defined

rational expression any algebraic expression that is a quotient of two other algebraic expressions

rational number any number which can be expressed as a quotient of two integers (division by zero excluded).

rationalizing the denominator the process of removing the radicals (or fractional exponents) from the denominator

real number a non imaginary number. It includes the rational numbers as well as the imaginary numbers all integers and natural numbers.

reciprocal multiplicative inverse of a number. For example, the reciprocal of 5 is 1/5.

rectangle a parallelogram all of wthose angles are right angle

rectangular hyperbola a hyperbola with length of semi-transverse axis, "a" equals the length of the semi-conjugate axis, "b". Eccentricity of this hyperbola is square root of 2.

rectangular parallelepiped a polyhedron whose six faces are all rectangles

rectilinear pertaining to straight line.

reduction the process of converting a fraction into a decimal form.

redundant equation any equation which, because of some mathematical process, has acquired an extra root

reflex angle any angle greater than 180 degrees but less than 360 degrees

regular polygon a polygon with all sides equal and all angles equal. A regular polygon is equiangular and equilateral. Also, a regular polygon is convex.

regular pyramid a pyramid whose base is a regular polygon and whose center coincides with the root of the perpendicular dropped from the vertex to the base

relation is any set of ordered pairs (x,y)

relative density (see specific gravity)

relative error a measure of the difference between a number and an estimate.

remainder the amount left when a quantity cannot be divided exactly by a divisor.

resultant the single vetor quantity which is the sum of two or more vector quantities.

rhombus (syn. diamond, lozenge) a parallelogram all of whose sides are congruent

right angle angle equivalent to 90 degrees

right triangle a triangle having one right angle

rigid body a body which will not be affected or deformed when an extremely large or extremely small load or temperature is applied.

Roman numerals (see numerals)

root value that satisfy a given equation.

rounding (of numbers) replacing it with another number to produce fewer significant decimal digit. For an integer, fewer value carrying (non-zero) digit. Example: 3.14159 is rounded off to four decimal places as 3.1416. For numbers ...5, the rule states that it should be rounded off to the nearest even rounding boundary to minimize the systematic rounding error.

rows the numbers in order which appears horizontally in a matrix

S

sample any subset of a population

sample space the set of all possible outcomes of an experiment.

salvage value the cost recovered or which could be recovered form a used property when removed, sold or scrapped. It is sometimes referred to as second hand value

scalar quantity a physical quantity that is described by a single number only, the *magnitude*. It does not have a direction in space.

scalar product of A and B is denoted as A.B. Because of this notation, scalar product is also called as dot product

scalene triangle a triangle having all sides of unequal lengths

scattergram the relation between two variables is shown by a series of dots plotted on a graph

scientific notation a number represent using powers-of-10 notations used to described a very large small numbers.

score another term used for the number twenty (20)

scrap value the value of an equipment if disposed as junk. This is sometimes referred to as junk value.

secant a line which intersects the circle in two points. The reciprocal of the trigonometric function, tangent.

second originally defined in 1889 as the fraction 1/86400 of the mean solar day, and redefined in 1967 as the duration of 9,192,631,770 periods of the radiation of a certain state of the cesium-133 atom

Second proposition of Pappus The volume of any solid of revolution is equal to the generating area times the circumference of the circle described by the centroid of the area.

section of a solid the plane figure cut from the solid by passing a plane through it

sector a part of a circle bounded by the radii and an arc.

segment part of a circle bounded by an arc and a chord.

septagon a polygon of seven sides

sequence (syn. progression) a succession of numbers in which one number id designed as first, another as second, another as third and so on

series sum of a finite of infinite sequence.

set *(syn. class)* a collection of objects.

sexagesimal pertaining to the number 60.

sexagesimal number system a number system using a place value of 60. This was used by the Babylonians or Mesopotamians and is considered be the oldest number system which dates back to 2 millenium B.C.

shear stress a stress that is caused by forces acting along or parallel to the area resisting the force

short radius the shortest distance from the center of a regular polygon to any of its sides.

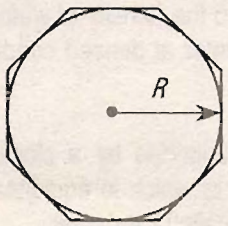


Figure shows R, the short radius

significant figures /digits the meaningful digits in a number. A number is considered significant unless it is used to the place a decimal point.

simple interest the interest charges under the condition that interest in any time is only charged on the principal

sine curve (syn. sine wave) a curve with equation y = sin x.

sinking fund method a method of depreciation where a fixed sum of money is regularly deposited at compound interest in a real or imaginary fund in order to accumulate an amount equal to the total depreciation of an asset at the end of the asset's estimated life.

skew lines two lines that are not coplanar

slant height (syn. element) the length of a generator of a circular cone.

slope ratio of the vertical distance to horizontal distance. It also refers to the tangent function of the angle of inclination. Also refers to rise over run. In Analytic

Geometry, slope of line is denoted as m. m $= \frac{y_2 - y_1}{x_2 - x_1}$

solid is any limited portion of space, bounded by surfaces

space a set of all points

specific gravity the ratio of the density of the substance to the density of water. Specific gravity of water at densed condition (4°C) = 1.0

sphere a solid bounded by a closed surface every point of which is equidistant from a fixed point called the center

spherical angle the opening between two great circle arcs. A spherical angle is measured by the plane angle formed by the tangents to the arcs at their point of intersection

spherical excess the sum of the angles of a polygon over (n-2) 180° with n as the number of sides of the polygon

spherical polygon the portion of a spherical surface bounded by three or more great circle arcs

spherical pyramid the portion of a sphere bounded by a spherical polygon and the plane of the sides

spherical sector a solid generated by rotating a sector of a circle about an axis which passes through the center of the circle but which contains no point inside the sector

spherical segment a solid bounded by a zone and the planes of the zone's bases

spherical triangle a triangle extracted from the surface of a sphere.

spherical wedge the portion bounded by a lune and the planes of two great circles

spheroid another term for ellipsoid.

spherometer an instrument for measuring the curvature of a surface.

square a rectangular all of whose sides are congruent

square matrix (syn. determinant) a matrix with the same number of rows columns

square numbers (see figurate numbers)

square pyramidal numbers (see figurate numbers)

standard deviation a quantitative measure defining the extent to which scores are dispersed throughout in relation to the arithmetic mean. This is also equal to the square root of the variance.

statically indeterminate the condition exists in structures where the reactive forces or the internal resisting forces over the estimated life of the asset in terms of the periods or units of output

steradian *(sr)* a unit of measure of solid angle. Then maximum value for a solid angle is a full sphere which is equal to 4π steradians.

strain the change of relative positions of points due to stress. Unit strain is equal to the ratio of the deformation to the total length.

stress a force per unit area

subset a set that contains some of the elements of a given set

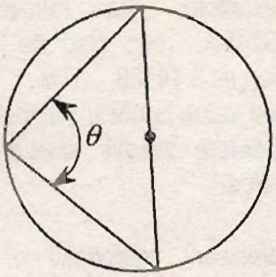
subtrahend the number to be subtracted. Example: 7 - 5 = 2, 5 is the subtrahend.

sum the result of addition.

sum-of-years' digits method (syn. SYD method) a method of computing depreciation in which the amount for any

year is based on the ratio: (years of remaining life/(1 + 2 + 3 + ..+ n), with n being the total anticipated life of the equipment

supplementary chords two chords which join a point on a circle to the end points of a diameter.



Angle θ is the supplemental angle

supplementary angle two angle whose sum is equal to 180°

surd an irrational number which is a root of a positive integers or fraction or it is a radical expressing an irrational number.

Types of surds:	Example
Quadratic	$\sqrt{2}$
Cubic	$\sqrt[3]{2}$
Quartic	$\sqrt[4]{2}$

The type of surd is named after the index of the radical.

Pure surd is a surd that contains no rational number (i.e. all its factors or terms are surds). Example: $\sqrt{2}$, $\sqrt{3} + \sqrt{2}$

Mixed surd is a surd that contains at least one rational term. Example: $2 + \sqrt{3}$

T

table compilation of values such as trigonometric table, logarithmic table, etc.

tangent a line (in the same plane) which intersect the curve in one and only one point. In trigonometry, it is the ratio of the side opposite to side adjacent in a right triangle.

tangent plane of a sphere a plane which intersects the sphere in exactly one point

tension force in longitudinal direction.

terminal speed the final speed v_t attained by the falling body. The principle is that when a body first start to move, v = 0, the resisting force is zero and the initial acceleration is a = g. As it speed increases, the resisting force also increase until finally it equals the weight in magnitude. At this time, the acceleration becomes zero and there is no more increase in its speed.

ternary a number system using a place value notation with 3 as the base.

tetrahedral numbers (see figurate numbers)

theorem a statement of truth of which must be established by proof

time value of money the cumulative effect of elapsed time on the money value of an event, based on the earning power of equivalent invested funds capital should or will earn

ton a mass of 1000 kilograms.

torque *(syn. moment of force)* a force times a moment arm.

torr a unit of pressure which is equivalent to mm of mercury (Hg). 1 torr = 1 mm of Hg. 1 torr is equivalent to 133 Pascals.

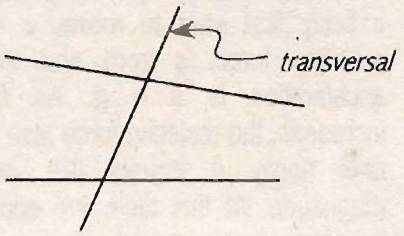
torus (syn. anchor ring or doughnut) a solid formed by revolving a circle about a line not intersecting it

trajectory the path followed by a projectile. It is a graph of a parabola.

translation a parallel displacement of the original system along one or more of its axes.

transpose to transfer to the other side of the equation. When a term is transpose, the sign must be changed.

transversal the intersecting line of two parallel or non parallel lines



transverse axis the axis of the hyperbola which passes through the foci, vertices and center

trapezium (syn. trapezoid) commonly used term in United Kingdom rather than trapezoid. In United States of America, the term trapezoid is used.

trapezoid a quadrilateral in which one and only one pair of opposite sides are parallel. The parallel sides of the trapezoids are called bases.

triangular numbers (see figurate numbers)

triangular region is the union of a triangle and its interior

trigonometry branch of mathematics which deals with triangles and trigonometric functions.

trinomial having three terms.

trivial considering the values of all the variables as zero.

truncated prism the portion of a prism included between the base and a plane not parallel to the base cutting all the edges or elements

truncated value the value of number when written with the further digits have been suppressed and replaced with three dots. For example, the number π = 3.141592653... can also be written by truncation as 3.14159... The truncated value is not a rounded off value and therefore always smaller than the exact value.

truss a framework composed of members joined at their ends to form a rigid structure.

twin primes prime numbers that appear in pair and differ by 2. Examples are 3 and 5, 11 and 13,...

U

undecagon polygon of eleven sides.

union a set consisting of all elements that appear at least once in the original set.

union of two sets a set of all objects that belong to one or both sets

unit circle a circle of radius one unit and is used to determine the sign of all trigonometric functions in all quadrants

unit vector a vector having a magnitude of unity, with no units and is used only to described a direction in space

unity referring to one.

universe (statistics, see population)

V

variable an expression than is assigned a certain set of values.

variance a measure of the dispersion of scores in a distribution away from the arithmetic mean. The mean of the squared deviations about the mean.

Varignon's Theorem The moment of the resultant or two concurrent forces with respect to the center in their planes is equal to the algebraic sum of the moments of the components with respect to the same center. Named after the French mathematician, Pierre Varignon (1654-1722).

vector quantity a physical quantity the described a *magnitude* ("how much" or "how big") and the *direction* in space

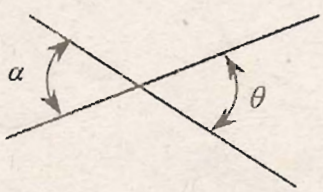
vector product of A and B is denoted as A x B. Because of this notation, vector product is also called the cross product.

velocity rate of change of displacement.

Venn diagram a pictorial description of the probability concepts of independent and dependent events. This was named after English logician, John Venn (1834 – 1923).

vertex point of intersection of two sides of a polygon.

vertical angles angles that are opposite to each other and formed by two intersecting straight lines. Vertical angles are equal.



Vertical angles ($\alpha = \theta$)

vigesimal pertaining to the number 20.

vigesimal number system a number system using the base 20

volume space occupied by a solid. Volume is expressed in cubic units.

W

watt the SI unit of power. This is equivalent to joules per second.

weight the force of the earth's gravitational attraction for the body. It is a downward force acting at the centroid or center of gravity of the body.

Weight = mass x gravitational acceleration

whole number another term for natural number

word problems (worded problems) real problems that are usually given orally or written in words

work the force time a distance.

X

x-axis the horizontal axis of the rectangular coordinate system.

x-intercept the value of the abscissa of the point where the curve crosses the x-axis

Y

y-axis the vertical axis of the rectangular coordinates system.

y-intercept the value of the ordinate of the point where the curve crosses the y-axis

Young's modulus (see modulus of elasticity)

Z

zenith a point in the celestial sphere directly above the observer.

zero (syn. cipher) void, emptiness or nothing. Zero is derived from Hindu word "*sunya*"

zone a portion of the surface of a sphere included between two parallel planes

Appendix B: Conversion

Unit	SI	English
1 Acre	0.40468 hectares	43,560 sq.feet
1 Are	100 sq. meters	119.599 sq. yds.
1 Barrel, Petroleum	158.98729 liters	42 gallons
1 Barrel, Proof Spirit, US	151.41647 liters	40 gallons
1 Barrel, beer	117.34777 liters	31 gallons
1 Bushel	35.23907 liters	4 pecks
1 Cable	219.456 meters	120 fathoms
1 Chain, Surveyor's	20.1168 meters	66 feet
1 Cord (wood)	3.6246 cu. Meters	128 cu. Feet
1 Cup	0.23659 liters	8 ounces
1 Dram, advp	1.77184 grams	0.06255 ounces
1 Dram, troy	3.88793 grams	0.125 ounces
1 Dram, liquid	3.69669 ml	0.125 ounces
1 Fathom	1.8288 meters	6 feet
1 Foot	30.48 cm	12 inches
1 Foot^2	929.0302 cm^2	144 inches^2
1 Foot^3	28.31684 liters	7.48052 gallons
1 Furlong	201,168 meters	220 yards
1 Gallon, US	3.78541 liters	4 quarts
1 Gill	118.29412 ml	4 ounces
1 Grain	64.79891 mg	0.00229 ounces
1 Gram	1,000 milligrams	0.035274 ounces
1 Hand	10.16 centimeters	4 inches
1 Hectare	10,000 sq. meters	2.471054 acres
1 Hundredweight (long)	50.80234 kg	112 pounds
1 Hundredweight (short)	45.35923 kg	100 pounds
1 Inch	2.54 centimeters	0.08333 foot
1 Kilogram	0.001 metric ton	2.204623 pounds
1 Kilometer	1,000 meters	0.621 statute mi
1 Kilometer^2	100 hectares = 1,000,000 sq. m	247.105 acres 0.386 statute mi
1 Knot (Nautical mile per hour)	1.852 kilometers per hour	1.151 statute mile per hour
1 League, nautical	5.559552 km	3 nautical miles
1 League, statute	4.828032 km	3 statute miles
1 Link, surveyor's	20.1168 cm	7.92 inches
1 Liter	0.001 cu. Meter	61.02374 inches^3
1 meter	100 centimeters	1.093613 yards
1 meter^2	10,000 cm^2	1.19599 yards^2
1 meter^3	1,000 liters	1.307951 yards^3
1 micron	0.000001 meter	0.0000394 inch
1 mil	0.0254 mm	0.001 inch

1 mile, nautical	1.852 kilometers	6,080 feet
1 mile, statute	1.609 kilometers	5,280 feet
1 Minim	0.06161 ml	0.002083 ounces
1 Ounce, advp	28.349523 grams	437.5 grains
1 Ounce, liquid	29.57353 ml	0.0625 pints
1 Ounce, troy	31.103477 grams	480 grains
1 Pace	76.2 centimeters	30 inches
1 Peck	8.8097675 liters	8 quarts
1 Pennyweight	1.5551738 grams	24 grains
1 Pint, dry (US)	0.55061947 liters	0.5 quarts, dry
1 Pint, liquid	0.47317647 liters	0.5 quarts, liquid
1 Point (typographical)	0.3514598 mm	0.13837 inch
1 Pound, advp	453.59237 grams	16 ounces, advp
1 Pound, troy	373.24172 grams	12 ounces, troy
1 Quart, dry	1.101221 liters	2 pints, dry
1 Quart, liquid	0.946353 liters	2 pints, liquid
1 Quintal	100 kilograms	220.4623 pounds
1 Rod	5.0292 meters	5.5 yards
1 Scruple	1.29598 grams	20 grains
1 Section (US)	2.5899881 km^2	1 statute $mile^2$
1 Span	22.86 centimeters	9 inches
1 Stere	1 cubic meter	1.30795 cu. Yards
1 Tablespoon	14.78676 ml	3 teaspoon
1 Teaspoon	4.928922 ml	0.3333 tablespoon
1 Ton, metric	1,000 kilograms	2,204.623 pounds
1 Ton, register	2.831685 $meter^3$	100 $feet^3$
1 Ton, short	907.185 kilograms	2,000 pounds
1 Township (US)	93.239572 km^2	36 statute $mile^2$
1 Yard	0.9144 meters	3 feet

Appendix C: Physical constants

Name	Symbol	Value
Absolute zero	0° K	-237.15° C
Acceleration due to gravity	g	9.80665 m/s^2
Atmospheric pressure, standard	p	760 mm of mercury = 14.7 psi
Atomic mass unit	1 u	1.66054×10^{-27} kg
Avogadro's number	No	6.02486×10^{26} / kg-mol
Boltzmann's constant	k	1.381×10^{-23} J/°K
Charge of electron	e	1.60218×10^{-19} C
Charge-to-mass ratio of electron	e/m	1.75890×10^{11} C/kg
Density of water (maximum)	ρ	0.99997×10^{3} kg/ m^3
Density of mercury (maximum)		13.5950×10^{3} kg/ m^3
Electron volt	1 eV	1.602×10^{-19} J
Electron rest energy	mc^2	0.511 MeV
Energy equivalent to 1 u	Mc^2	931.494 MeV
Faraday's constant	F	9.65219×10^{7} C/kg-mol
Gas constant	R	8.31451 J/mol-°K
Gravitational const.	G	6.67259×10^{-11} N-m^2/kg^2
Mass of electron	m_e	9.10939×10^{-31} kg
Mass of neutron	m_n	1.67262×10^{-27} kg
Mass of proton	m_p	1.67492×10^{-27} kg
Mechanical equivalent of heat		4.186 J/cal (15° calorie)
Permeability of free space	μ_o	$4\pi \times 10^{-7}$ Wb/A·m
Permittivity of free space	ε_o	8.854×10^{-12} C^2/N-m^2
Planck'c constant	h	6.62608×10^{-34} J-s
Speed of light	c	2.99792×10^{8} m/s
Volume of Ideal gas (0°C, 1 atm)	V	22.4207 m^3/kg-mol

Appendix D: Power of 10

Prefix	Symbol	Power of 10
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
teta	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
hectokilo	hk	10^{5}
myria	ma	10^{4}
kilo	k	10^{3}
hecto	h	10^{2}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
decimilli	dm	10^{-4}
centimilli	cm	10^{-5}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

Appendix E: Numeration

Base	System
2	binary or dyadic
3	ternary
4	quaternary
5	quintary
6	senary
7	septenary
8	octal or octenary
9	nonary
10	decimal or decadic or denary
11	undenary
12	duodecimal or duodenary
16	hexadecimal or hexadecadic
20	vigesimal or vicenary
60	sexagesimal

Appendix F: Math Notation

Prefix of 10		Name
10^{18}	1,000,000,000,000,000,000	1 quintillion
10^{15}	1,000,000,000,000,000	1 quadrillion
10^{12}	1,000,000,000,000	1 trillion
10^{9}	1,000,000,000	1 billion
10^{6}	1,000,000	1 million
10^{3}	1,000	1 thousand
10^{2}	100	1 hundred
10^{1}	10	ten
10^{0}	1	one
10^{-1}	0.1	1 tenth
10^{-2}	0.01	1 hundredth
10^{-3}	0.001	1 thousandth
10^{-6}	0.000001	1 millionth
10^{-9}	0.000000001	1 billionth
10^{-12}	0.000000000001	1 trillionth
10^{-15}	0.000000000000001	1quadrillionth
10^{-18}	0.000000000000000001	1quintillionth

Appendix G: Greek Alphabets

Alphabeth	Symbol	
Alpha	A	α
Beta	B	β
Gamma	Γ	γ
Delta	Δ	δ
Epsilon	E	ε
Zeta	Z	ζ
Eta	H	η
Theta	Θ	θ
Iota	I	ι
Kappa	K	κ
Lambda	Λ	λ
Mu	M	μ
Nu	N	ρ
Xi	Ξ	ξ
Omicron	O	o
Pi	Π	π
Rho	P	ρ
Sigma	Σ	σ
Tau	T	τ
Upsilon	Y	ν
Phi	Φ	φ
Chi	X	χ
Psi	Ψ	ψ
Omega	Ω	ω

Appendix H: Divisibility Rules



An integer is evenly divisible by

2 - if it is even or it ends with an even number

3 - if the sum of its digits is divisible by 3

4 - if the number formed by the last two digits is divisible by 4

5 - if it ends with either 0 or 5

6 - if it is divisible by 2 and 3

7 - if the number formed after cancellation of the units digit and subtraction of twice the value of the units digit is divisible by 7

8 - if the number formed by the last three digits is divisible by 8

9 - if the sum of its digit is divisible by 9

10 - if it ends with zero

11 - if the difference between the cross sums of alternate digits is divisible by 11

12 - if it divisible by 3 and 4